PRAYAS

JEE 2025

Lecture-02 | Mathematics

Relation & Functions

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Topics *to be covered*

1. Types of Relations
2. Practice Problems

Recap *of previous lecture*

1. If $n(A) = p$ & $n(B) = q$ then $n(A \times B) =$ pq, $n(B \times A) =$ pq
   also if $n(A \cap B) = r$ then $n((A \times B) \cap (B \times A)) =$ r^2

2. $A \times (B \cup C) =$ $(A \times B) \cup (A \times C)$, $A \times (B \cap C) =$ $(A \times B) \cap (A \times C)$, $A \times (B - C) =$ $(A \times B) - (A \times C)$

3. If $n(A \cap C) = 3$, $n(B \cap D) = 4$ then $n((A \times B) \cap (C \times D)) =$ $12.$

   $n(A \cap C) \cdot n(B \cap D)$

Recap of previous lecture

4. $A \times B$ is not equal to $B \times A$ in general.

5. $R \times R \times R =$ R^3 & it denotes the entire 3D space

6. $R \times R =$ R^2 & it denotes the entire 2D space.

QUESTION

If P, Q and R subsets of a set A then $R \times (P' \cup Q')' =$ $R \times ((P')' \cap (Q')')$

$= R \times (P \cap Q)$

$= (R \times P) \cap (R \times Q)$

A. $(R \times P) \cap (R \times Q)$

B. $(R \times Q) \cap (R \times P)$

C. $(R \times P) \cup (R \times Q)$

D. none of these

Relation: Any subset of $A \times B$ is said to be a relation from A to B.

★ If $n(A) = n$, $n(B) = m \Rightarrow n(A \times B) = nm \Rightarrow$ No: of Relation from A to B $= 2^{mn}$

★ If $n(A) = n$, $n(B) = m \Rightarrow$ no: of non empty relations from A to B $= 2^{mn} - 1$.

Ex: $A = \{1, 2, 3\}$, $B = \{2, 4\}$ $\Rightarrow A \times B = \{(1,2)\ (1,4)\ (2,2)\ (2,4)\ (3,2), (3,4)\}$

$\Downarrow$

No: of Relations from A to B $= 2^6$

$R_3 = \phi$ (void Relation)

$R_1 = \{(1,2), (1,4), (2,4)\}$

$R_2 = \{(3,2)\ (1,4)\}$

$(1,2) \in R_1 \Leftrightarrow 1 R_1 2$

$(3,2) \in R_2 \Leftrightarrow 3 R_2 2$

$(a,b) \in R \iff aRb$

'b' is called image of 'a' under R

'a' is called preimage of 'b' under R

Domain of a Relation : set of all 1st coord in ordered pairs in R is said to be Domain of R

$R_1 = \{(1,2), (1,4) (2,4)\}$ Domain $= \{1,2\}$.

$R_2 = \{(3,2), (1,4)\}$ Domain: $\{3,1\}$

Range of Relation : set of all 2nd coordinates in ordered pairs in R is said to be Range of R.

Range of $R_1 = \{2,4\}$

Range of $R_2 = \{2,4\}$.

if R is a relation from A to B then B is called codomain of R

Ex: $A = \{1,2,3,4,5,6\}$.

$B = \{3,4,9,10\}$

$R_1 : A \to B$ — codomain

Ex: $R_1 = \{(1,3)(1,9)(3,9)(4,10)\}$

- Domain $= \{1,3,4\}$
- Range $= \{3,9,10\}$
- Image of 1 $= 3, 9$
- preimage of 9 $= 1, 3$.
- $4 R_1 10$

Any subset of $A \times A$ is called a Relation from A to A or it is said to be a Relation on A

$R : A \to A$

If $n(A) = n \Rightarrow$ No: of Relations on A $= 2^{n^2}$

Ex: $A = \{1,2,3\}$

$R_1 = \{(1,1)(1,2)\}$

- Domain $= \{1\}$
- Range $= \{1,2\}$
- Image of $1 = 1, 2$
- preimage of $2 = 1$

codomain $= A$

$R_2 = \{(2,2)(1,1)(3,3)\}$

Relation

Every subset of $A \times B$ defined a relation from set A to set B. If R is relation from $A \to B$

NOTE :

If $(a\ b) \in R$ then

(i) 'b' is called image of 'a' under R.

(ii) 'a' is called pre-image of 'b'

(iii) $(a\ b) \in R \Leftrightarrow a\,R\,b$

Relation

Domain of Relation

Set of all first entries of all ordered pairs that occur in R

Range of Relation

Set of all second entries in R.

Relation

NOTE :

If $n(A) = p$ $n(B) = q$ then

(i) number of relations from A to B = 2^{pq}

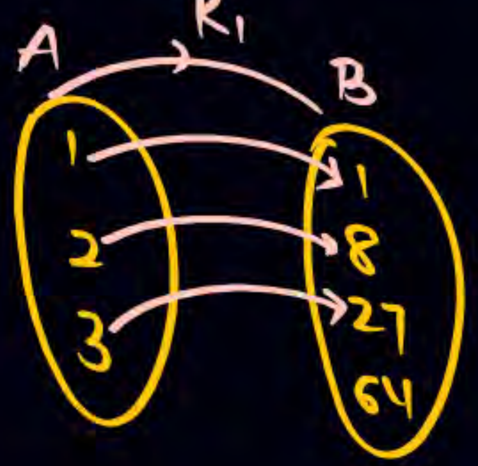
(ii) number of non empty relations from A to B = $2^{pq} - 1$

Representation of a Relation

$A = \{1, 2, 3\}$ $B = \{1, 8, 27, 64\}$

$R_1 = \{(1,1), (2,8), (3,27)\}$ → All elements are listed separated by commas ⟹ Roster form.

$R_1 = \{(a, b) \mid b = a^3, a \in A, b \in B\}$. → Property satisfied by all elements only is mentioned ⟹ set Builder form.



Arrow Diagram

Representation of a Relation

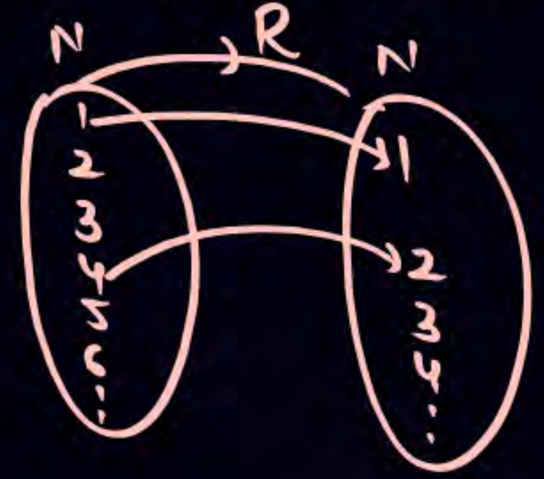
- Roster Form
- Set Builder Form
- Arrow Diagram

Ex: $a R b \iff a = b^2 \quad a, b \in N.$

① R is Relation from $\underline{N}$ to $\underline{N}$

② $R = \{(a, b) \mid a = b^2, a, b \in N\}$ — Set Buider form

$R = \{(1,1), (4,2), (9,3), (16,4) \ldots\}$ — Roster form.



(Arrow Diagram)

QUESTION

A relation R is defined on a set $A = \{1, 2, 3, 4, 5\}$ defined by

$R = \{(x, y) : |x^2 - y^2| < 20$ then find :

R is a subset of $A \times A$

(a) R

(b) Domain of R $= \{1, 2, 3, 4, 5\}$

(c) Range of R $= \{1, 2, 3, 4, 5\}$.

$R = \{(1,1)\ (1,2)\ (2,1)\ (1,3)\ (3,1)\ (1,4)\ (4,1)$
$(2,2)\ (2,3)\ (3,2)\ (2,4)\ (4,2)$
$(3,3)\ (3,4)\ (4,3)\ (3,5)\ (5,3)$
$(4,4)\ (4,5)\ (5,4)$
$(5,5)\}$

QUESTION

Tan 1

If $R = \{(x, y) \mid x^2 + y^2 \leq 4 \mid$ where $x, y \in Z\}$ is a relation on Z then

(A) Domain of R is $\{0, 1, 2\}$

(B) Domain of R is $\{-2, -1, 0, 1, 2\}$

(C) Domain of R = range of R

(D) $n(R) = 13$

QUESTION

Let $X = \{1\ 2\ 3\ 4\}$ and $Y = \{1\ 3\ 5\ 7\ 9\}$. Which of the following is relation from X to Y -

$R_1 = \{(1,3)\ (3,5)\}$

(A) $R_1 = \{(x, y) \mid y = 2 + x, x \in X, y \in Y\}$

(B) $R_2 = \{(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)\}$ Not a Relation $X \longrightarrow Y$ b'coz $R_2 \not\subseteq X \times Y$.

(C) $R_3 = \{(1, 1), (1, 3), (3, 5), (3, 7), (5, 7)\}$ $R_3 \not\subseteq X \times Y$

(D) $R_4 = \{(1, 3), (2, 5), (2, 4), (7, 9)\}$ $R_4 \not\subseteq X \times Y$

QUESTION [JEE Mains 2024 (1 Feb)]

Let A = {1, 2, 3, ..., 20}. Let R_1 and R_2 two relation on A such that

$R_1 = \{(a, b) : b \text{ is divisible by } a\}$

$R_2 = \{(a, b) : a \text{ is an integral multiple of } b\}$.

a is divisible by b.

Then, number of elements in $R_1 - R_2$ is equal to

$R_1 = \{(1,1)\ (1,2)\ (1,3)\ ---\ (1,20) \longrightarrow$ 20 elements

$(2,2)\ (2,4)\ (2,6)\ ---\ (2,20) \longrightarrow$ 10 elements.

$(3,3)\ (3,6)\ (3,9)\ ----\ (3,18) \longrightarrow$ 6 elements.

$(4,4)\ (4,8)\ ------\ (4,20) \longrightarrow$ 5 elements.

$(5,5)\ (5,10)\ (5,15)\ (5,20) \longrightarrow$ 4 elements

$(6,6)\ (6,12)\ (6,18) \longrightarrow$ 3 elements.

$(7,7)\ (7,14) \longrightarrow$ 2 elements

$(8,8)\ (8,16) \longrightarrow$ 2 elements

$(9,9)\ (9,18) \longrightarrow$ 2 elements

$(10,10)\ (10,20) \longrightarrow$ 2 elements

$\underbrace{(11,11)\ (12,12)\ --\ (20,20)}_{10}$ 10 elements

$n(R_1) = 66$

If $(x,y) \in R_1$ then $(y,x) \in R_2 \Rightarrow n(R_2) = 66$.

Ans. 46

$$R_1 - R_2 = R_1 - (R_1 \cap R_2)$$

$$(R_1 \cap R_2) = \{(1,1)(2,2)(3,3) \cdots (20,20)\}$$

$$n(R_1 \cap R_2) = 20$$

$$n(R_1 - R_2) = n(R_1) - n(R_1 \cap R_2)$$

$$= 66 - 20 = 46 \text{ Ans}$$

QUESTION [JEE Mains 2022]

Let R be a relation from the set {1, 2, 3, ..., 60} to itself such that R = {(a, b) : b = pq, where p, q ≥ 3 are prime numbers}. Then the number of elements in R is :

(A) 600

(B) 660

(C) 540

(D) 720

$b = 3\times3,\ 5\times5,\ 7\times7$

$3\times5,\ 3\times7,\ 3\times11,\ 3\times13,\ 3\times17,\ 3\times19$

$5\times7,\ 5\times11$

11 options for b.

(_ , _)

60 ways   11 ways $= 60\times11 = 660$ Ans.

QUESTION [JEE Mains 2023 (6 April)]

Tan2

Let $A = \{1, 2, 3, 4, \ldots, 10\}$ and $B = \{0, 1, 2, 3, 4\}$. The number of elements in the relation $R = \{(a, b) \in A \times B : 2(a-b)^2 + 3(a-b) \in B\}$ is ______

Ans. 18

★ If R is a Relation on $A \Rightarrow R \subseteq A \times A$ — $(a,b) \in R$

★ If R is a Relation from A to $B \Rightarrow R \subseteq A \times B$ — $(a,b) \in R$

★ If R is a Relation on $A \times B \Rightarrow R \subseteq (A \times B) \times (A \times B)$

$$\big((a,b),(c,d)\big) \in R$$

★ If R is a relation on $A \times A \Rightarrow R \subseteq (A \times A) \times (A \times A)$

Ex: $A = \{1,2\}$

$R = \{(1,2)\,(2,1),(2,2)\}$

$A \times A = \{(1,1)\,(1,2)\,(2,1)\,(2,2)\}$

16 elements

$(A \times A) \times (A \times A) = \{((1,1),(1,1))\,((1,1),(1,2))\,((1,1),(2,1))\,((1,1),(2,2))$
$((1,2),(1,1))\,((1,2),(1,2))\,((1,2),(2,1))\,((1,2),(2,2))$ - - - - - - - - $\}$

QUESTION [JEE Mains 2024 (9 April)]

$2+6+3+2+8+4=25$ elements

Let A = {2, 3, 6, 7} and B = {4, 5, 6, 8). Let R be a relation defined on A × B by $(a_1, b_1)\ R\ (a_2, b_2)$ if and only if $a_1 + a_2 = b_1 + b_2$. Then the number of elements in R is

$((a_1, b_1), (a_2, b_2)) \in R$ if $a_1 + a_2 = b_1 + b_2$.

$2+2=4$
$2+3=5=3+2$
$2+6=8=6+2$
$2+7=9=7+2$
$3+3=6$
$3+6=9=6+3$
$3+7=10=7+3$
$6+7=13=7+6$
$7+7=14$
$6+6=12$

$4+4=8$
$4+5=9=5+4$
$4+6=10=6+4$
$4+8=12=8+4$
$5+5=10$
$5+6=11=6+5$
$5+8=13=8+5$
$6+6=12$
$6+8=14=8+6$
$8+8=16$

$((2, \underline{4}), (6, \underline{4}))$

	a_1+a_2	b_1+b_2
$8=$	$2+6$, $6+2$	$= 4+4$

2×1

$((7, \_), (3, \_))$

$10 = 7+3$, $3+7$; $4+6$, $6+4$, $5+5$

2×3 = 6 elements.

$12 = 6+6$; $6+6$, $4+8$, $8+4$

1×3 = 3 elements.

$14 = 7+7$; $6+8$, $8+6$

1×2 = 2 elements.

8 elements. 4×2

$9 = 2+7 = 4+5$
$7+2$; $5+4$
$3+6$
$6+3$

	a_1+a_2	b_1+b_2
$13=$	$6+7$, $7+6$	$8+5$, $5+8$

4 elements $= 2\times2$.

Ans. 25

Inverse of a Relation

$A = \{1, 2, 3\}$, $B = \{2, 4, 5\}$

$R_1 : A \longrightarrow B$ $\quad R_1 = \{(1,2)(2,4)(2,5)(3,2)\} \longrightarrow R_1^{-1} = \{(2,1)(4,2),(5,2),(2,3)\}$

$R_2 : A \longrightarrow B$ $\quad R_2 = \{(1,5)(2,4)(3,5)\} \longrightarrow R_2^{-1} = \{(5,1)(4,2)(5,3)\}$.

if $(a,b) \in R$ then $(b,a) \in R^{-1}$

Domain of $R_1 = \{1, 2, 3\}$ = Range of R_1^{-1}

Range of $R_1 = \{2, 4, 5\}$ = Domain of R_1^{-1}

Domain of R = Range of R^{-1}

Range of R = Domain of R^{-1}

Inverse of a Relation

Let A, B be two sets and let R be a relation from a set A to a set B. Then the inverse of R denoted by R^{-1} is a relation from B to A and is defined by $R^{-1} = \{(b\ a) : (a\ b) \in R\}$

Clearly $(a\ b) \in R \Leftrightarrow (b\ a) \in R^{-1}$.

Range $(R) = \mathrm{Dom}(R^{-1})$.

Domain of $R =$ Range of R^{-1}

QUESTION

Tah3

The relation R defined in $A = \{1, 2, 3\}$ by a R b if $|a^2 - b^2| \leq 5$. Which of the following is false?

(A) $R = \{(1, 1), (2, 2), (3, 3), (2, 1), (1, 2), (2, 3), (3, 2)\}$

(B) $R^{-1} = R$

(C) Domain of $R = \{1, 2, 3\}$

(D) Range of $R = \{5\}$

Types of Relation

Let R be relation on A i.e. $R : A \to A$ then

- **Identity Relation :**
  A relation defined on a set A is said to be an identity relation if each & every element of A is related to itself & only to itself.

- **Reflexive :**
  A relation defined on a set A is said to be reflexive relation if each & every element of A is related to itself.

Let R be a Relation on $A \Rightarrow R \subseteq A \times A$ then R is said to be

1) Identity Relation: If each & every element of A is related to itself and only to itself by R.

$A = \{1, 2, 3\}$

$R = \{(1,1)\ (2,2)\ (3,3)\} \longrightarrow$ identity Relation.

$R = \{(1,1)\ (2,2)\} \longrightarrow$ Not an identity Relation. b'coz $(3,3)$ is missing

$R = \{(1,1), (2,2), (3,3), (1,3)\}$ Not an identity Relation $\longrightarrow$ b'coz $(1,3) \in R$

2) Reflexive Relation : If each & every element of A is related to itself by R

(a, a), a ∈ A type ke saaray elements R mai honay chahiyay phir kuch aur elements hoo yaa nahoo

Ex: $A = \{1, 2, 3\}$

$R = \{(1,1), (2,2), (3,3)\}$ → Reflexive = Identity

$R = \{(1,1) (2,2) (1,3)\}$ — Not Reflexive → (3,3) is missing

$R = \{(1,1) (2,2) (3,3) (2,3)\}$ — Reflexive

$R = \{(1,1) (2,2) (3,2)\}$ — Not Reflexive.

Every identity relation is Reflexive but not the converse.

Identity relation on a set is unique.

③ Symmetric Relation : If $(a,b) \in R$ then (b,a) also lies in R

$A = \{1, 2, 3\}$

$R_1 = \{(1,1), (1,2), (1,3), (3,1)\}$ → Not symmt. $(1,2) \in R$ but $(2,1) \notin R$

$R_2 = \{(1,1)\ (1,2)\ (2,1)\ (3,3)\}$. → symmt.

$R_3 = \{(1,3)\ (1,2)\ (2,1)\ (3,1)\ (2,2)\}$ → symmt.

(4) Transitive Relation : If (a,b) & $(b,c) \in R$ then (a,c) should also lie in R.

$A = \{1, 2, 3\}$.

$R = \{(1,1)\ (1,2)\ (2,3)\ (1,3)\}$ → (Transitive)

$R = \{(1,1)\ (2,3)\}$ → (Transitive)

Empty Relation on a Set A.
- Reflexive ✗
- Symmt. ✓
- Transitive ✓

Agar (a,b) & (b,c) belong to R then (a,c) should also belong to R.

$(a,a)\ (a,b)$ check karne ki zaroorat nahi

(5) Antisymmetric Relation : If (a,b) & $(b,a) \in R$ then $a=b$.

$A = \{1,2,3\}$

$R = \{(1,2)(1,3)(3,3)\}$ → Antisymmt

$R = \{(1,1)(2,2)(3,3)\}$ → symmt ✓✓, Antisymmt ✓✓

$R = \{(1,1)(2,3)(3,2),(3,3)\}$ → symmt ✓, Antisymmt ✗

Kisi bhi element kaa reverse R mai nahi honaa chahiyay except for elements of type (a,a)

★ If a relation is symmetric then it can not be antisymmt (False)

★ If a relation is antisymmt. then it can not be symmt. False

(6) Equivalence Relation : If it is Reflexive, Symmt as well as transitive

Types of Relation

➢ **Symmetric :**

A relation defined on a set is said to be symmetric if a R b ⇒ b R a.

If (a b) ∈ R then (b a) must be necessarily there in the same relation.

Types of Relation

➢ **Antisymmetric Relation :**

A relation on a set A is said to be antisymmetric if $(a\ \ b) \in R$ & $(b\ \ a) \in R$ then $a = b$

➢ **Equivalence Relation :**

If a relation is Reflexive symmetric and transitive then it is said to be an equivalence relation.

QUESTION

Let L denote the set of all straight lines in a plane. Let a relation R be defined $\alpha R \beta \Leftrightarrow \alpha \perp \beta, \alpha \beta \in L$. Then R is

$L = \{ L_\alpha, L_\beta, L_\gamma \ldots \}$

$R : L \to L$ s.t $L_\alpha R L_\beta \iff L_\alpha \perp L_\beta$

Reflexive Relation ✗

since $(L_\alpha, L_\alpha) \notin R$

b'coz no line is ⊥ar to itself

symmt ✓

If $(L_\alpha, L_\beta) \in R \Rightarrow L_\alpha \perp L_\beta$

$\Downarrow$

$(L_\beta, L_\alpha) \Leftarrow L_\beta \perp L_\alpha$

also lies in R.

Transitive.

(L_1, L_2) & $(L_2, L_3) \in R$

$L_1 \perp L_2, L_2 \perp L_3 \Rightarrow L_1 \parallel L_3 \Rightarrow (L_1, L_3) \notin R$

Not transitive

- To say some statement is true we have to prove it
- To say some statement is false we just need one counter example to say it is false

QUESTION

Tah

Relation R in the set of A of human beings in a town at a particular time given by

(A) R = {(x y) : x and y work at the same place}

Reflex ✓
symm ✓
Transt ✓

(B) R = {(x y) : x and y live in the same locality}

Ref ✓
symm ✓
Trans ✓

same locality

$(x, y), (y, z)$

(x, z)

(C) R = {(x y) : x is exactly 7 cm taller than y}

(D) R = {(x y) : x is wife of y}

(E) R = {(x y) : x is father of y}

QUESTION [AIEEE 2006]

Let W denote the words in the English dictionary. Define the relation R by:
$R = \{(x,y) \in W \times W \mid$ the words x and y have at least one letter in common$\}$.
Then R is

(A) reflexive symmetric and not transitive

(B) reflexive symmetric and transitive

(C) reflexive not symmetric and transitive

(D) not reflexive symmetric and transitive

$(x, y) \in R$ if x & y have atleast one alphabet common

word

Reflexive ✓ $(x, x) \in R \;\forall\; x \in W$

symmt ✓ if $(x, y) \in R \Rightarrow x$ & y have atleast one common alphabet
$\Downarrow$
$(y, x) \in R$

✗ Transitive:
(pen, egg), (egg, gut) $\in R$
but (pen, gut) $\notin R$

QUESTION [JEE Mains 2022 (28 June)]

Let $R_1 = \{(a\,b) \in N \times N : |a-b| \le 13\}$ and $R_2 = \{(a\,b) \in N \times N : |a-b| \ne 13\}$. Then on N :

(A) ~~Both R_1 and R_2 are equivalence relations~~

(B) ~~Neither R_1 nor R_2 is an equivalence relation~~

(C) ~~R_1 is an equivalence relation but R_2 is not~~

(D) R_2 is an equivalence relation but R_1 is not

$R_1 : N \to N$

R_1:
- Reflexive ✓ $\because (a,a) \in R$ since $|a-a| \le 13, \forall a \in N$
- Symmt ✓ If $(a,b) \in R \Rightarrow |a-b| \le 13 \Downarrow (b,a) \in R \Leftarrow |b-a| \le 13$
- Transitivity $(1,7),(7,15) \in R$ But $(1,15) \notin R$ b'coz $|1-15| = 14 \nleq 13$

R_2:
- Reflexive ✓ $\because |a-a| = 0 \ne 13 \Rightarrow (a,a) \in R_2 \; \forall a \in N$
- Symmt ✓ If $(a,b) \in R \Rightarrow |a-b| \ne 13 \Rightarrow |b-a| \ne 13 \Downarrow (b,a) \in R$
- ✗ Transitive : $(1,7),(7,14) \in R$ But $(1,14) \notin R$

Sabse Important Baat Yaad Rahe

Sabhi Class Illustrations Retry Karnay hai...

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with good Rank

Class illustrations

Module,DPP

KTK,TAH CHALLENGER

QUESTION

Paragraph

If $A_0 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ and $B_0 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

$B_n = \text{adj}(B_{n-1}), n \in N$ and I is an identity matrix of order 3 then answer the following questions.

det. $(A_0 + A_0^2 B_0^2 + A_0^3 + A_0^4 B_0^4 + \ldots\ldots 10 \text{ terms})$ is equal to

(A) 1000

(B) -800

(C) 0

(D) -8000

Ans. C

QUESTION

Paragraph

If $A_0 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ and $B_0 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

$B_n = \text{adj}(B_{n-1}), n \in N$ and I is an identity matrix of order 3 then answer the following questions.

$B_1 + B_2 + ... + B_{49}$ is equal to

(A) B_0

(B) $7B_0$

(C) $49B_0$

(D) $49I$

Ans. C

QUESTION

Paragraph

If $A_0 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ and $B_0 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

$B_n = \text{adj}(B_{n-1}), n \in N$ and I is an identity matrix of order 3 then answer the following questions.

For a variable matrix X the equation $A_0X = B_0$ will have

A unique solution

B infinite solution

C finitely many solution

D no solution

Ans. D

QUESTION

Consider a system of linear equation $3x + y - z = 0$, $x - \frac{py}{4} + z = 2$ and $2x - y + 2z = q$ where $p, q \in I$ and $p, q \in [1, 10]$, then identify the correct statement(s).

List-I		List-II	
(I)	Number of ordered pairs (p, q) for which system of equation has unique solution is	(P)	1
(II)	Number of ordered pairs (p, q) for which system of equation has no solution is	(Q)	9
(III)	Number of ordered pairs (p, q) for which system of equation has infinite solution is	(R)	91
(IV)	Number of ordered pairs (p, q) for which system of equation has atleast one solution is	(S)	90

QUESTION

Which one of the following option is correct?

(A) $I \rightarrow P, II \rightarrow R, III \rightarrow S, IV \rightarrow R$

(B) $I \rightarrow Q, II \rightarrow S, III \rightarrow P, IV \rightarrow R$

(C) $I \rightarrow S, II \rightarrow Q, III \rightarrow P, IV \rightarrow R$

(D) $I \rightarrow Q, II \rightarrow P, III \rightarrow S, IV \rightarrow P$

Ans. C

Homework from Module

Chapter: Matrices

Prarambh: COMPLETE

Prabal : Complete

(Revision Practice Problems)

QUESTION

RPP 1

Let $a, b, c, d \in R$; $a + b + c + d = 10$, the minimum value of $a^2 \cot 9^\circ + b^2 \cot 27^\circ + c^2 \cot 63^\circ + d^2 \cot 81^\circ$ is $\sqrt{n}$; $n \in N$, then 'n' is

(A) even

(B) odd

(C) prime

(D) divisible by 5

Ans. B, D

QUESTION

RPP 2

If the number of solutions of the equation $\cos^2\left(\frac{\pi}{4}(\cos x+\sin x)\right)-\tan^2\left(x+\frac{\pi}{4}\tan^2 x\right)=1$ in $[-2\pi, 2\pi]$ is 'k', then $\frac{3k}{25}$ equals

Ans. 0.48

QUESTION

RPP 3

Let $f_n(\theta) = \sum_{r=0}^{n} \frac{1}{4^r} \cdot \sin^4(2^r\theta)$, then

(A) $f_2\left(\frac{\pi}{4}\right) = \frac{\pi}{\sqrt{2}}$

(B) $f_3\left(\frac{\pi}{8}\right) = \frac{2+\sqrt{2}}{4}$

(C) $f_4\left(\frac{3\pi}{2}\right) = 1$

(D) $f_5(\pi) = 0$

Ans. C, D

Previous TAH

Solutions

QUESTION [JEE Mains 2022 (26 June)]

Let $A = \{n \in N : \text{H.C.F.}\ (n, 45) = 1\}$ and let $B = \{2k : k \in \{1, 2, \ldots\ldots, 100\}\}$.

Then the sum of all the elements of $A \cap B$ is ____

Ans. 5264

REDO BY BHAVISHYA KASHYAP

JM 2022

Q1 $A = \{n \in \mathbb{N} : HCF(n, 45) = 1\}$
$B = \{2k : k \in \{1, 2, \ldots, 100\}\}$

Then the sum of all the elements of $A \cap B$ is ___

Soln ⇒ A = A consist of elements which have HCF = 1 with 45.

⇓

If $\alpha \in A \Rightarrow \alpha$ is not divisible by 3 or 5

$45 = 3^2 \cdot 5$

$B = \{2, 4, 6, 8, \ldots, 200\}$

$A \cap B$

elements in B s.t HCF $(\alpha, 45) = 1$

$M_3 = \{6, 12, 18, \ldots, 198\}$
$M_5 = \{10, 20, 30, \ldots, 200\}$
$M_3 \cap M_5 = \{30, 60, 90, 120, \ldots, 180\}$

* $n(M_3)$ $\quad 6 + (P-1)6 = 198$
$P = 33$

$S(M_3) = \frac{33}{2}(6 + 198) = 3366$

* $n(M_5)$ $\quad 10 + (P-1)10 = 200$
$P = 20$

$S(M_5) = \frac{20}{2}(10 + 200) = 2100$

* $n(M_3 \cap M_5)$ $\quad 30 + (P-1)30 = 180$
$P = 6$

$S(M_3 \cap M_5) = \frac{6}{2}(30 + 180) = 630$

Sum of elements divisible by 3 or 5
$= S_{M_3} + S_{M_5} - S_{M_3 \cap M_5}$
$= 3366 + 2100 - 630$
$= 4836$

Sum of all elements in set B
$= \frac{100}{2}(2 + 200) = (5050)2$
$= 10100$

∴ Final ans
$S_{A \cap B} = 10100 - 4836$
$= 5264$

Ques ⇒ Let $A = \{n \in N : HCF(n, 45) = 1\}$ and Let $B = \{2k : k \in \{1, 2, \ldots, 100\}\}$. Then the sum of all the elements of $A \cap B = ?$

Mains 2022 (26 June)

$B = \{2, 4, 6, \ldots\ldots, 200\} \rightarrow S = 100 \times 101 = 10100$

A consist of elements which have HCF = 1 with 45

If $\alpha \in A \rightarrow \alpha$ is not divisible of 3 or 5.

$A \cap B$ — elements in B such that $HCF(\alpha, 45) = 1$

$M_3 = \{6, 12, 18, \ldots\ldots, 198\}$

$M_5 = \{10, 20, 30, \ldots\ldots, 200\}$

$M_3 \cap M_5 = \{30, 60, 90, 120, 150, 180\}$

Sum of elements divisible by 3 or 5

$= S_{M_3} + S_{M_5} - S_{M_3 \cap M_5}$

$= \frac{33}{2}(6 + 198) + \frac{20}{2}(10 + 200) - \frac{6}{2}(30 + 180)$

$= 33 \times 102 + 10 \times 210 - 3 \times 210$

$= 3366 + 210(10 - 3) = 4836$

Sum of elements not divisible by 3 or 5

$=$ Total sum $-$ Sum of elements divisible by 3 or 5.

$= 10100 - 4836 = 5264$ Ans.

Shweta from UP

QUESTION [JEE Mains 2020]

A survey shows that 63% of the people in a city reads newspaper A whereas 76% read newspaper B. If x% of the people read both the newspaper, then a possible value of x can be

(A) 55

(B) 65

(C) 29

(D) 37

TAH - 1

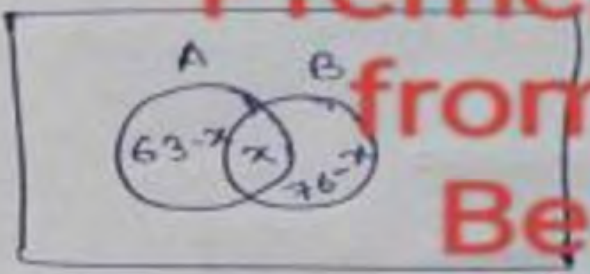
A survay shows that 63% of the people in a city reads newspaper A, whereas 76% read newspaper B. If x% of the people read both the news-paper. then a possible value of x can be.

Premendu Gain from West Bengal

$n(U) = 100$

$n(A) = 63$

$n(B) = 76$



$63 - x \geq 0$

$x \leq 63$

$76 - x \geq 0$

$x \leq 76$

$n(A \cup B) \leq 100$

$63 + 76 - x \leq 100$

$139 - x \leq 100$

$x \geq 39$

$\therefore \; 39 \leq x \leq 63$

(A) 55 Ans.

(B) 65

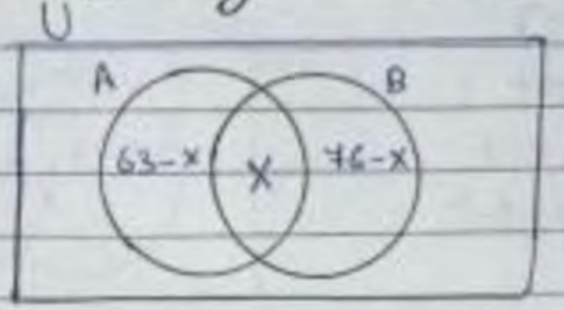
(C) 29

(D) 37

Que [JEE Main 2020] Tah-1

A Survey shows that 63% of the people in a city reads newspaper A where 76% read newspaper B. If x% of the people read both the the newspaper, then a possible value of x can be.

(A) 55
(B) 65
(C) 29
(D) 37.



$$63 - x \geq 0 \Rightarrow x \leq 63$$
$$76 - x \geq 0 \Rightarrow x \leq 76$$
$$x \geq 0 \Rightarrow x \geq 0$$
$$\Downarrow$$
$$0 \leq x \leq 63 \quad \text{(I)}$$

$$n(A \cup B) \leq 100$$
$$63 + 76 - x \leq 100$$
$$139 - x \leq 100$$
$$x \geq 39 \quad \text{(II)}$$

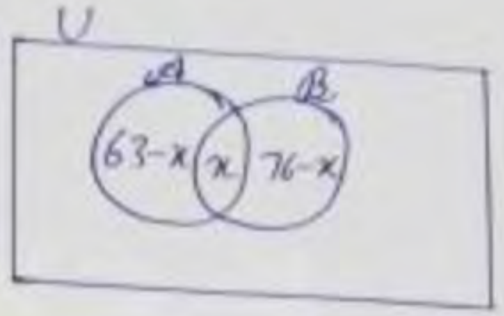
$$\Rightarrow 39 \leq x \leq 63$$

$\therefore$ Possible Value of $x \Rightarrow 55$ Ans

Shreya Sahu
from
Madhya Pradesh

TAH-1

Sol^n: $n(U) = 100$
$n(A) = 63$
$n(B) = 76$



We know that,
$$63 - x \geq 0 \Rightarrow x \leq 63$$
$$76 - x \geq 0 \Rightarrow x \leq 76$$
$$x \geq 0$$
$$\Rightarrow 0 \leq x \leq 63 \quad \text{— (1)}$$

$$n(A \cup B) \leq 100$$
$$63 + 76 - x \leq 100$$
$$x \geq 39 \quad \text{— (2)}$$

$$\boxed{39 \leq x \leq 63}$$

$\therefore$ Option (A) 55 is correct.

Name- Bhumika Sharma
From- Sri Ganganagar, Rajasthan

QUESTION [JEE Mains (July) 2021]

Out of all the patients in a hospital 89% are found to be suffering from heart aliment and 98% are suffering from lungs infection. If k% of them are suffering from both aliments, the K can not belong to the set :

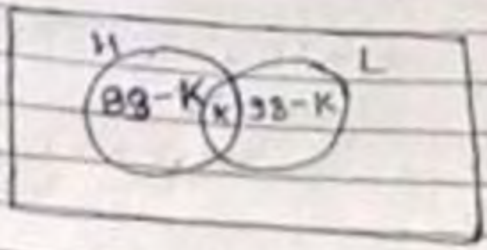
(A) $\{80, 83, 86, 89\}$

(B) $\{84, 86, 88, 90\}$

(C) $\{79, 81, 8, 85\}$

(D) $\{84, 87, 90, 93\}$

JAH(2)



$$\left.\begin{aligned} K &\le 89 \\ K &\le 98 \\ K &\ge 0 \end{aligned}\right\} \Rightarrow 0 \le K \le 98 \quad \text{(1)}$$

$n(H \cup L) \le 100$

$89 + 98 - K \le 100$

$87 \le K$ (ii)

From (1) & (ii)

$87 \le K \le 98$

$K = \{87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98\}.$

(C) $K \notin \{79, 81, 8, 85\}$

Manik kumhar

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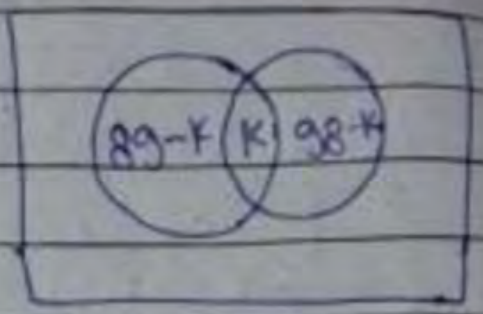
Tan2

$n(P) = 89$

$n(\text{lungs infe}) = 98$



$89 - K \ge 0 = x \le 89$

$98 - K \ge 0 = x \le 98$

$x \ge 0$

$0 \le x \le 89$ — (1)

$n(P \cup L) \le 100$

$89 + 98 - K \le 100$

$187 - K \le 100$

$K \ge 87$ — (ii)

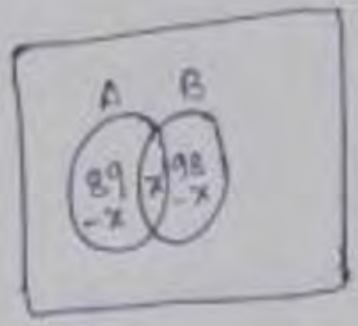
Sumit kar from Bihar

$= 87 \le x \le 89$

$= \{80, 83, 86, 89\}$ Ans

tah - 2

TAH-2



$n(U) = 100$

$n(A) = 89$

$n(B) = 98$

$89 - x \geq 0 \qquad 98 - x \geq 0$

$x \leq 89 \qquad x \leq 98$

$x \geq 0$

$n(A \cup B) \leq 100 \qquad 0 \leq x \leq 89$

$89 + 98 - x \leq 100$

$187 - x \leq 100$

$x \geq 87$

Premendu Gain from West Bengal

$\therefore \quad 87 \leq x \leq 89$

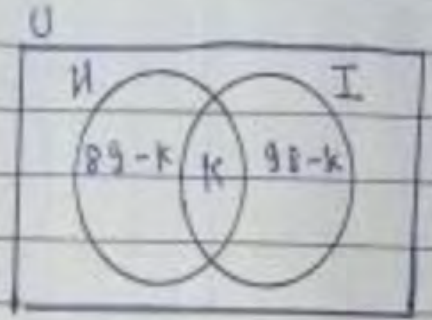
Ans. (c) $\{79, 81, 8, 85\}$

tah - 7

Tah 2 [JEE Main (July 2021)]

Que. Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If K% of them are suffering from both ailments, then K can not belong to the set:

(a) $\{80, 83, 86, 89\}$
(b) $\{84, 86, 88, 90\}$
(c) $\{79, 81, 83, 85\}$
(d) $\{84, 87, 90, 92\}$



$\Rightarrow \quad 89 - K \geq 0 \Rightarrow K \leq 89$

$98 - K \geq 0 \Rightarrow K \leq 98$

$K \geq 0 \Rightarrow K \geq 0$

$\Downarrow$

$0 \leq x \leq 89$ (i)

$n(H \cup I) \leq 100$

$89 + 98 - K \leq 100$

$187 - K \leq 100$

$K \geq 87$ (ii)

Shreya Sahu from Madhya Pradesh

$87 \leq x \leq 89$

(c) $\{79, 81, 83, 85\}$, K can not belong to the set.

QUESTION [JEE Mains 2019]

In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics, course, those whose number is divisible by 3 opted Physics course and those whose number is divisible by 5 opted Chemistry course. Then the number of students who did not opt for any of the three courses is.

(A) 42

(B) 1

(C) 102

(D) 38

Date: / /

Tah3

$M = \{2, 4, 6, ---- 140\} = 70$
$P = \{3, 6, 9, ---- 138\} = 46$
$C = \{5, 10, 15, ---140\} = 28$

$n(M) = 70$
$n(P) = 46$
$n(C) = 28$

Even no. divisible by $3 = \{6, 12, -- 138\} = 23$
" $5 = \{10, 20 - 140\} = 14$
" $3\&5 = \{30, 60, 90, 120\} = 4$
no. divisible by $(3\&5) = \{15, 30, --- 135\} = 9$

$n(M \cup P \cup C) = n(M) + n(P) + n(C) - (n(m \cup P) + n(P \cap C) + n(C \cap m)) + n(m \cap P \cap C)$

$\Rightarrow 70 + 46 + 28 - 23 - 14 - 9 + 4 = 102$

$=$ total Student $- n(M \cup P \cup C)$
$140 - 102$
38 Ans

Sumit Kumar
from Bihar

Page:

Tah-3:-

$n(M) \rightarrow$ those numbered students which are even i.e divisible by 2 & selected Mathematics course.

$P \rightarrow$ those numbered students which are divisible by 3 & opted physics course.

$C \rightarrow$ those numbered students which are divisible by 5 & opted Chemistry course.

$n(U) = 140$
$n(M) = 70$
$n(P) = 46$
$n(C) = 28$
$n(M \cap P) = 23$
$n(M \cap C) = 14$
$n(P \cap C) = 9$
$n(M \cap P \cap C) = 4$

$\because M = \{2, 4, 6, \ldots, 140\}$
$P = \{3, 6, 9, \ldots, 138\}$
$C = \{5, 10, 15, \ldots, 140\}$
$M \cap P = \{6, 12, \ldots, 138\}$
$M \cap C = \{10, 20, \ldots, 140\}$
$P \cap C = \{15, 30, \ldots, 135\}$
$M \cap P \cap C = \{30, 60, \ldots, 120\}$

$\Rightarrow n(M \cup P \cup C) = n(M) + n(P) + n(C) - (n(M \cap P) + n(M \cap C) + n(P \cap C)) + n(M \cap P \cap C).$

$= 70 + 46 + 28 - (23 + 14 + 9) + 4$
$n(M \cup P \cup C) = 148 - (46)$
$= 102$

$\Rightarrow n(M \cup P \cup C)' = 140 - 102$
$= 38$ Ans

Vishal Yadav
Tah 3

Ques ⇒ In a class of 140 students numbered 1 to 140, all even numbered students opted Mathematics course, those whose number is divisible by 3 opted physics and those whose number is divisible by 5 opted chemistry course. Then the number of students who did not opt for any of the three course = ?

$n(M\cup P\cup C) = n(M)+n(P)+n(C)-(n(M\cap P)+n(P\cap C)+n(C\cap M))+n(M\cap P\cap C)$

$M=\{2,4,6,\ldots,140\} \Rightarrow n(M)=70$

$P=\{3,6,9,\ldots,138\} \Rightarrow n(P)=46$

$C=\{5,10,15,\ldots 140\} \Rightarrow n(C)=28$

$M\cap P=\{6,12,18\ldots 138\} \Rightarrow n(M\cap P)=23$

$P\cap C=\{15,30,45\ldots 135\} \Rightarrow n(P\cap C)=9$

$C\cap M=\{10,20,30\ldots 140\} \rightarrow n(C\cap M)=14$

$M\cap P\cap C=\{30,60,90,120\} \longrightarrow n(M\cap P\cap C)=4$

$n(M\cup P\cup C)=70+46+28-(23+9+14)+4$

$= 144-46+4 = 102$

TAH 3

No. of students who did not opt for any of the three cou

= Total students − $n(M\cup P\cup C)$

= $140-102 = 38$ Ans

Shweta from UP

QUESTION

From 51 students taking examinations in Mathematics, Physics and Chemistry, 37 passed, Mathematics, 24 Physics and 43 Chemistry. At most 19 passed Mathematics and Physics, at most 29 passed Mathematics and Chemistry and at most 20 passed Physics and Chemistry. The largest possible number that could have passed all three examination is-

(A) 11

(B) 15

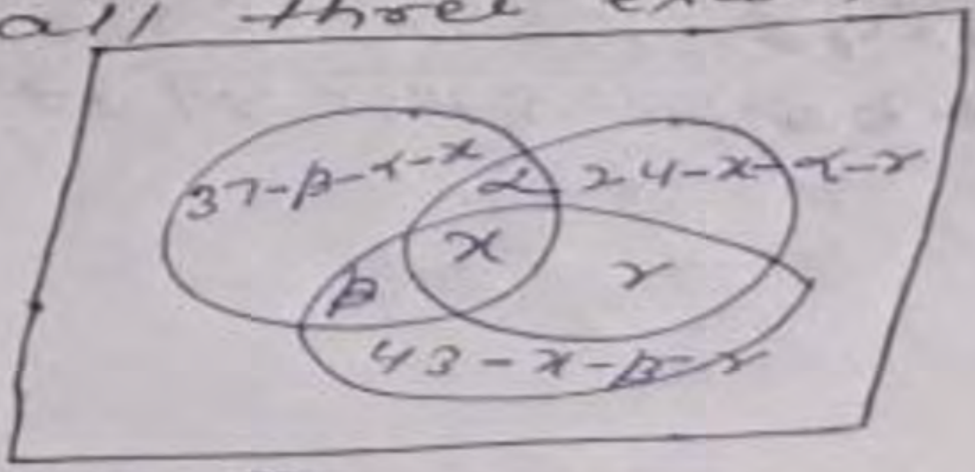
(C) 16

(D) 14

Q37. From 51 students taking examinations in mathematics, physics and chemistry, 37 passed mathematics, 24 physics & 43 chemistry. At most 19 passed mathematics & physics, at most 29 passed maths & chemistry and at most 20 passed physics & chemistry. The largest possible number that could have passed all three examinations.

(a) 11
(b) 15
(c) 16
(d) 14

$M = 37$
$P = 24$
$C = 43$



$n(M \cap P) \le 19$
$\Rightarrow \alpha + x \le 19$ — (I)

$n(M \cap C) \le 29 \rightarrow \beta + x \le 29$ — (II)

$n(P \cap C) \le 20 \rightarrow \gamma + x \le 20$ — (III)

$\alpha + \beta + \gamma \le 68 - 3x$ — (III)

$37 - \cancel{\beta} - \cancel{\alpha} - \cancel{x} + x + \cancel{\alpha} + \cancel{\beta} + \cancel{\gamma} + 24 - x - \alpha - \cancel{\gamma} + 43 - x - \beta - \gamma \le 51$

$61 + 43 - 2x - (\alpha + \beta + \gamma) \le 51$

$104 - 2x - 68 - 3x \le 51$

$x \le 51 - 36$

$x \le 15$

$\boxed{x_{max} = 15}$

Tah-4

Gautam from muzaffarpur bihar

QUESTION

In a class of 200 students, 70 played cricket, 60 played hockey and 80 played football. 30 played cricket and football, 30 played hockey and football, 40 played cricket and hockey. Find the maximum number of people playing all the three games and also the minimum number of people playing at least one game?

(A) 200, 100

(B) 30, 110

(C) 30, 120

(D) 20, 110

QUESTION

If $n(A) = 7$, $n(B) = 8$ and $n(A \cap B) = 4$, then which of the following columns.

(i)	$n(A \cup B)$	(a)	56
(ii)	$n(A \times B)$	(b)	16
(iii)	$n((B \times A) \times A)$	(c)	392
(iv)	$n(A \times B) \cap (B \times A))$	(d)	96
(v)	$n((A \times B) \cup (B \times A)$		

Tah 7 :- If $n(A)=7$, $n(B)=8$, & $n(A\cap B)=4$, then which of the following columns.

(i)	$n(A\cup B)$	(a) 56
(ii)	$n(A\times B)$	(b) 16
(iii)	$n((B\times A)\times A)$	(c) 392
(iv)	$n((A\times B)\cap(B\times A))$	(d) 96.
(v)	$n((A\times B)\cup(B\times A))$	(e) 11

Vishal Yadav

Tah 7

(i) $n(A\cup B) = n(A)+n(B)-n(A\cap B)$
$= 7+8-4$
$= \underline{11}$

(ii) $n(A\times B) = n(A)\cdot n(B)$
$= 7\cdot 8$
$= 56.$

(iii)
$n(B\times A) = 56$
$\Rightarrow n((B\times A)\times A) = 56\times 7$
$= 392$

(iv) $n((A\times B)\cap(B\times A)) = (n(A\cap B))^2$
$= 16$

Date:
Page:

(v)

$$n((A\times B)\cup(B\times A)) = n(A\times B) + n(B\times A) - n((A\times B)\cap(B\times A))$$
$$= 56 + 56 - 16$$
$$= \underline{\underline{96}}$$

Tah 7

Q.A.H. 7 A/Q $n(A)=7,\ n(B)=8,\ n(A\cap B)=4$

(i) $n(A\cup B) = n(A)+n(B)-n(A\cap B)$
$= 11$

(ii) $n(A\times B) = n(A)\cdot n(B) = 56$

(iii) $n((B\times A)\times A) = n(B\times A)\cdot n(A)$
$= n(B)\cdot n(A)\cdot n(A)$
$= 392$

(iv) $n((A\times B)\cap(B\times A)) = (n(A\cap B))^2 = 4^2 = 16$

(v) $n((A\times B)\cup(B\times A))$
$= n(A\times B) + n(B\times A) - n((A\times B)\cap(B\times A))$
$= 56 + 56 - 16$
$= 96$

By: Mayank Raj From: Jharkhand

TAH 7 If $n(A) = 7$, $n(B) = 8$ and $n(A \cap B) = 4$, then which of the following columns.

(i) $n(A \cup B)$ - None a) 56

(ii) $n(A \times B)$ - (a) b) 16

(iii) $n((B \times A) \times A)$ - (c) c) 392

(iv) $n((A \times B) \cap (B \times A))$ - (b) d) 96

(v) $n((A \times B) \cup (B \times A))$ - (d)

Somya Bansal

(i) $7 + 8 - 4 = 11$

(ii) $n(A) \cdot n(B) = 7 \times 8 = 56$

(iii) $n(B \times A) \cdot n(A)$

$56 \times 7 = 392$

(iv) $(n(A \cap B))^2 = 4^2 = 16$

(v) $n(A \times B) + n(B \times A) - n((A \times B) \cap (B \times A))$

$56 + 56 - 16$

$= 96$

QUESTION

If $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$ and $C = \{1, 5, 4, 3\}$ then find the number of elements in

(i) $A \times B \times C$

(ii) $(A \times B) \cap (B \times C)$

(iii) $(A \times B \times C) \cap (B \times C \times A)$

Th:- 8:→ If $A=\{1,2,3\}$, $B=\{2,3,4\}$ and $C=\{1,5,4,3\}$ then find the no. of element in

(I) $A\times B\times C$

$n(A)=3$ $\quad n(A\cap B)=2$

$n(B)=3$ $\quad n(B\cap C)=2$

$n(C)=4$ $\quad n(A\cap C)=2$

→ $n(A\times B\times C)=3\times3\times4=36$.

(ii) $(A\times B)\cap(B\times C)=n(A\cap B)\cdot n(B\cap C)$

$=2\cdot2=4$

Vishal Yadav

Tah 8

(iii) $(A\times B\times C)\cap(B\times C\times A)$

if $(\alpha,\beta,\gamma)\in(A\times B\times C)\cap(B\times C\times A)$

$\Rightarrow(\alpha,\beta,\gamma)\in(A\times B\times C)$ & $(\alpha,\beta,\gamma)\in(B\times C\times A)$.

$\Rightarrow\alpha\in A$ & B $\Rightarrow\alpha\in A\cap B$

$\beta\in B$ & C $\quad \beta\in B\cap C$

$\gamma\in C$ & A $\quad c\in C\cap A$

$\Rightarrow n((A\times B\times C)\cap(B\times C\times A))=n(A\cap B)\cdot n(B\cap C)\cdot n(C\cap A)$

$=2\cdot2\cdot2=8$.

Tah-8

If $A=\{1,2,3\}$, $B=\{2,3,4\}$

$C=\{1,5,4,3\}$

Find no. of elements in ⟶

$n(A)=3$, $n(B)=3$, $n(C)=4$

i) $A\times B\times C$.

$n(A\times B\times C)=n(A).n(B).n(C)$

$=3\times3\times4$

$n(A\times B\times C)=36$

ii) $(A\times B)\cap(B\times C)$

$n(\underbrace{(A\times B)\cap(B\times C)}_{2\quad 2})=2\times2=4$

iii) $(A\times B\times C)\cap(B\times C\times A)$

$n[\underbrace{(A\times B\times C)\cap(B\times C\times A)}_{2\times 2\quad 2}]=2^3\times2\times2=8$

(Solution to KTK)

QUESTION

Paragraph

There exists a matrix Q such that $PQP^T = N$, where $P = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Given N is a diagonal matrix of form $N = \text{diag.}(n_1, n_2, n_3)$ where n_1, n_2, n_3 are satisfying the equation $\det.(P - nI) = 0$, $n_1 < n_2 < n_3$.

[Note: I is an identity matrix of order 3×3]

The value of det. (adj N) is equal to

[Note : adj M denotes the adjoint of a square matrix M]

(A) 4

(B) $\frac{1}{4}$

(C) $\frac{1}{9}$

(D) 9

Ans. D

KTK01. $PQP^T = N$, $P = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, $N = \text{diag}\begin{bmatrix} n_1 & 0 & 0 \\ 0 & n_2 & 0 \\ 0 & 0 & n_3 \end{bmatrix}$

$Q = N$ $\quad \det(P - nI) = 0$, $n_1 < n_2 < n_3$

$P - nI = \begin{bmatrix} 1-n & 2 & 0 \\ 2 & 1-n & 0 \\ 0 & 0 & 1-n \end{bmatrix}$

$|P - nI| = (1-n)(1-n)^2 - 2(2(1-n)) = 0$

$= (1-n)\{(1-n)^2 - 4\} = 0$

$= 1-n = 0 \qquad (1-n-2)(1-n+2) = 0$

$n = 1 = n_2 \qquad n = -1 = n_1 \qquad n = 3 = n_3$

(a) $\det(\text{adj } N)$ value

$= N = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

$= \text{adj } N = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix} = -3I$

$= |\text{adj } N| = |-3I|^{3-1}$

$= |-3|^2 = 9$ Ans

KTK-1

$P = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad PQP^T = N \quad , (P - nI) = 0$

$P^T = P$

$$PP^T = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 0 \\ 4 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$(P - nI) = 0$ taking det in bot sides

$|P - nI| = 0$

$$\begin{vmatrix} 1-n & 2 & 0 \\ 2 & (1-n) & 0 \\ 0 & 0 & (1-n) \end{vmatrix} = 0 \Rightarrow (1-n)\{(1-n)^2 - 4\} = 0$$

$(1-n) = 0 \quad , (1-n-2)(1-n+2) = 0$

$n = 1 \; , (-1-n)(3-n) = 0$

$n = 1 , \; n = -1 \; , n = 3$

$\therefore n_1 = -1, n_2 = 1, n_3 = 3$

$$N = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}, |(\text{adj } N)| = |N|^{3-1} = |N|^2$$

$$= (-1+4)^2 = 3^2 = 9$$

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KTK-1. There exist a matrix Q such that $PQP^T = N$, Where $P = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Paragraph-1

Given N is diagonal matrix of form $N = \text{diag}(n_1, n_2, n_3)$, Where n_1, n_2, n_3 are satisfying the eq^n $\det(P - nI) = 0$, $n_1 < n_2 < n_3$.

[Note: I is an identity matrix of order 3×3]

KTK I A

The value of $\det(\text{adj } N)$ is equal to—

[Note: adj M denotes the adjoint of a square matrix M]

$\Rightarrow P = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

N is diagonal matrix of form $N = \text{diag}(n_1, n_2, n_3)$ Where, n_1, n_2, n_3 are satisfying the eq^n $\det(P - nI) = 0$

$$\det(P - nI) = \begin{vmatrix} 1-n & 2 & 0 \\ 2 & 1-n & 0 \\ 0 & 0 & 1-n \end{vmatrix} = 0$$

$\Rightarrow (1-n)[n^2 - 2n + 1 - 4] = 0$

$\Rightarrow (n-1)(n^2 - 2n - 3) = 0$

$\Rightarrow (n-1)(n+1)(n-3) = 0$

$\therefore n = -1, 1, 3$

$n_1 = -1, \; n_2 = 1, \; n_3 = 3$ (as $n_1 < n_2 < n_3$)

$$N = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

$\det(N) = -3$.

Now, $\det(\text{adj } N) = |N|^{3-1}$

$= (-3)^2 = (9)$ Ans.

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QUESTION

Paragraph

There exists a matrix Q such that $PQP^T = N$, where $P = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Given N is a diagonal matrix of form $N = \text{diag.}(n_1, n_2, n_3)$ where n_1, n_2, n_3 are satisfying the equation $\det.(P - nI) = 0$, $n_1 < n_2 < n_3$.

[Note: I is an identity matrix of order 3×3]

If $Q^T = Q + \alpha I$, then the value of α is equal to

(A) -1

(B) 0

(C) 1

(D) $\frac{-1}{3}$

Ans. B

QUESTION

Paragraph

There exists a matrix Q such that $PQP^T = N$, where $P = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Given N is a diagonal matrix of form $N = \text{diag.}(n_1, n_2, n_3)$ where n_1, n_2, n_3 are satisfying the equation $\det.(P - nI) = 0$, $n_1 < n_2 < n_3$.

[Note: I is an identity matrix of order 3×3]

The trace of matrix P^{2012} is equal to

[Note: The trace of a matrix is the sum of its diagonal entries]

(A) $3^{2011} + 2$

(B) 3^{2012}

(C) $3^{2012} + 2$

(D) 3^{2011}

Ans. C

Paragraph-3 The trace of matrix P^{2012} is equal to —
[Note: The trace of a matrix is the sum of its diagonal entries]

KTK I C

$\rightarrow$ $P = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$$P^2 = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 4 & 0 \\ 4 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore tr(P^2) = 11 = (3^2+2) \cdot\cdot$$

$$P^4 = P^2 \cdot P^2 = \begin{bmatrix} 5 & 4 & 0 \\ 4 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 5 & 4 & 0 \\ 4 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 41 & 40 & 0 \\ 40 & 41 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$tr(P^4) = 41+41+1 = 83 = (3^4+2).$$

$$P^6 = P^4 \cdot P^2 = \begin{bmatrix} 41 & 40 & 0 \\ 40 & 41 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 5 & 4 & 0 \\ 4 & 5 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 365 & 364 & 0 \\ 364 & 365 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$tr(P^6) = 365+365+1 = 731 = (3^6+2).$$

$\vdots$

General form, for P^{2n} matrix, where, $n \in N$

$$tr(P^{2n}) = 3^{2n}+2.$$

Now, $tr(P^{2012}) = (3^{2012}+2)$ Ans

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(Solution to RPP)

QUESTION

(RPP 1)

Consider the equation $3x^4 - 18x^3 + px^2 - 8qx + 3q = 0$. The equation has only positive real roots then the value of $\frac{p}{q}$ is

(A) $\frac{1}{8}$

(B) 4

(C) $\frac{1}{4}$

(D) 8

Ans. D

RPP-1

$$3x^4-18x^3+px^2-8qx+3q=0 \quad \alpha,\beta,\gamma,\delta \qquad , \alpha,\beta,\gamma,\delta \in R^+$$

$$s_1=\alpha+\beta+\gamma+\delta=\frac{-(-18)}{3}=6.$$

$$s_2=\sum\alpha\beta=\alpha\beta+\alpha\gamma+\alpha\gamma+\beta\gamma+\beta\gamma+\gamma\delta=\frac{p}{3}$$

$$s_3=\sum\alpha\beta\gamma=\alpha\beta\gamma+\alpha\gamma\delta+\beta\gamma\delta+\beta\delta\alpha=\frac{-(8q)}{3}=\frac{8q}{3}$$

$$s_4=\alpha\beta\gamma\delta=\frac{3q}{3}=q.$$

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Now $\quad \dfrac{s_3}{s_4}=\dfrac{1}{\alpha}+\dfrac{1}{\beta}+\dfrac{1}{\gamma}+\dfrac{1}{\delta}=\dfrac{8}{3}$

AM HM ineq, $\quad \dfrac{4}{\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}}=\dfrac{\alpha+\beta+\gamma+\delta}{4}$

$$\frac{4}{8/3}\leq\frac{6}{4}\Rightarrow 3/2\leq 3/2\Rightarrow HM=AM \text{ satisfy}$$

$$\therefore \alpha=\beta=\gamma=\delta=3/2$$

Now $\quad s_2=\dfrac{p}{3}=6\cdot\left(\dfrac{3}{2}\right)^2=\dfrac{6\times 9}{4}\Rightarrow p=\dfrac{81}{2}$

$$s_4=q=\left(\frac{3}{2}\right)^4=\frac{81}{16}$$

$$\therefore \frac{p}{q}=\frac{81/2}{81/16}=16/2=8.$$

RPP-1 Consider the eq^n $3x^4-18x^3+px^2-8qx+3q=0$. The eq^n has only positive real roots then the value of p/q is —

RPP 1

$\rightarrow$ $3x^4-18x^3+px^2-8qx+3q=0$ $\alpha, \beta, \gamma, \delta \rightarrow R^+$

$S_1 = \alpha+\beta+\gamma+\delta = 18/3 = 6$.

$S_2 = \sum\alpha\beta = \alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta = \frac{p}{3}$.

$S_3 = \sum\alpha\beta\gamma = \alpha\beta\gamma+\alpha\gamma\delta+\alpha\beta\delta+\beta\gamma\delta = \frac{8q}{3}$.

$S_4 = \alpha\beta\gamma\delta = \frac{3q}{3} = q$.

Now, $\frac{S_3}{S_4} = \frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta} = \frac{8}{3}$

from AM–HM inequality,

$$\frac{4}{\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}+\frac{1}{\delta}} \le \frac{\alpha+\beta+\gamma+\delta}{4}$$

$$\rightarrow \frac{4}{8/3} \le \frac{6}{4}$$

$$\rightarrow \frac{3}{2} \le \frac{3}{2} \rightarrow HM = AM.$$

$$\boxed{\alpha=\beta=\gamma=\delta}$$

$$\alpha+\beta+\gamma+\delta = 6$$

$$\boxed{\alpha=\beta=\gamma=\delta=\frac{3}{2}}$$

from S_2,

$$\frac{p}{3} = 6\cdot\left(\frac{3}{2}\right)^2$$

$$\rightarrow \boxed{p = 18\times\frac{9}{4} = \frac{81}{2}}$$

from S_4, $q = \left(\frac{3}{2}\right)^4 = \frac{81}{16}$

Now, $\frac{p}{q} = \frac{81/2}{81/16} = (8)$ Ans.

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QUESTION

(RPP 2)

If both the roots of the equation $x^2 - 6ax + 2 - 2a + 9a^2 = 0$ exceed 3 then

(A) $a < \frac{1}{2}$

(B) $a > \frac{1}{2}$

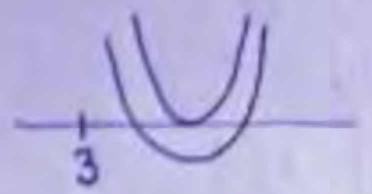
(C) $a < 1$

(D) $a > \frac{11}{9}$

Ans. D

RPP 02

$x^2 - 6ax + 2 - 2a + 9a^2 = 0$



$f(3) > 0$ & $-\frac{b}{2a} > 3$ & $D \geq 0$

$f(3) > 0$

$9 - 18a + 2 - 2a + 9a^2 > 0$

$9a^2 - 20a + 11 > 0$

$(a-1)\left(a - \frac{11}{9}\right) > 0$

$\boxed{a \in (-\infty, 1) \cup \left(\frac{11}{9}, \infty\right)}$

$\frac{6a}{2} > 3$

$\boxed{a > 1}$

$36a^2 - 4(2 - 2a + 9a^2) > 0$

$\cancel{36a^2} - 8 + 8a - \cancel{36a^2} > 0$

$(a-1) > 0$

$\boxed{a > 1}$

'$\cap$'

$a \in \left(\frac{11}{9}, \infty\right)$

$\boxed{a > \frac{11}{9}}$ Ans

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RPP-2 If both the roots of the eq^n $x^2 - 6ax + 2 - 2a + 9a^2 = 0$ exceed 3 then –

RPP 2

→

(i) $f(3) > 0$

(2) $-\frac{b}{2a} > 3$

(3) $D \geq 0$

(1) $f(3) > 0$

$\Rightarrow 9 - 18a + 2 - 2a + 9a^2 > 0$

$\Rightarrow 9a^2 - 20a + 11 > 0$

$\Rightarrow 9a^2 - 11a - 9a + 11 > 0$

$\Rightarrow (9a - 11)(a - 1) > 0$

$a \in (-\infty, 1) \cup \left(\frac{11}{9}, \infty\right)$ — (i).

(2) $-\frac{B}{2A} > 3$

$\Rightarrow \frac{6a}{2} > 3$

$\Rightarrow 3a > 3$

$\boxed{\therefore a > 1}$

$\therefore a \in (1, \infty)$ — (ii).

(3) $D \geq 0$

$\Rightarrow 36a^2 - 4(2 - 2a + 9a^2) \geq 0$

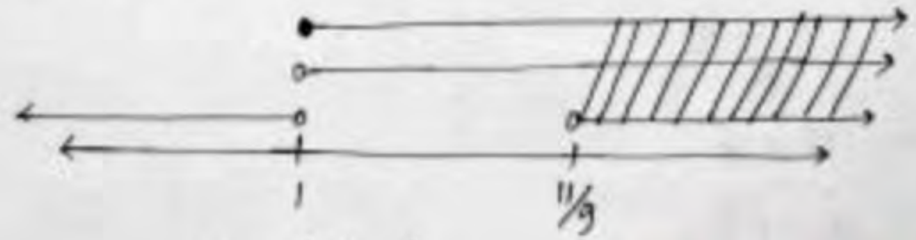
$\Rightarrow \cancel{36a^2} - 8 + 8a - \cancel{36a^2} \geq 0$

$\Rightarrow 8a - 8 \geq 0$

$\Rightarrow a - 1 \geq 0$

$\therefore a \in [1, \infty)$ — (iii).

(i) $\cap$ (ii) $\cap$ (iii) $\Rightarrow$



$a \in \left(\frac{11}{9}, 0\right)$

$\boxed{\therefore a > \frac{11}{9}}$ Ans

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QUESTION

(RPP 3)

If $f(x) = ax^2 + 6x - a$ has maximum value 10 then sum of all possible value(s) of 'a' is

(A) -20

(B) -10

(C) 10

(D) 20

Ans. B

RPP 03

$f(x) = ax^2 + 6x - a$

Max value exist if $a<0$

$f(x) = ax^2 + 6x - a$

$f\left(\frac{-b}{2a}\right) = 10$

$f\left(\frac{-6}{2a}\right) = a\left(\frac{-6}{2a}\right)^2 + 6\left(\frac{-6}{2a}\right) - a = 10$

$\frac{36}{4a} - \frac{36}{2a} - a = 10$

$\frac{-36}{4a} - a = 10$

$36 + 4a^2 = -40a$

$4a^2 + 40a + 36 = 0$

$a^2 + 10a + 9 = 0$

$(a+1)(a+9) = 0$

$a = -1, -9$

Sum $= -1 - 9 = -10$

Ans!

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RPP 3 If $f(x) = ax^2 + 6x - a$ has maximum value 10 then Sum of all possible value(s) of 'a' is—

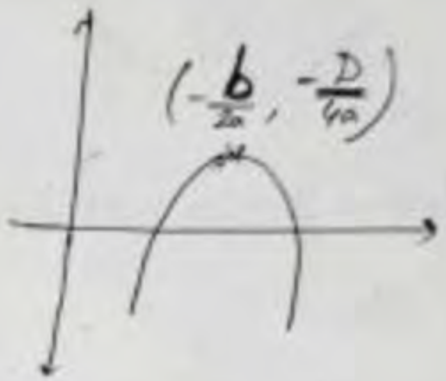
RPP 3

→ for a quadratic equation maximum and minimum value occurs at vertex.

$f(x) = ax^2 + 6x - a.$

↳ for occuring maximum value

$a<0 \rightsquigarrow$ downward parabola



Now, $-\frac{b}{2a} = -\frac{3}{a}.$

$f\left(-\frac{3}{a}\right) = a\left(-\frac{3}{a}\right)^2 + 6\left(-\frac{3}{a}\right) - a$

$\Rightarrow 10 = \frac{9}{a} - \frac{18}{a} - a$

$\Rightarrow 10a = -9 - a^2$

$\Rightarrow a^2 + 10a + 9 = 0$

$a = \frac{-10 \pm \sqrt{100 - 36}}{2}$

$\Rightarrow a = \frac{-10 \pm 8}{2}$

$\Rightarrow a = -5 \pm 4$

$\therefore a = -9, -1$

$\therefore$ The sum of all possible values of 'a' $= (-9-1) = (-10)$ Ans.

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QUESTION

(RPP 4)

Let α and β are roots of equation $7x^2 - 5x - 1 = 0$, then

$$\lim_{n\to\infty}\sum_{r=0}^{n}\left(\frac{1}{(7\alpha-5)^r}+\frac{1}{(7\beta-5)^r}\right)$$ is

(A) 9

(B) -3

(C) 3

(D) $\frac{19}{13}$

Ans. A

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$\boxed{\text{RPP-4}}$ $7x^2-5x-1=0 \begin{cases}\alpha\\ \beta\end{cases}$

$$\lim_{n\to\infty}\sum_{r=0}^{n}\left(\frac{1}{(7\alpha-5)^r}+\frac{1}{(7\beta-5)^r}\right)$$

$7\alpha^2-5\alpha-1=0$

$7\alpha^2-5\alpha=1$

$\alpha=\frac{1}{7\alpha-5}$

$$\Rightarrow \lim_{n\to\infty}\sum_{r=0}^{n}\left(\alpha^r+\beta^r\right).$$

$$\Rightarrow \frac{1}{1-\alpha}+\frac{1}{1-\beta}=\frac{1-\beta+1-\alpha}{(1-\alpha)(1-\beta)}=\frac{2-(\alpha+\beta)}{1-(\alpha+\beta)+\alpha\beta}$$

lly $\beta=\frac{1}{7\beta-5}$

$$\Rightarrow \frac{2-5/7}{1-5/7-1/7}=\frac{\frac{14-5}{7}}{\frac{7-5-1}{7}}=9$$

RPP-4 Let α and β are roots of eqⁿ $7x^2-5x-1=0$, then $\lim_{n\to\infty}\sum_{r=0}^{n}\left(\frac{1}{(7\alpha-5)^r}+\frac{1}{(7\beta-5)^r}\right)$ is -

RPP 4

$\rightarrow$

$$7x^2-5x-1=0 \begin{matrix}\nearrow \alpha \\ \searrow \beta\end{matrix}$$

$$7\alpha^2-5\alpha-1=0$$

$$\rightarrow \boxed{\frac{1}{7\alpha-5}=\alpha}$$

$$\text{lly}, \boxed{\frac{1}{7\beta-5}=\beta}$$

Now, $\lim_{n\to\infty}\sum_{r=0}^{n}\left(\frac{1}{(7\alpha-5)^r}+\frac{1}{(7\beta-5)^r}\right)$

$$=\lim_{n\to\infty}\sum_{r=0}^{n}(\alpha^r+\beta^r)$$

$$=\frac{1}{1-\alpha}+\frac{1}{1-\beta}=\frac{1-\beta+1-\alpha}{(1-\alpha)(1-\beta)}$$

$$=\frac{2-(\alpha+\beta)}{1-(\alpha+\beta)+\alpha\beta}$$

$$=\frac{2-\frac{5}{7}}{1-\frac{5}{7}-\frac{1}{7}}=\frac{\frac{14-5}{7}}{\frac{7-5-1}{7}}$$

$$=(9)\ \underline{\text{Ans}}$$

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RPP-4

$7x^2 - 5x - 1 = 0 <^{\alpha}_{\beta}$ then $\lim_{n\to\infty} \sum_{r=0}^{n} \left(\frac{1}{(7\alpha-5)^r} + \frac{1}{(7\beta-5)^r} \right)$

Soln:- $S = \alpha + \beta = \frac{5}{7}$, $\alpha\beta = \frac{-1}{7}$

$$S = \underbrace{\lim_{n\to\infty} \sum_{r=0}^{n} \left(\frac{1}{(7\alpha-5)^r} \right)}_{\text{infinite GP}} + \underbrace{\lim_{n\to\infty} \sum_{r=0}^{n} \left(\frac{1}{(7\beta-5)^r} \right)}_{\text{infinite GP}}$$

$$= \left(1 + \frac{1}{7\alpha-5} + \left(\frac{1}{7\alpha-5}\right)^2 + \dots \infty \right) + \left(1 + \frac{1}{7\beta-5} + \left(\frac{1}{7\beta-5}\right)^2 + \dots \infty \right)$$

$$= \frac{1}{1 - \frac{1}{7\alpha-5}} + \frac{1}{1 - \frac{1}{7\beta-5}} = \frac{7\alpha-5}{7\alpha-6} + \frac{7\beta-5}{7\beta-6}$$

$$= \frac{2 \times 49(\alpha\beta) - 42(\alpha+\beta) - 35(\alpha+\beta) + 60}{49\alpha\beta - 42(\alpha+\beta) + 36} = \frac{-14 - 30 - 25 + 60}{-37 + 36}$$

$$= \frac{-9}{-1} = \underline{\underline{9}}$$

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QUESTION

(RPP 5)

If α, β are the roots of the equation $x^2 - 2x + 3$, then the value of

$\sum_{r=1}^{10}(r + \alpha)(r + \beta)$ is equal to

(A) -525

(B) -305

(C) 305

(D) 525

Ans. D

RPP-5.

If α, β are the roots of the eq^n x^2-2x+3, then the value of $\sum_{r=1}^{10}(r+\alpha)(r+\beta)$ is equal to —

RPP 5

$\Rightarrow$ $x^2-2x+3=0 \;\begin{matrix}\alpha \\ \beta\end{matrix}$

$\alpha+\beta=2, \;\alpha\beta=3.$

$(r+\alpha)(r+\beta) = r^2+r(\alpha+\beta)+\alpha\beta$

$= (r^2+2r+3).$

Now, $\sum_{r=1}^{10}(r+\alpha)(r+\beta) = \sum_{r=1}^{10}(r^2+2r+3)$

$= \sum_{r=1}^{10}(r^2) + 2\sum_{r=1}^{10}(r) + 3\times 10$

$= \frac{10(10+1)(20+1)}{6} + \cancel{2}\cdot\frac{10\times 11}{\cancel{2}} + 30$

$= \frac{\cancel{10}^{5}\times 11\times \cancel{21}^{7}}{\cancel{6}\,\cancel{2}} + 110 + 30$

$= (525)$ - Ans.

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RPP-5

if α, β are the roots of the equation $x^2 - 2x + 3$.

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$\alpha + \beta = 2$

$\alpha\beta = 3$

$$\sum_{r=1}^{10} (r+\alpha)(r+\beta)$$

$= r^2 + r(\alpha+\beta) + \alpha\beta$

$= r^2 + r \cdot 2 + 3$

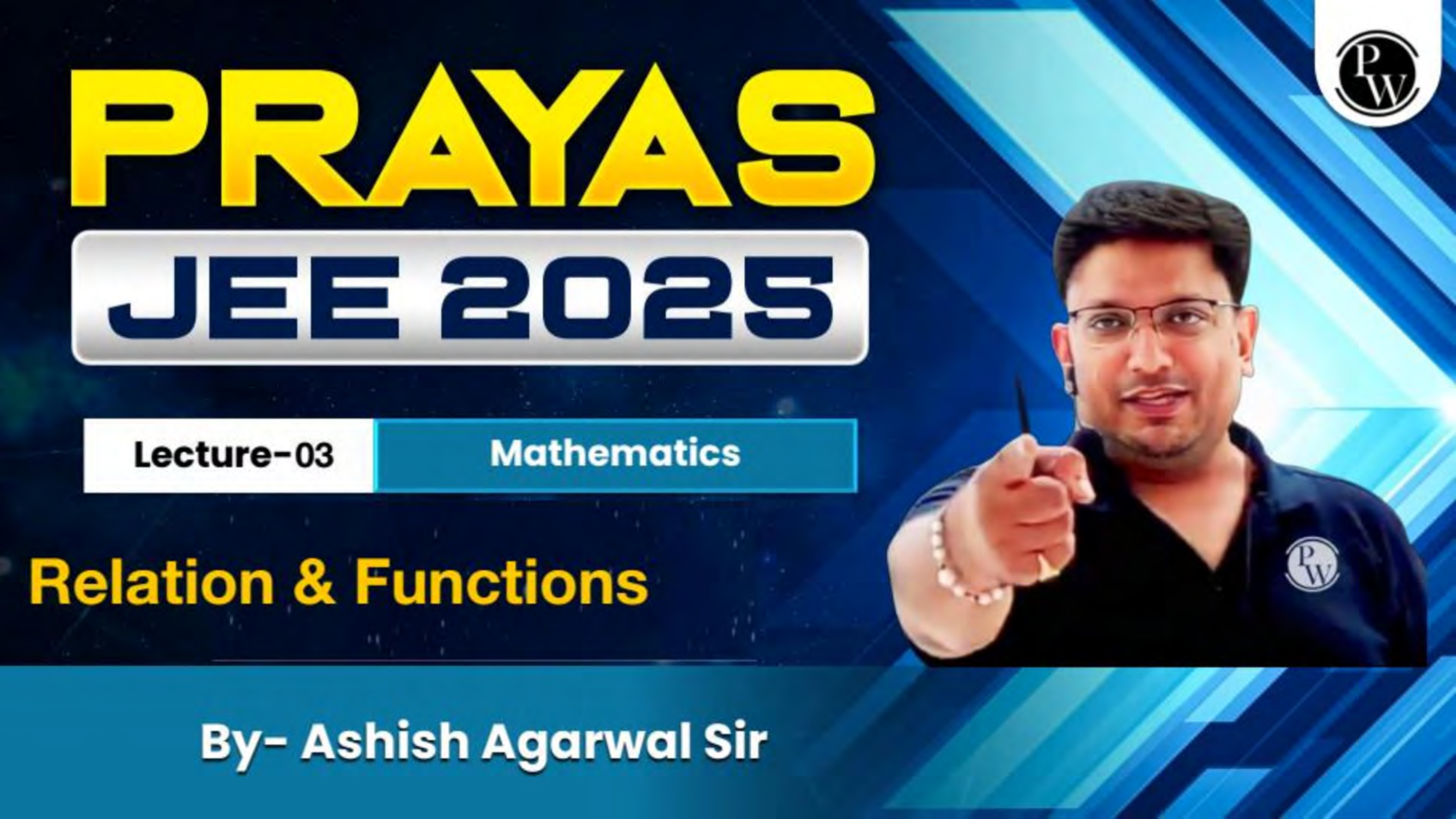
$= \frac{\cancel{10}^{5} \cdot 11 \cdot \cancel{21}^{7}}{\cancel{6}\,\cancel{3}} + \frac{\cancel{2} \cdot 10 \cdot 11}{\cancel{2}} + 30$

$= 35 \times 11 + 110 + 30$

$= 385 + 140$

$= \underline{525}$ Ans.


THANK
YOU


PRAYAS
JEE 2025
Lecture-03
Mathematics
Relation & Functions
By- Ashish Agarwal Sir

Topics *to be covered*

1. Practice Problems
2. Introduction to functions

Recap *of previous lecture*

1. $n((A \times B) \cap (B \times A)) = (n(A \cap B))^2$

2. $n((A \times B \times C) \cap (P \times Q \times R)) = n(A \cap P) \cdot n(B \cap Q) \cdot n(C \cap R)$

3. A relation from A to B is subset of $A \times B$ while a relation on A is a subset of $A \times A$ & relation on B is a subset of $B \times B$.

4. Every identity relation is reflexive. (T/F) (True)

5. Every relation which is not symmetric is antisymmetric. (T/F) (False)

Recap *of previous lecture*

6. A relation R on A is

   (i) Reflexive if $(a,a) \in A$ for every $a \in A$

   (ii) Symmetric if $(a,b) \in R$ then (b,a) should also lie in R, $(a, b \in A)$

   (iii) Transitive if (a,b) & $(b,c) \in R$ then (a,c) should also lie in R. $(a,b,c \in A)$

   (iv) Antisymmetric if (a,b) & $(b,a) \in R$ then $a=b$.

   (v) Equivalence if it is Reflexive, symmt & Transitive.

Recap *of previous lecture*

7. $n(A \times B) =$ $n(A) \cdot n(B)$

8. $n((A \times B) \cup (B \times A)) =$ $n(A\times B) + n(B\times A) - (n(A\cap B))^2$

9. If $n(A) = 4$, $n(B) = 2$ then number of relations from A to B having atleast three elements is $({}^8C_0 + {}^8C_1 + {}^8C_2 + {}^8C_3 + {}^8C_4 + {}^8C_5 + \text{—} + {}^8C_8) - {}^8C_0 - {}^8C_1 - {}^8C_2$

$n(A\times B) = 8$

$= 2^8 - 1 - 8 - \frac{8\times 7}{2!} = 2^8 - 9 - 28 = 2^8 - 37.$

QUESTION

$70-(40-x+x+30-x)$ Tah 5

In a class of 200 students, 70 played cricket, 60 played hockey and 80 played football. 30 played cricket and football, 30 played hockey and football, 40 played cricket and hockey. Find the maximum number of people playing all the three games and also the minimum number of people playing at least one game?

$n(U)=200$

(A) 200, 100

(B) 30, 110

(C) 30, 120

(D) 20, 110

$40-x \geq 0 \rightarrow x \leq 40$

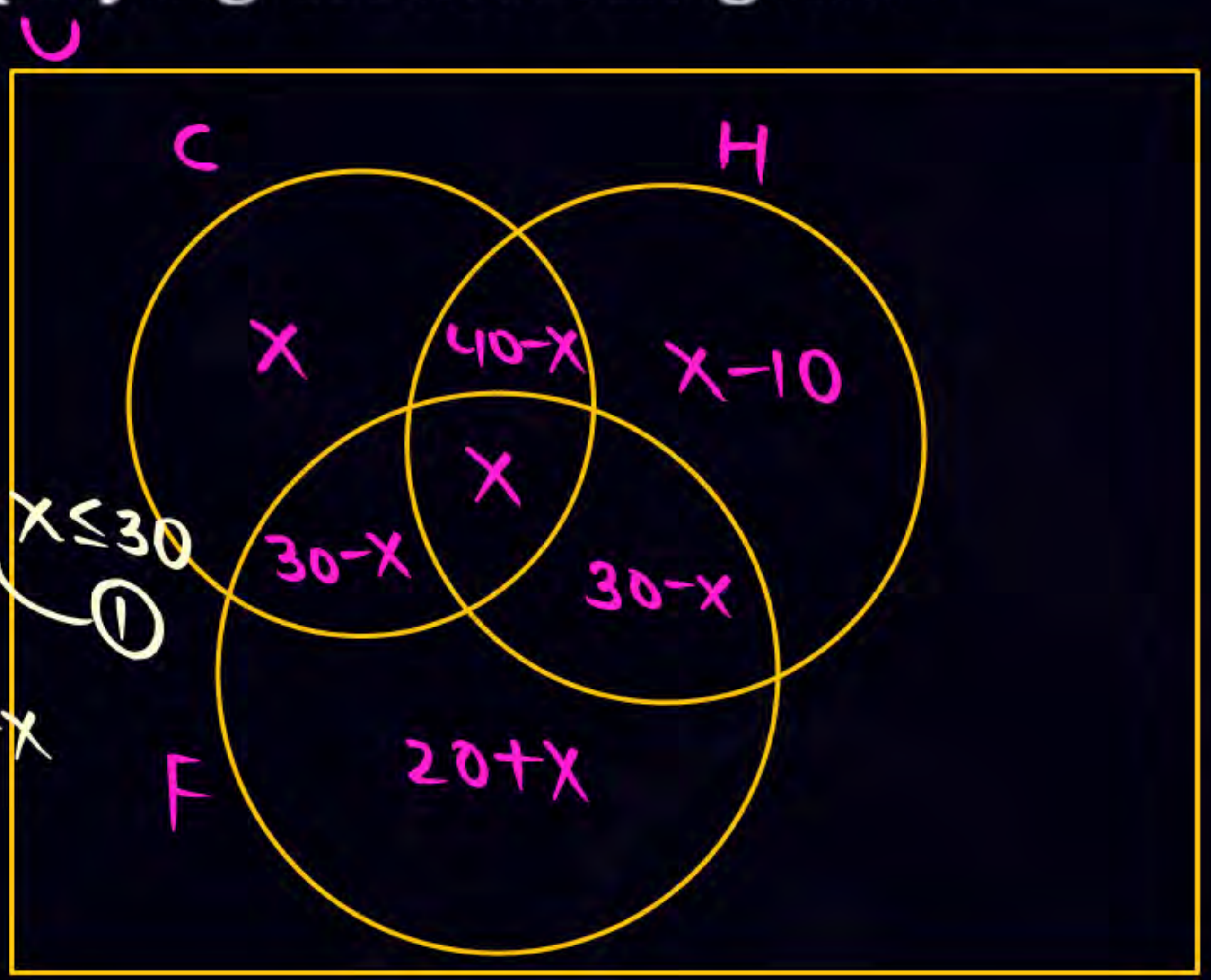
$30-x \geq 0 \rightarrow x \leq 30$

$x \geq 0 \rightarrow x \geq 0$

$x-10 \geq 0 \rightarrow x \geq 10$

$10 \leq x \leq 30$ ①

$n(C \cup F \cup H) = 70+x-10+30-x+20+x$
$= 110+x$



$n(C \cup F \cup H) = 110 + x \leq 200$.

$x \leq 90$ — (II)

from (I) & (II)

$10 \leq x \leq 30$.

$x_{MAX} = 30$

No: of people playing atleast one game$\Big|_{MIN}$

$n(C \cup H \cup F) = 110 + x\Big|_{MIN} = 110 + 10 = \underline{\underline{120}}$

QUESTION

$P^T = P \frown P$ is symmt.

$N = PQP$

Paragraph

There exists a matrix Q such that $PQP^T = N$, where $P = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Given N is a diagonal matrix of form $N = \text{diag.}(n_1, n_2, n_3)$ where n_1, n_2, n_3 are satisfying the equation det. $(P - nI) = 0$, $n_1 < n_2 < n_3$.

[Note: I is an identity matrix of order 3×3]

The value of det. (adj N) is equal to

[Note : adj M denotes the adjoint of a square matrix M]

$\det(P - nI) = \begin{vmatrix} 1-n & 2 & 0 \\ 2 & 1-n & 0 \\ 0 & 0 & 1-n \end{vmatrix} = 0$

$n = 1, -1, 3.$

(A) 4

(B) $\frac{1}{4}$

(C) $\frac{1}{9}$

(D) 9

$N = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \text{diag}(-1, 1, 3)$

$\det(\text{adj} N) = |N|^2 = (1 \cdot -1 \cdot 3)^2 = \underline{\underline{9}}.$

Ans. D

QUESTION

$P^T = P$ $(A^{-1})^T = (A^T)^{-1}$

Paragraph

There exists a matrix Q such that $PQP^T = N$, where $P = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

$\det P = 1(1-0) - 2(2-0)$

$\det P = -3 \neq 0$

P is invertible

Given N is a diagonal matrix of form N = diag. (n_1, n_2, n_3) where n_1, n_2, n_3 are satisfying the equation det. $(P - nI) = 0$, $n_1 < n_2 < n_3$.

[Note: I is an identity matrix of order 3 × 3]

If $Q^T = Q + \alpha I$, then the value of α is equal to

M1

$PQP^T = N$

$PQP = N$

$N = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 3 \end{pmatrix} \Rightarrow N^T = N$

(A) -1

(B) 0

(C) 1

(D) $\frac{-1}{3}$

$P^{-1}(PQP)P^{-1} = P^{-1}NP^{-1}$

$Q = P^{-1}NP^{-1}$

$Q^T = (P^{-1}NP^{-1})^T = (P^{-1})^T N^T (P^{-1})^T$

$Q^T = (P^T)^{-1} N^T (P^T)^{-1} = P^{-1} N^T P^{-1}$

$Q^T - Q = P^{-1}N^TP^{-1} - P^{-1}NP^{-1}$

$= P^{-1}(N^T - N)P^{-1} = P^{-1} O P^{-1} = O = 0 \cdot I$

Ans. B

M② $PQP^T = N$

$PQP = N$ — $(PQP)^T = N^T$

$P^T Q^T P^T = N^T = N.$

$PQ^T P = N$

$PQ^T P - PQP = N - N$

$P(Q^T - Q)P = 0$

$\det P \neq 0 \Rightarrow P$ is invt.

$P^{-1}\left(P(Q^T - Q)P\right)P^{-1} = 0$

$Q^T - Q = 0 = 0 \cdot I = 0$

M③ $Q^T = Q + \alpha I.$

$(Q^T)^T = (Q + \alpha I)^T$

$Q = Q^T + \alpha I.$

$\cancel{Q} = \cancel{Q} + \alpha I + \alpha I$

$0 = 2\alpha I$

$\Downarrow$

$\alpha = 0.$

QUESTION

Observe the pattern

Paragraph

There exists a matrix Q such that $PQP^T = N$, where $P = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Given N is a diagonal matrix of form $N = \text{diag.}\ (n_1, n_2, n_3)$ where n_1, n_2, n_3 are satisfying the equation $\det.\ (P - nI) = 0$, $n_1 < n_2 < n_3$.

[Note: I is an identity matrix of order 3×3]

The trace of matrix P^{2012} is equal to

[Note: The trace of a matrix is the sum of its diagonal entries]

(A) $3^{2011} + 2$

(B) 3^{2012}

(C) $3^{2012} + 2$

(D) 3^{2011}

Ans. C

QUESTION

RPP 2

KCLS

If the number of solutions of the equation $\cos^2\left(\frac{\pi}{4}(\cos x+\sin x)\right)-\tan^2\left(x+\frac{\pi}{4}\tan^2 x\right)=1$ in $[-2\pi, 2\pi]$ is 'k', then $\frac{3k}{25}$ equals

$$\underbrace{\cos^2\left(\pi/4(\cos x+\sin x)\right)}_{\le 1}=\underbrace{1+\underbrace{\tan^2\left(x+\frac{\pi}{4}\tan^2 x\right)}_{\ge 0}}_{\ge 1}$$

$\cos^2\left(\pi/4(\cos x+\sin x)\right)=1$ & $\tan^2\left(x+\pi/4\tan^2 x\right)=0$

$\cancel{\pi}/4(\cos x+\sin x)=n\cancel{\pi}\pm 0$

$\cos x+\sin x=4n, n\in I$

Ans. 0.48

$n=0$ is only possible

$\tan x = \tan(-\pi/4)$

$x = n\pi - \pi/4$

$\cos x + \sin x = 0$

$\tan x = -1$

$\Downarrow$

$x = 3\pi/4, \frac{7\pi}{4}, -\pi/4, -\frac{5\pi}{4}$

$\Downarrow$

4 soln

$\tan^2(x + \frac{\pi}{4}\tan^2 x) = 0$

put here

$A \cap B \subseteq A, B$

$k=4 \Rightarrow \frac{3k}{25} = \frac{12}{25} = 0.48$

QUESTION

RPP 3

(KCLS)

Let $f_n(\theta) = \sum_{r=0}^{n} \frac{1}{4^r} \cdot \sin^4(2^r\theta)$, then

(A) $f_2\left(\frac{\pi}{4}\right) = \frac{\pi}{\sqrt{2}}$

(B) $f_3\left(\frac{\pi}{8}\right) = \frac{2+\sqrt{2}}{4}$

(C) $f_4\left(\frac{3\pi}{2}\right) = 1$

(D) $f_5(\pi) = 0$

$$T_r = \frac{1}{4^r}\sin^4(2^r\theta) = \frac{1}{4^r}\sin^2(2^r\cdot\theta)\cdot\sin^2(2^r\cdot\theta)$$

$$= \frac{1}{4^r}\sin^2 2^r\theta\,(1-\cos^2 2^r\theta) = \frac{1}{4^r}\sin^2 2^r\theta - \frac{1}{4^r}\sin^2(2^r\theta)\cdot\cos^2(2^r\theta)$$

$$= \frac{1}{4}\sin^2(2^r\theta) - \frac{(2\sin(2^r\theta)\cdot\cos(2^r\cdot\theta))^2}{4^r\cdot 4}$$

$$= \frac{1}{4^r}\sin^2(2^r\theta) - \frac{\sin^2(2\cdot 2^r\theta)}{4^{r+1}} = \frac{1}{4^r}(\sin(2^r\theta))^2 - \frac{1}{4^{r+1}}\left(\sin(2^{r+1}\theta)\right)^2$$

Ans. C, D

QUESTION

Determine whether each of the following relations are reflexive, symmetric and transitive :

(i) Relation R in the set A = {1, 2, 3, ..., 13, 14} defined as

$R = \{(x, y) : 3x - y = 0\}$ — $3x = y$ ① Reflex ✗ ② ~~Symmt~~ : $(1,3) \in R$ But $(3,1) \notin R$

(ii) Relation R in the set N of natural numbers defined as ③ Transitive ✗

$R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$

$(1,3), (3,9) \in R$

$(1,9) \notin R$

(iii) Relation R in the set A = {1, 2, 3, 4, 5, 6} as

$R = \{(x, y) : y \text{ is divisible by } x\}$

(iv) Relation R in the set Z of all integers defined as

$R = \{(x, y) : x - y \text{ is an integer}\} = Z \times Z$ — Reflex, symm Transitive

(ii) $R = \{(1,6), (2,7), (3,8)\}$

① Reflexive ✗

② Symmt ✗

③ Transitive ✓

(iii) $R = \{(1,2)(1,1)(1,3)(1,4)(1,5)(1,6)(4,4)(2,2)(2,4)(2,6)(3,3)(3,6)(5,5)(6,6)\}$

Reflex ✓

Symmt ✗

Transitive ✓

A, B finite sets

Relation A → B

$R \subseteq A \times B$

$\Downarrow$

largest Relation from A to B $= A \times B$

Relation on A

or $R \subseteq A \times A$

$\Downarrow$

largest relation on $A = A \times A$ — Ref ✓, symm ✓, Transitive ✓

$A = \{1, 2, 3\}$

	1	2	3
1	(1,1)	(1,2)	(1,3)
2	(2,1)	(2,2)	(2,3)
3	(3,1)	(3,2)	(3,3)

QUESTION [JEE Mains 2023 (1 Feb)]

R is a relation on R

Let R be a relation on $\mathbb{R}$ given by

$R = \{(a,b) : 3a - 3b + \sqrt{7}$ is an irrational number$\}$. Then R is

$(a,b) \in R \iff 3a - 3b + \sqrt{7} \in Irr.$

(A) an equivalence relation

(B) reflexive and symmetric but not transitive

(C) reflexive and transitive but not symmetric

(D) reflexive but neither symmetric nor transitive

① Reflexive ✓

$(a,a) \in R \quad \forall\, a \in R$

since $3a - 3a + \sqrt{7} = \sqrt{7} \in Irr.$

② Symmetric : ✗

$\left(\frac{\sqrt{7}}{3}, 1\right) \in R$ But $\left(1, \sqrt{7}/3\right) \notin R$

$3 \cdot \frac{\sqrt{7}}{3} - 3 \cdot 1 + \sqrt{7} = 2\sqrt{7} - 3 \in Irr.$

$3 \cdot 1 - 3 \cdot \frac{\sqrt{7}}{3} + \sqrt{7} = 3 \notin Irr.$

$3a - 3b + \sqrt{7} \in Irr.$

③ Transitive

$\left(1, \; 2\sqrt{7}\right)$

$3 \cdot 1 - 3 \cdot 2\sqrt{7} + \sqrt{7} \in Irr$

$\left(2\sqrt{7}, \; \frac{\sqrt{7}}{3}\right)$

lie in R

$3 \cdot 2\sqrt{7} - 3 \cdot \frac{\sqrt{7}}{3} + \sqrt{7} \in Irr$

$(1, 2\sqrt{7})\left(2\sqrt{7}, \frac{\sqrt{7}}{3}\right) \in R$ But $\left(1, \frac{\sqrt{7}}{3}\right) \notin R$

$\Downarrow$

(Not Transitive)

QUESTION [JEE Mains 2022 (27 July)]

Tah 1

$(a,b) \in R_1 \quad a \cdot b \geq 0$

Let R_1 and R_2 be two relations defined on $\mathbb{R}$ by $a\,R_1\,b \Leftrightarrow ab \geq 0$ and $a\,R_2\,b \Leftrightarrow a \geq b$. Then

(A) R_1 is an equivalence relation but not R_2

(B) R_2 is an equivalence relation but not R_1

(C) both R_1 and R_2 are equivalence relations

$(a,b) \in R \quad a \geq b.$

(D) neither R_1 nor R_2 is an equivalence relation

QUESTION [JEE Mains 2023 (31 Jan)]

KCLS

Reflexive : ✓

Among the relations

$S = \left\{(a,b): a,b \in \mathbb{R} - \{0\}, 2 + \frac{a}{b} > 0\right\}$ and $T = \left\{(ab): ab \in \mathbb{R} a^2 - b^2 \in \mathbb{Z}\right\}$

$(a,b) \in S \Leftrightarrow 2 + \frac{a}{b} > 0 \quad a, b \in R - \{0\}.$

(A) S is transitive but T is not

(B) both S and T are symmetric

(C) neither S nor T is transitive

(D) T is symmetric but S is not

(1) Symmt :

$(-2, 4) \in S$ But $(4, -2) \notin S$

$2 + \frac{-2}{4} = \frac{3}{2} > 0 \qquad 2 + \frac{4}{-2} = 0 \not> 0$

(2) Transitive : lie in S $\left(\frac{4}{\ }, \frac{3}{\ }\right) - 2 + \frac{4}{3} > 0$, $\left(\frac{3}{\ }, \frac{-2}{\ }\right) - 2 + \frac{3}{-2} > 0$

$(4,3)(3,-2) \in S$ But $(4,-2) \notin S \Rightarrow$ Not Transitive

$a, b \in R$

$(a, b) \in T \Rightarrow a^2 - b^2 \in Z$

① Symmt: If $(a, b) \in T$ then $a^2 - b^2 \in Z$

$-(b^2 - a^2) \in Z$

$b^2 - a^2 \in Z$ — $(b, a) \in T$

② Transitive: let $(a, b)\ (b, c) \in T$

$a^2 - b^2 \in Z$

$b^2 - c^2 \in Z$

$a^2 - b^2 + b^2 - c^2 \in Z$

$a^2 - c^2 \in Z$

$\Downarrow$

$(a, c) \in T$

QUESTION [JEE Mains 2023 (24 Jan)]

(KCLS)

$a, b \in I$

The relation $R = \{(a, b) : \gcd(a, b) = 1, 2a \neq b, a, b \in \mathbb{Z}\}$ is :

$(a, b) \in R \Rightarrow \gcd(a, b) = 1, 2a \neq b$

(A) reflexive but not symmetric

(B) transitive but not reflexive

(C) symmetric but not transitive

(D) neither symmetric nor transitive

① Reflexive ✗ $(2, 2) \notin R$ $\gcd(2, 2) = 2$.

② Symmt: $(2, 1) \in R$ $\gcd(2, 1) = 1$

But $(1, 2) \notin R$ $\because 2 \cdot 1 = 2$

i.e $2a = b$.

$\Downarrow$

Not symmt.

③ Transitive: ✗

$(2, 3)$, $(3, 4)$ $\in R$ But $(2, 4) \notin R$.

QUESTION [JEE Mains 2023 (29 Jan)]

(KCLS) $a, b \in N$

Let R be a relation defined on $\mathbb{N}$ as a R b if 2a + 3b is a multiple of 5, $a, b \in \mathbb{N}$. Then R is

$(a, b) \in R \iff 2a+3b$ is a multiple of 5

(A) an equivalence relation

(B) non reflexive

(C) symmetric but not transitive

(D) transitive but not symmetric

① Reflexive: Yes

$(a, a) \in R \ \forall a \in N$ since $2a+3a=5a$ is a multiple of 5.

② Symmteric: Yes

Let $(a, b) \in R$ $\quad 2a+3b=5\lambda.$

let $\quad 3a+2b=x.$

$$5a+5b=5\lambda+x$$

$$x=5(a+b-\lambda)$$

x is a multiple of 5.

3a+2b is a multiple of 5

$\Downarrow$

$(b, a) \in R$

(3) Transitive. Yes.

let (a,b) & $(b,c) \in R$

$$2a+3b=5\lambda$$

$$2b+3c=5\mu$$

$$2a+5b+3c=5(\lambda+\mu)$$

$$2a+3c=5(\lambda+\mu-b)$$ multiple of 5.

$$\Downarrow$$

$$(a,c) \in R$$

QUESTION [JEE Mains 2024 (29 Jan)]

If R is the smallest equivalence relation on the set {1, 2, 3, 4} such that {(1, 2), (1, 3)} ⊂ R then the number of elements in R is

$\{1,2,3,4\}$ $\{(1,2)(1,3)\} \subset R$

(A) 15

(B) 10

(C) 12

(D) 8

$R = \{(1,2)(1,3), (1,1)(2,2)(3,3)$
$(4,4), (2,1), (3,1)$
$(3,2), (2,3)\}$

(Transitive)

$(3,1)(1,2) \Rightarrow (3,2) \in R$

	1	2	3	4
1	(1,1)	(1,2)	(1,3)	(1,4)
2	(2,1)	(2,2)	(2,3)	(2,4)
3	(3,1)	(3,2)	(3,3)	(3,4)
4	(4,1)	(4,2)	(4,3)	(4,4)

Ans. B

QUESTION [JEE Mains 2023 (8 April)]

TAH 2

Let $A = \{0, 3, 4, 6, 7, 8, 9, 10\}$ and R be the relation defined on A such that $R = \{(x, y) \in A \times A : x - y$ is odd positive integer or $x - y = 2\}$. The minimum number of elements that must be added to the relation R, so that it is a symmetric relation, is equal to ________

$(x, y) \in R$ if $x - y =$ odd +ve Integer

$x - y = 2$.

Ans. 19

QUESTION [JEE Mains 2024 (31 Jan)]

Let A = {1, 2, 3, ..., 100}. Let R be a relation on A defined by $(x, y) \in R$ if and only if $2x = 3y$. Let R_1 be a symmetric relation on A such that $R \subset R_1$ and the number of elements in R_1 is n. Then, the minimum value of n is

$x, y \in \{1, 2, \ldots 100\}$, $(x, y) \in R \Rightarrow 2x = 3y$

$R \subset R_1$

$x = \frac{3}{2}y$

$y = 2, 4, 6, 8, \ldots, 66.$

$x = 3, 6, 9, 12, \ldots, 99$

$1 \le \frac{3}{2}y \le 100.$

33 elements.

$\frac{2}{3} \le y \le \frac{200}{3} = 66.6\ldots$

$R = \{\overbrace{(3,2)(6,4)(9,6)(12,8)\ldots(99,66)}^{33\text{ elements}}\}$

$R \subset R_1 \Rightarrow R_1 = \{(3,2)(6,4)(9,6)\ldots(99,66)(2,3)(4,6)\ldots(66,99)\}$

symm Relation

$\Downarrow$

$n(R_1) = 66.$

Ans. 66

QUESTION [JEE Mains 2024 (31 Jan)]

Tan 3

Let $A = \{1, 2, 3, 4\}$ and $R = \{(1,2), (2, 3), (1, 4)\}$ be a relation on A. Let S be the equivalence relation on A such that $R \subset S$ and the number of elements in S is n. Then, the minimum value of n is

Ans. 16

QUESTION [JEE Mains 2020]

Tany

Let R_1 and R_2 be two relations defined as follows:

$R_1 = \{(ab) \in R^2 : a^2 + b^2 \in Q\}$ and $R_2 = \{(ab) \in R^2 : a^2 + b^2 \notin Q\}$

where Q is the set of all rational numbers. Then :

(A) R_1 is transitive but R_2 is not transitive.

(B) R_1 and R_2 are both transitive.

(C) R_2 is transitive but R_1 is not transitive.

(D) Neither R_1 nor R_2 is transitive.

QUESTION [JEE Mains 2024 (29 Jan)]

Tan 5

Let R be the relation on $Z \times Z$ defined by $(a, b)\ R\ (c, d)$ if and only if $ad - bc$ is divisible by 5. Then R is

(A) Reflexive and transitive but not symmetric

(B) Reflexive and symmetric but not transitive

(C) Reflexive but neither symmetric nor transitive

(D) Reflexive, symmetric and transitive

Ans. B

QUESTION [JEE Mains 2024 (1 Feb)]

Tan 6

Consider the relations R_1 and R_2 defined as $a R_1 b \Leftrightarrow a^2 + b^2 = 1$ for all $a, b \in R$ and $(a, b) R_2 (c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in N \times N$. Then:

(A) R_1 and R_2 both are equivalence relations

(B) Only R_1 is an equivalence relation

(C) Only R_2 is an equivalence relation

(D) Neither R_1 nor R_2 is an equivalence relation

Ans. C

$A = \{1, 2, 3, - - , n\}$

	1	2	3 - - -	n
1	$(1,1)$	$(1,2)$	$(1,3)$ - - -	$(1,n)$
2	$(2,1)$	$(2,2)$	$(2,3)$ - - -	$(2,n)$
3	$(3,1)$	$(3,2)$	$(3,3)$ - - -	$(3,n)$
⋮	⋮	⋮	⋮	⋮
n	$(n,1)$	$(n,2)$	$(n,3)$ - - -	(n,n)

$\frac{n^2-n}{2}$

n elements.

$\frac{n^2-n}{2}$

for Reflexive relation: $\underbrace{1\times1\times - - \times 1}_{\text{diagonal elements}} \times \underbrace{\overset{\text{Above diagonal}}{2}\times2\times - - \times2}_{\frac{n^2-n}{2}\text{ times}} \times \overbrace{\underbrace{(2\times2\times - - \times2)}_{\frac{n^2-n}{2}\text{ times.}}}^{\text{below diagonal}}$

$\Rightarrow$ No: of Reflexive Relations

$= 2^{\frac{n^2-n}{2}} \cdot 2^{\frac{n^2-n}{2}} = 2^{n^2-n}.$

$A = \{1, 2, 3, \ldots, n\}$

	1	2	3	---	n
1	(1,1)	(1,2)	(1,3)	---	(1,n)
2	(2,1)	(2,2)	(2,3)	---	(2,n)
3	(3,1)	(3,2)	(3,3)	---	(3,n)
⋮	⋮	⋮	⋮		⋮
n	(n,1)	(n,2)	(n,3)	---	(n,n)

$\frac{n^2-n}{2}$

n elements.

Married couples
$(1,2)\ (2,1) \to 2$
$(1,3)\ (3,1) \to 2$
⋮
$(n, n-1),\ (n-1, n) \to 2$

Bachelors: $(1,1)\ (2,2) \ldots (n,n)$

for symmetric relation: $\underbrace{\overbrace{2 \times 2 \times \ldots \times 2}^{\frac{n^2-n}{2}\text{ times}}}_{\text{Married couples}} \cdot \times \underbrace{\overbrace{2 \times 2 \times \ldots \times 2}^{n\text{ times}}}_{\text{Bachelors}}$

No: of Symmt Relations
$= 2^{\frac{n^2-n}{2}} \cdot 2^n = 2^{\frac{n^2-n}{2}+n} = 2^{\frac{n^2+n}{2}} = 2^{\frac{n(n+1)}{2}}$

Number of Reflexive & Symmetric Relations

If $|A| = n$ then

(1) Number of reflexive relations on $A = 2^{n^2-n}$

(2) Number of symmetric relations on $A = 2^{\frac{n^2+n}{2}}$

Number of transitive relations on a set A

- If $n(A) = 1$ then Number of transitive relations = 2
- If $n(A) = 2$ then Number of transitive relations = 13
- If $n(A) = 3$ then Number of transitive relations = 171
- If $n(A) = 4$ then Number of transitive relations = 3994

$n(A) = 1$

(1) $A = \{a\}$

$A \times A = \{(a,a)\}$

$R_1 = \{(a,a)\}$

$R_2 = \phi$

Transitive.

(2) $n(A) = 2$

$A = \{a, b\}$

$A \times A = \{(a,a)(a,b)(b,b)(b,a)\}$.

Total no: of Relations = 16.

$R_1 = \{(a,b)(b,a)\}$

$R_2 = \{(a,b),(b,a),(b,b)\}$

$R_3 = \{(a,b),(b,a),(a,a)\}$

not Transitive

No: of Transitive Relations $= 16 - 3 = \underline{\underline{13}}$.

QUESTION [JEE Mains 2024 (30 Jan)]

$\{(1,1)(2,2)(3,3)(4,4)$
$(1,2)(2,1)\}$

The number of symmetric relations defined on the set {1, 2, 3, 4} which are not reflexive is ________

$A = \{1,2,3,4\}$

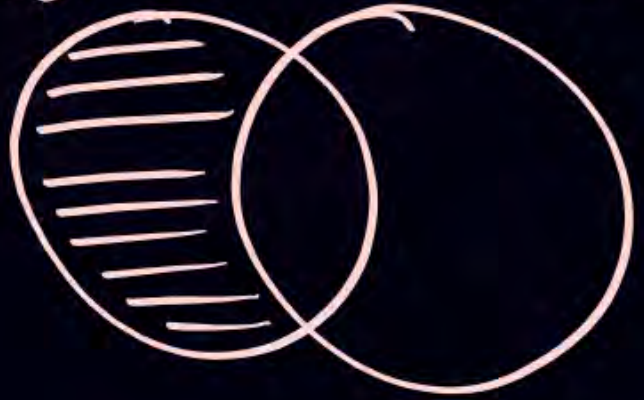
$n(S) =$ no: of Symmt Relations

$\;\;\;\;\;\;\;\;\;\;\;\;\;\;\;\; \| $

$2^{\frac{n(n+1)}{2}} = 2^{\frac{4\cdot 5}{2}} = 2^{10} = 1024.$

Symmt Relations which are not Reflexive $=$ Symmt Relations $-$ Relations which are Reflexive as well as symmt

Symm Reflexive



Now we want Relations which are Reflexive as well as symmt

	1	2	3	4
1	(1,1)	(1,2)	(1,3)	(1,4)
2	(2,1)	(2,2)	(2,3)	(2,4)
3	(3,1)	(3,2)	(3,3)	(3,4)
4	(4,1)	(4,2)	(4,3)	(4,4)

$n(S \cap T)$
$\|$

$\underbrace{1\times1\times1\times1}_{\text{diagonal.}}\times2\times2\times2\times2\times2\times2 = 2^6 = 64 =$ no: of Reflex as well as Symmt Relations

Ans: $1024 - 2^6 = 1024 - 64$

$= 960$

Sabse Important Baat Yaad Rahe

Sabhi Class Illustrations Retry Karnay hai...

Today's KTK

$$\text{No Selection} \xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}} \text{Selection with good Rank}$$

Class illustrations

Module,DPP

KTK,TAH CHALLENGER

QUESTION [JEE Mains 2023 (15 April)]

(KTK 1)

Let $A = \{1, 2, 3, 4\}$ and R be a relation on the set AA defined by
$R = \{((a, b), (c, d)) : 2a + 3b = 4c + 5d\}$.
Then the number of elements in R is ________

Ans. 6

QUESTION [JEE Mains 2018 (15 April)]

(KTK 2)

Consider the following two binary relations on the set $A = \{a, b, c\}$:

$R_1 = \{(c, a), (b, b), (a, c), (c, c), (b, c), (a, a)\}$ and

$R_2 = \{(a, b), (b, a), (c, c), (c, a), (a, a), (b, b), (a, c))$.

Then:

A) both R_1 and R_2 are not symmetric.

B) R_1 is not symmetric but it is transitive.

C) R_2 is symmetric but it is not transitive.

D) both R_1 and R_2 are transitive.

Ans. C

QUESTION [JEE Mains 2022 (28 June)]

(KTK 3)

Let R_1 and R_2 be relations on the set (1, 2, ..., 50) such that
$R_1 = \{(p, p^n) : p$ is a prime and $n \geq 0$ is an integer) and
$R_2 = \{(p, p^n) : p$ is a prime and $n = 0$ or 1).
Then, the number of elements in $R_1 - R_2$ is ______

Ans. 8

QUESTION [JEE Mains 2020 (2 Sept)]

(KTK 4)

If $R = \{(x, y) : x, y \in Z, x^2 + 3y^2 \leq 8\}$ is a relation on the set of integers Z, then the domain of R^{-1} is:

(A) $\{0, 1\}$

(B) $\{-2, -1, 1, 2\}$

(C) $\{-1, 0, 1\}$

(D) $\{-2, -1, 0, 1, 2\}$

Ans. C

QUESTION [JEE Mains 2023 (11 April)]

KTK 5

Let $A = \{1,3, 4, 6, 9\}$ and $B = \{2, 4, 5, 8, 10)$. Let R be a relation defined on $A \times B$ such that $R = \{((a_1, b_1), (a_2, b_2)) : a_1 \leq b_2$ and $b_1 \leq a_2)$. Then the number of elements in the set R is :

(A) 180

(B) 26

(C) 52

(D) 160

Ans. D

Homework from Module

Chapter: SETS

Prarambh: COMPLETE

Prabal : COMPLETE

(Revision Practice Problems)

QUESTION [WB JEE 2022]

(RPP 1)

If a, b are odd integers, then the roots of the equation $2ax^2 + (2a + b)x + b = 0, a \neq 0$ are

(A) rational

(B) irrational

(C) non-real

(D) equal

Ans. A

QUESTION [BITSAT 2023]

(RPP 2)

If A, B, C ? $[0, \pi]$ and A, B, C are in A.P., then $\dfrac{\sin A+\sin C}{\cos A+\cos C}$ is equal to

(A) $\sin B$

(B) $\cos B$

(C) $\cot B$

(D) $\tan B$

Ans. D

QUESTION [VITEEE 2022]

(RPP 3)

The roots of the equation $\cos x + \sqrt{3}\sin x = 2\cos 2x$, are

(A) $-2n\pi + \frac{\pi}{3}, n \in Z$

(B) $\frac{2n\pi}{3} + \frac{\pi}{9}, n \in Z$

(C) $2n\pi - \frac{\pi}{3}, n \in Z$

(D) $\frac{2n\pi}{3} - \frac{\pi}{9}, n \in Z$

Previous TAH

Solutions

QUESTION

If $R = \{(x, y)|^2 + y^2 \leq 4|$ where $x, y \in Z\}$ is a relation on Z then

(A) Domain of R is $\{0, 1, 2\}$

(B) Domain of R is $\{-2, -1, 0, 1, 2\}$

(C) Domain of R = range of R

(D) $n(R) = 13$

Date: / /
Page No.:

4th July

Tah-(01)

If $R=\{(x,y) \mid x^2+y^2 \le 4 \mid$ where x, y ~~and~~ $\in Z\}$ is a relation on z then -

$R=\{(x,y) \mid x^2+y^2 \le 4 \mid$

$R=\{(0,0)\ (0,1)\ (0,-1)\ (0,2)\ (0,-2)\ (-1,1)\ (1,-1)\ (1,1)$
$(-1,-1)\ (1,0)\ (-1,0)\ (2,0)\ (-2,0)\}$

So. Domain of $R=\{0,1,2,-1,-2\}$
Range of $R=\{0,1,2,-1,-2\}$
$n(R)=13$

Done by:-
Kalpana Tiwari

Options: a) Domain of R is $\{0,1,2\}$
b) Domain of R is $\{-2,-1,0,1,2\}$ ✓
c) Domain of R = range of R ✓
d) $n(R)=13$ ✓

Option (b) (c) & (d) are correct

Tah-1

Q) If $R = \{(x, y) \mid x^2 + y^2 \le 4 \mid \text{where } x, y \in z\}$ is a relation on z then.

$$R = \{(-2,0), (0,-2), (1,1), (-1,-1), (0,0)$$
$$(0,-1), (-1,1) (1,-1), (-1,0), (2,0),$$
$$(0,2), (0,1) (1,0)\}$$

Domain of $R = \{-2, -1, 0, 1, 2\}$

Rang of $R = \{-2, -1, 0, 1, 2\}$

$n(R) = 13$

Ans : (B), (c), (D)

Anima Sen

QUESTION [JEE Mains 2023 (6 April)]

Let $A = \{1, 2, 3, 4, ..., 10\}$ and $B = \{0, 1, 2, 3, 4\}$. The number of elements in the relation $R = \{(a, b) \in A \times A : 2(a - b)^2 + 3(a - b) \in B\}$ is ______

Ans. 18

TAH-2.

Let $A = \{1, 2, 3, 4, \dots 10\}$ and $B = \{0, 1, 2, 3, 4\}$. The number of elements in the relation $R = \{(a,b) \in A \times A : 2(a-b)^2 + 3(a-b) \in B\}$ is:

TAH 2

$A = \{1, 2, 3, 4, \dots 10\}$

$B = \{0, 1, 2, 3, 4\}$.

$R = \{(a,b) \in A \times A : 2(a-b)^2 + 3(a-b) \in B\}$

Now, for $(a,a) \in A$.

$(2 \times 0 + 3 \times 0) \in A \Rightarrow \{(1,1), (2,2), \dots (10,10)\}$

$\Downarrow$

10 elements

sourik Maiti
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Also, $\{(1,3), (2,4), (3,5), (4,6), (5,7), (6,8), (7,9), (8,10)\}$ will also work

$\therefore$ The total number of elements in $R = 10 + 8 = 18$.

QUESTION

The relation R defined in $A = \{1, 2, 3\}$ by a R b if $|a^2 - b^2| \le 5$. Which of the following is false?

A) $R = \{(1, 1), (2, 2), (3, 3), (2, 1), (1, 2), (2, 3), (3, 2)\}$

B) $R^{-1} = R$

C) Domain of $R = \{1, 2, 3\}$

D) Range of $R = \{5\}$

Kalpana...

Tah-(03)

The relation R defined in $A = \{1,2,3\}$ by a R b if $|a^2 - b^2| \leq 5$. Which of the following is false?

A) $R = \{(1,1),(2,2),(3,3),(2,1)(1,2)(2,3)(3,2)\}$

B) $R^{-1} = R$

C) Domain of $R = \{1,2,3\}$

D) Range of $R = \{5\}$ ✓

Soln:- $A = \{1,2,3\}$ $|a^2 - b^2| \leq 5$

- $R = \{(1,2)(2,1)(2,3)(3,2)(1,1)(2,2)(3,3)\}$

- $R^{-1} = \{(2,1)(1,2)(3,2)(2,3)(1,1)(2,2)(3,3)\} = R$

So, $R^{-1} = R$

- Domain of $R = \{1,2,3\}$

- Range of $R = \{1,2,3\}$

So the false statement is option (d)

TAH-3

The relation R defined in $A=\{1,2,3\}$ by aRb if $|a^2-b^2|\leq 5$. Which of the following is false?

(A) $R=\{(1,1),(2,2),(3,3),(2,1),(1,2),(2,3),(3,2)\}$

(B) $R^{-1}=R$

(C) Domain of $R=\{1,2,3\}$

(D) Range of $R=\{5\}$

Risha
From WB

(A) $R=\{(1,1),(2,2),(3,3),$
$(1,2),(2,1),(2,3),(3,2)\}$

(C) Domain of $R=\{1,2,3\}$

(B) $R^{-1}=R$

(D) Range of $R=\{1,2,3\}$

QUESTION

Relation R in the set of A of human beings in a town at a particular time given by

(A) R = {(x y) : x and y work at the same place}

(B) R = {(x y) : x and y live in the same locality}

(C) R = {(x y) : x is exactly 7 cm taller than y}

(D) R = {(x y) : x is wife of y}

(E) R = {(x y) : x is father of y}

Tah-(04)

Relation R in the set of A of human beings in a town at a particular time given by

(C) $R = \{(x, y) : x$ is exactly 7 cm taller than $y\}$

For reflexive: $\square$

if x is 7 cm taller than y

then y will 7 cm smaller than x

Not reflexive. ✗

Kalpana Tiwari....

Kalpana Tiwari...

for Symmetric:
x is 7 cm taller than y.
then y will not taller than x.
So, $(x,y), (y,x) \notin R$
Hence, not symmetric ✗

for Transitive
if x is 7 cm taller than y
and y is 7 cm taller than z
then z will 14 cm taller than x.

Not transitive ✗

(D) $R = \{(x,y) : x$ is wife of $y\}$

for reflexive:
$(x,x) \notin R$
bcz, x is not wife of herself

Not reflexive. ✗

for symmetric:
~~a is~~ x is wife of y i.e $(x,y) \in R$
but y is not wife of x i.e $(y,x) \notin R$

Not Symmetric. ✗

for Transitive:
x is wife of y but y can't be wife of z.
Since, it doesn't break the rule
so, it is transitive. ✓

(E) $R = \{(x,y) : x$ is father of $y\}$

for reflexive
$(x,x) \notin R$
Since, x can't be father of himself
Not reflexive

for symmetric
x is father of y, but y is son of x.
$(x,y) \in R$ but $(y,x) \notin R$
Not Symmetric

for Transitive
x is father of y and y is father of z.
but z is grandfather of x.
$(x,y)\ (y,z) \in R$ but $(x,z) \notin R$
Not Transitive.

Kalpana Tiwarii...

JAH-4

Relation R in the set of A of human beings in a town at a particular time given by —

(A) $R = \{(x, y) : x$ and y work at the same place$\}$

(B) $R = \{(x, y) : x$ and y live in the same locality$\}$

(C) $R = \{(x, y) : x$ is exactly 7 cm taller than $y\}$

(D) $R = \{(x, y) : x$ is wife of $y\}$

(E) $R = \{(x, y) : x$ is father of $y\}$

Risha From WB

	Symmetric	Transitive	Reflexive
(A) R	✓	✓	✓
(B) R	✓	✓	✓
(C) R	×	×	×
(D) R	×	✓	×
(E) R	×	×	×

(Solution to KTK)

QUESTION

Paragraph

If $A_0 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ and $B_0 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

$B_n = \text{adj}(B_{n-1}), n \in N$ and I is an identity matrix of order 3 then answer the following questions.

det. $(A_0 + A_0^2 B_0^2 + A_0^3 + A_0^4 B_0^4 + \ldots\ldots$ 10 terms) is equal to

(A) 1000

(B) -800

(C) 0

(D) -8000

Ans. C

KTK - I

Paragraph—

(1) if $A_0 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ and $B_0 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

$B_n = adj(B_{n-1}), n \in N$

$\det(A_0 + A_0^2 B_0^2 + A_0^3 + A_0^4 B_0^4 + \ldots 10 \text{ terms}) = ?$

$B_0^2 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix} \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

so now,

$\det(A_0 + A_0^2 B_0^2 + A_0^3 + A_0^4 B_0^4 + \ldots 10 \text{ terms})$

$= \det(A_0 + A_0^2 + A_0^3 + A_0^4 + \ldots A_0^{10})$

$A_0^2 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$

$A_0^2 = A_0$

so $\det(\underbrace{A_0 + A_0 + A_0 + \ldots A_0}_{10 \text{ times}})$

$\det(10A_0)$

$10^3 |A_0|$

$\because |A_0| = \begin{vmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{vmatrix} = 2(-9+8) + 2(3-4) - 4(2-3)$

$= 4 - 4$

$= 0$

so $\boxed{Ans = 0}$

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Mr. Anoop Mishra (UP)
Mrs. smita mishra

KTK-IA

If $A_0 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ and $B_0 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

$B_n = adj(B_{n-1}), n \in N$ and I is an identity matrix of order 3 then answer the following questions—

KTK I A

$\det(A_0 + A_0^2 B_0^2 + A_0^3 + A_0^4 B_0^4 + \ldots 10 \text{ terms})$ is equal to—

$\Rightarrow A_0^2 = A_0 \cdot A_0 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix} = A_0$

$\therefore A_0^2 = A_0$

$\therefore A_0^3 = A_0^2 \cdot A_0 = A_0 \cdot A_0 = A_0^2 = A_0$

$\therefore A_0^4 = A_0^2 \cdot A_0^2 = A_0 \cdot A_0 = A_0$

$\vdots$

$\therefore A_0^{10} = A_0$

Also, $B_0^2 = B_0 \cdot B_0 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix} \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$\therefore B_0^2 = I$

$\therefore B_0^4 = B_0^2 \cdot B_0^2 = I \cdot I = I^2 = I$

$\vdots$

$B_0^{10} = I$

Now, $\det(A_0 + A_0^2 B_0^2 + A_0^3 + A_0^4 B_0^4 + \ldots\ldots + A_0^9 + A_0^{10} B_0^{10})$

$= \det(A_0 + A_0 + A_0 + \ldots \text{ upto } 10 \text{ terms})$

$= \det(10A_0)$

$= 10^3 \det(A_0) = 10^3 \begin{vmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{vmatrix}$ $\begin{matrix} C_1' \to C_1 + C_2 \\ C_3' \to C_3 - 2C_2 \end{matrix}$

$= 10^3 \begin{vmatrix} 0 & -2 & 0 \\ 2 & 3 & -2 \\ -1 & -2 & 1 \end{vmatrix} = 10^3 \cdot (-2) \begin{vmatrix} 2 & -2 \\ -1 & 1 \end{vmatrix}$

$= \underline{(0)}$ Ans

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QUESTION

Paragraph

If $A_0 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ and $B_0 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

$B_n = \text{adj}(B_{n-1}), n \in N$ and I is an identity matrix of order 3 then answer the following questions.

$B_1 + B_2 + ... + B_{49}$ is equal to

(A) B_0

(B) $7B_0$

(C) $49B_0$

(D) $49I$

Ans. C

Date ______
Page ______

(II) $B_1 + B_2 + \dots + B_{49} = ?$

$$B_0 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix} \qquad \text{cofator} = \begin{bmatrix} -4 & +1 & 4 \\ -3 & 0 & 4 \\ -3 & 1 & 3 \end{bmatrix}$$

$$\text{adj } B_0 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$$

$\text{adj } B_0 = B_0$ $\quad$ Illy $\text{adj } B_1 = B_1 = \text{adj} B_0 = B_0$

So $\quad B_1 + B_2 + B_3 + \dots + B_{49}$

$= B_0 + B_0 + \dots + B_0$

$= 49 B_0$ Ans

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KTK-1B. $B_1 + B_2 + \dots + B_{49}$ is equal to —

KTK 1 B

$\rightarrow$ $$B_0 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$$

$$\text{adj}(B_0) = \begin{bmatrix} -4 & 1 & 4 \\ -3 & 0 & 4 \\ -3 & 1 & 3 \end{bmatrix}^T = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$$

$$\boxed{\text{adj}(B_0) = B_0}$$

Now, $B_1 = \text{adj } B_0 = B_0$

$B_2 = \text{adj}(\text{adj } B_0) = B_0$

$B_3 = \text{adj}(\text{adj}(\text{adj } B_0)) = B_0$

$\vdots$

$B_{49} = \text{adj}(\text{adj} \dots \text{adj}(B_0)) = B_0$

$\therefore B_1 + B_2 + \dots + B_{49}$

$$= \underbrace{B_0 + B_0 + \dots + B_0}_{49 \text{ times}}$$

$= (49 B_0)$ Ans

QUESTION

Paragraph

If $A_0 = \begin{bmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{bmatrix}$ and $B_0 = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

$B_n = \text{adj}(B_{n-1})$, $n \in N$ and I is an identity matrix of order 3 then answer the following questions.

For a variable matrix X the equation $\overset{3\times3}{A_0}\underset{3\times3}{X} = \overset{3\times3}{B_0}$ will have

(A) unique solution

(B) infinite solution

(C) finitely many solution

(D) no solution

$A_0 X = B_0.$

$|A_0 X| = |B_0|$

$|A_0||X| = |B_0|$

$0 \cdot |X| = 1$

$0 = 1$ (N.P)

Ans. D

KTK-1C For a variable matrix X. the eq^n $A_0X = B_0$ will have —

KTK 1 C

$\rightarrow$ $\det(A_0) = \begin{vmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{vmatrix} = 0$

Now, $\operatorname{adj}(A_0) = \begin{bmatrix} -1 & 1 & -1 \\ 2 & -2 & 2 \\ 4 & -4 & 4 \end{bmatrix}^T = \begin{bmatrix} -1 & 2 & 4 \\ 1 & -2 & -4 \\ -1 & 2 & 4 \end{bmatrix}$

Now, $\operatorname{adj}(A_0).B_0 = \begin{bmatrix} -1 & 2 & 4 \\ 1 & -2 & -4 \\ -1 & 2 & 4 \end{bmatrix} \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$

$= \begin{bmatrix} 22 & 19 & 17 \\ -22 & -19 & -17 \\ 22 & 19 & 17 \end{bmatrix} \neq 0$

$|A_0| = 0 \Rightarrow (\operatorname{adj} A_0).B_0 \neq 0$

$\Downarrow$

$\boxed{\text{No solution}}$ Ans (D).

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QUESTION

Consider a system of linear equation $3x + y - z = 0$, $x - \frac{py}{4} + z = 2$ and $2x - y + 2z = q$ where $p, q \in I$ and $p, q \in [1, 10]$, then identify the correct statement(s).

List-I		List-II	
(I)	Number of ordered pairs (p, q) for which system of equation has unique solution is	(P)	1
(II)	Number of ordered pairs (p, q) for which system of equation has no solution is	(Q)	9
(III)	Number of ordered pairs (p, q) for which system of equation has infinite solution is	(R)	91
(IV)	Number of ordered pairs (p, q) for which system of equation has atleast one solution is	(S)	90

QUESTION

Which one of the following option is correct?

(A) I → P, II → R, III → S, IV → R

(B) I → Q, II → S, III → P, IV → R

(C) I → S, II → Q, III → P, IV → R

(D) I → Q, II → P, III → S , IV → P

Ans. C

KTK 2 .

Consider a system of linear eq^n $3x+y-z=0$, $x-\frac{py}{4}+z=2$ and $2x-y+2z=q$. Where $p,q \in I$ and $p,q \in [1,10]$, then identify the correct statement(s).

KTK 2 (PART 1)

(I) Number of ordered pairs (p,q) for which system of eq^n has unique solution is -

$\Rightarrow$

$3x+y-z=0$ — (i)

$x-\frac{py}{4}+z=2$ — (ii)

$2x-y+2z=q$ — (iii)

(ii) $\times 2$ − (iii) $\Rightarrow$

$$\boxed{\left(1-\frac{p}{2}\right)y=(4-q)} \text{ — (iv)}$$

for unique sol^n $\rightarrow$

$$1-\frac{p}{2}\neq 0$$

$$\boxed{p\neq 2}$$

$\therefore$ p should not equal to 2. $p\in[1,10]-\{2\}$, $n(p)=9$

$\therefore$ q should be $\{1,2,3,\dots 10\}$. $n(q)=10$

$\therefore$ No. of ordered pair $=9\times 10=(90) \longrightarrow$ (S).

(II) Number of ordered pairs (p,q) for which system of eq^n has no solution is -

for no solution $\rightarrow$

$1-\frac{p}{2}=0$; $4-q\neq 0$

$\boxed{p=2}$; $\boxed{\therefore q\neq 4}$

$\therefore$ p should be only '2', $n(p)=1$

$\therefore$ q should not equal to '4' $\rightarrow q\in[1,10]-\{4\}$, $n(q)=9$.

$\therefore$ No: of ordered pair $=1\times 9=(9) \longrightarrow$ (Q).

(III) Number of ordered pairs (p,q) for which system of eq^n has infinite solution is -

for infinite solution $\rightarrow$

$1-\frac{p}{2}=0$; $4-q=0$

$\boxed{p=2}$; $\boxed{\therefore q=4}$

$\therefore$ 'p' should be only '2'. $n(p)=1$

$\therefore$ 'q' should be only '4'. $n(q)=1$

$\therefore$ No: of ordered pair $=1\times 1=(1) \longrightarrow$ (P).

(IV) Numbers of ordered pairs (p,q) for which system of eq^n has atleast one solution is -

$\rightarrow$ atleast one solution = possiblities of unique solution + possiblities of infinite solution

KTK 2 (PART 2)

$$=(90+1)=(91) \longrightarrow (R).$$

$$\boxed{I\rightarrow S,\ II\rightarrow Q,\ III\rightarrow P,\ IV\rightarrow R}$$ (C) Ans.

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(Solution to RPP)

QUESTION

RPP 1

Let $a, b, c, d \in R$; $a + b + c + d = 10$, the minimum value of $a^2 \cot 9^\circ + b^2 \cot 27^\circ + c^2 \cot 63^\circ + d^2 \cot 81^\circ$ is $\sqrt{n}$; $n \in N$, then 'n' is

(A) even

(B) odd

(C) prime

(D) divisible by 5

Ans. B, D

RPP-1. Let $a, b, c, d \in R$; $a+b+c+d = 10$. the minimum value of $a^2\cot 9° + b^2\cot 27° + c^2\cot 63° + d^2\cot 81°$ is $\sqrt{n}$; $n \in N$. then 'n' is—

$\Rightarrow a^2\cot 9° + b^2\cot 27° + c^2\cot(90° - 27°) + d^2\cot(90° - 9°)$

$= a^2\cot 9° + b^2\cot 27° + c^2\tan 27° + d^2\tan 9°. = R.$

RPP 1

Now, $$\frac{a^2\cot 9° + b^2\cot 27° + c^2\tan 27° + d^2\tan 9°}{4} \geq (a^2b^2c^2d^2)^{1/4}$$

$$\Rightarrow \frac{R}{4} \geq (abcd)^{1/2}. \quad \text{—(i)}$$

Now, $$\frac{a+b+c+d}{4} \geq (abcd)^{1/4}$$

$$\Rightarrow \frac{10}{4} \geq (abcd)^{1/4}$$

$$\Rightarrow \frac{100}{16} \geq (abcd)^{1/2} \quad \text{—(ii)}$$

from (i) & (ii) $\Rightarrow$

$$\frac{R}{4} \geq \frac{100}{16}$$

$$\boxed{R \geq 25}$$

sourik Maiti
West Bengal

Now, we need the min. value of $R = 25$.

$\sqrt{n} = 25$

$\boxed{\therefore n = 625}$

$\therefore$ n is odd & divisible by 5. (B) & (C).

Rpp 1

Q. Let $a, b, c, d \in R$; $a+b+c+d = 10$, the minimum value of $a^2 \cot 9° + b^2 \cot 27° + c^2 \cot 63° + d^2 \cot 81°$ is $\sqrt{n}$; $n \in N$, then 'n' is.

$\longrightarrow a^2 \cot 9° + b^2 \cot 27° + c^2 \cot 63° + d^2 \cot 81°$

$\Rightarrow a^2 \tan 81° + d^2 \cot 81 + b^2 \cot 27° + c^2 \tan 27°$

using AM $\geq$ G.M

$$\frac{a^2 \tan 81° + d^2 \cot 81° + b^2 \cot 27° + c^2 \tan 27°}{4} \geq (a^2 b^2 c^2 d^2)^{1/4} \quad \text{(I)}$$

also, $\frac{a+b+c+d}{4} \geq (abcd)^{1/4}$

Given $a+b+c+d = 10$

$\Rightarrow \frac{5}{2} \cancel{\frac{10}{4}} \geq (abcd)^{1/4}$

$\Rightarrow \frac{25}{4} \geq (abcd)^{1/2} \quad \text{(II)}$

Required Value.

$$\frac{a^2 \tan 81° + d^2 \cot 81° + b^2 \cot 27° + c^2 \tan 27°}{\cancel{4}} \geq \frac{25}{\cancel{4}}$$

The minimum value $= 25$; $\sqrt{n} = 25 \Rightarrow n = 625$

Aman Kumar From Katoria, Bihar

QUESTION

RPP 2

If the number of solutions of the equation $\cos^2\left(\frac{\pi}{4}(\cos x + \sin x)\right) - \tan^2\left(x + \frac{\pi}{4}\tan^2 x\right) = 1$ in $[-2\pi, 2\pi]$ is 'k', then $\frac{3k}{25}$ equals

Ans. 0.48

QUESTION

RPP 3

Let $f_n(\theta) = \sum_{r=0}^{n} \frac{1}{4^r} \cdot \sin^4(2^r\theta)$, then

A $f_2\left(\frac{\pi}{4}\right) = \frac{\pi}{\sqrt{2}}$

B $f_3\left(\frac{\pi}{8}\right) = \frac{2+\sqrt{2}}{4}$

C $f_4\left(\frac{3\pi}{2}\right) = 1$

D $f_5(\pi) = 0$

Ans. C, D

RPP 3. Let $S_n(\theta) = \sum_{r=0}^{n} \frac{1}{4^r} \cdot \sin^4(2^r\theta)$, then - RPP 3 (PART I)

$$t_r = \frac{1}{4^r}\cdot\sin^4(2^r\theta) = \frac{1}{4^r}\cdot\sin^2(2^r\theta)\cdot\sin^2(2^r\theta)$$

$$= \frac{1}{4^r}\sin^2(2^r\theta)\left[1-\cos^2(2^r\theta)\right]$$

$$= \frac{1}{4^r}\left[\sin^2(2^r\theta) - \frac{\left[2\sin(2^r\theta)\cdot\cos(2^r\theta)\right]^2}{4}\right]$$

$$= \frac{1}{4^r}\left[\sin^2(2^r\theta) - \frac{\left(\sin(2^{r+1}\theta)\right)^2}{4}\right]$$

$$\boxed{t_r = \left[\frac{\sin^2(2^r\theta)}{4^r} - \frac{\sin^2(2^{r+1}\theta)}{4^{r+1}}\right]}$$

If $r=0$. $t_0 = \frac{\sin^2(\theta)}{1} - \frac{\sin^2(2\theta)}{4}$

If $r=1$. $t_1 = \frac{\sin^2(2\theta)}{4} - \frac{\sin^2(2^2\theta)}{4^2}$

If $r=2$. $t_2 = \frac{\sin^2(2^2\theta)}{4^2} - \frac{\sin^2(2^3\theta)}{4^3}$

$\vdots$

If $r=n$. $t_n = \frac{\sin^2(2^n\theta)}{4^n} - \frac{\sin^2(2^{n+1}\theta)}{4^{n+1}}$

$$t_0+t_1+\cdots+t_n = \sum_{r=0}^{n} t_r = \left[\sin^2\theta - \frac{\sin^2(2^{n+1}\theta)}{4^{n+1}}\right]$$

$$\therefore S_n(\theta) = \left[\sin^2\theta - \frac{\sin^2(2^{n+1}\theta)}{4^{n+1}}\right]$$

(a) $S_2\left(\frac{\pi}{4}\right) = \left[\sin^2\left(\frac{\pi}{4}\right) - \frac{\sin^2\left(2^3\cdot\frac{\pi}{4}\right)}{2^3}\right] = \frac{1}{2} - 0 = \frac{1}{2}$.

(b) $S_3\left(\frac{\pi}{8}\right) = \left[\sin^2\left(\frac{\pi}{8}\right) - \frac{\sin^2\left(2^4\cdot\frac{\pi}{8}\right)}{2^4}\right] = \frac{1-\cos\frac{\pi}{4}}{2} = \frac{1-\frac{1}{\sqrt{2}}}{2}$

$= \left[\frac{\sqrt{2}-1}{2\sqrt{2}}\right]$


THANK
YOU

PRAYAS

JEE 2025

Lecture-04 | Mathematics

Relation & Functions

By- Ashish Agarwal Sir

Topics *to be covered*

1. Introduction to functions
2. Domain Range

Discussion of Homework of Previous Class

QUESTION [JEE Mains 2020]

TAH 4

Let R_1 and R_2 be two relations defined as follows:

$R_1 = \{(ab) \in R^2 : a^2 + b^2 \in Q\}$ and $R_2 = \{(ab) \in R^2 : a^2 + b^2 \notin Q\}$

where Q is the set of all rational numbers. Then :

$(a,b) \in R_1 \Rightarrow a^2+b^2 \in Q$

(A) R_1 is transitive but R_2 is not transitive.

(B) R_1 and R_2 are both transitive.

(C) R_2 is transitive but R_1 is not transitive.

(D) Neither R_1 nor R_2 is transitive.

belong to R_1: $(\underline{\sqrt{2}-1}, \sqrt{2}+1)$, $(\sqrt{2}+1, \underline{\sqrt{2}-1})$

$(\sqrt{2}-1)^2+(\sqrt{2}+1)^2 = 6 \in Q$

But $(\sqrt{2}-1, \sqrt{2}-1) \notin R$ since

$(\sqrt{2}-1)^2+(\sqrt{2}-1)^2 = 3-4\sqrt{2} \notin Q$

$\Downarrow$

R_1 is not transitive.

$(a, b) \in R_2$ $\quad a^2 + b^2 \notin Q$

$\left.\begin{matrix} (\sqrt{2}-1, & 3) \\ (3 & \sqrt{2}+1) \end{matrix}\right\}$ lie in R_2 $\quad$ But $(\sqrt{2}-1, \sqrt{2}+1) \notin R_2$

$\Downarrow$

R_2 is not transitive

QUESTION [JEE Mains 2024 (29 Jan)]

TAH 5

Let R be the relation on Z × Z defined by (a, b) R (c, d) if and only if ad – bc is divisible by 5. Then R is

$(c,d)\ (a,b)$

$R \subseteq (Z\times Z)\times(Z\times Z)$

(A) Reflexive and transitive but not symmetric

$((a,b),(c,d)) \in R$

$\Updownarrow$

$ad-bc$ is divisible by 5.

(B) Reflexive and symmetric but not transitive

(C) Reflexive but neither symmetric nor transitive

Reflexivity.

(D) Reflexive, symmetric and transitive

clearly $((a,b),(a,b)) \in R \ \forall\ a,b \in Z$.

b'coz $ab-ba=0$ is divisible by 5.

Symmetry: let $((a,b),(c,d)) \in R$

$\Rightarrow ad-bc=5\lambda \Rightarrow bc-ad=5(-\lambda) \Rightarrow ((c,d),(a,b)) \in R$

Ans. B

Transitivity

$$\left.\begin{matrix}((1,2)\ ,\ (5,5)) \\ ((5,5)\ \ (3,8))\end{matrix}\right\} \in R \quad \text{But } ((1,2),(3,8)) \notin R$$

$1\cdot 8 - 2\cdot 3 = 2$ not divisible by 5

$\Downarrow$

Not transitive.

QUESTION [JEE Mains 2023 (6 April)]

TAH 2 Lec-03

Let $A = \{1, 2, 3, 4, \ldots, 10\}$ and $B = \{0, 1, 2, 3, 4\}$. The number of elements in the relation $R = \{(a, b) \in A \times A : 2(a - b)^2 + 3(a - b) \in B\}$ is ______

$(a,b) \in R$ if $2(a-b)^2 + 3(a-b) \in B$ where $a, b \in A$ $R: A \to A$

$(a-b)(2(a-b)+3) \in B = \{0, 1, 2, 3, 4\}$.

Case (I) $(a-b)(2(a-b)+3) = 0$

$a-b=0 \to a=b \Rightarrow (1,1)(2,2)(3,3) - - - (10,10)$.

Case (II) $(a-b)(2(a-b)+3) = 1$

$a-b=1$ & $2(a-b)+3=1$ Not possible

or $a-b=-1$ & $2(a-b)+3=-1$

$2t^2+3t=1$

$2t^2+3t-1=0$

$t = \frac{3 \pm \sqrt{17}}{4}$

Case (III) $(a-b)(2(a-b)+3) = 2$

$a-b=2$ $2(a-b)+3=1$

$a-b=1$ $2(a-b)+3=2$ N.P

$a-b=-1$ $2(a-b)+3=-2$

$a-b=-2$ $2(a-b)+3=-1$

$a-b=-2$ — $(1,3)\ (2,4)\ (3,5) \ldots,(7,9)\ (8,10)$ — 8 elements

Case III $(a-b)(2(a-b)+3)=3$

M1

$a-b=3 \quad 2(a-b)+3=1$

$a-b=1 \quad 2(a-b)+3=3$

$(a-b)=-1 \quad 2(a-b)+3=-3$

$(a-b)=-3 \quad 2(a-b)+3=-1$

→ N.P

M2

$2(a-b)^2+3(a-b)=3$

$2t^2+3t-3=0$

$a-b=t=\dfrac{-3\pm\sqrt{9+24}}{4}$

$\Downarrow$

N.P

Case IV

$2(a-b)^2+3(a-b)=4$

$2t^2+3t-4=0$

$a-b=t=\dfrac{-3\pm\sqrt{9+32}}{2}$

$\Downarrow$

(N.P)

Pratham Vishwak...

3 hours ago

sir pls check rpp 1 solution there we can't apply am gm inequality ....

2

Pratham Vishwak...

3 hours ago

sir rpp 1 ka solution glt hai usme hm a,b,c,d me am gm inequality nhi lga skte haii ... pls check k

1

QUESTION

RPP 1

$$-|\bar{V}_1|\cdot|\bar{V}_2| \le \bar{V}_1\cdot\bar{V}_2 \le |\bar{V}_1||\bar{V}_2|$$

$$|\bar{V}_1\cdot\bar{V}_2| \le |\bar{V}_1||\bar{V}_2|$$

$$(\bar{V}_1\cdot\bar{V}_2)^2 \le |V_1|^2|\bar{V}_2|^2$$

Let $a, b, c, d \in R$; $a + b + c + d = 10$, the minimum value of $a^2\cot 9^\circ + b^2\cot 27^\circ + c^2\cot 63^\circ + d^2\cot 81^\circ$ is $\sqrt{n}$; $n \in N$, then 'n' is

(A) even

(B) odd

(C) prime

(D) divisible by 5

$$E = a^2\cot 9^\circ + b^2\cot 27^\circ + c^2\cot 63^\circ + d^2\cot 81^\circ$$

$$\bar{V}_1 = a\sqrt{\cot 9^\circ}\, i_1 + b\sqrt{\cot 27^\circ}\, i_2 + c\sqrt{\cot 63}\, i_3 + d\sqrt{\cot 81^\circ}\, i_4$$

$$\bar{V}_2 = \sqrt{\tan 9}\, i_1 + \sqrt{\tan 27}\, i_2 + \sqrt{\tan 63^\circ}\, i_3 + \sqrt{\tan 81}\, i_4$$

$$(\bar{V}_1\cdot\bar{V}_2)^2 \le |\bar{V}_1|^2|\bar{V}_2|^2$$

$$(a+b+c+d)^2 \le E\,(\tan 9^\circ + \tan 27^\circ + \tan 63^\circ + \tan 81^\circ)$$

$$100 \le E\,(\tan 9^\circ + \cot 9^\circ + \tan 27^\circ + \cot 27^\circ)$$

Ans. B, D

$$\frac{1}{\sin 54^\circ} = \frac{1}{\cos 36^\circ}$$

$$E\cdot\left(2\operatorname{cosec}18^\circ + 2\operatorname{cosec}54^\circ\right) \geq 100$$

$$E\left(\frac{8}{\sqrt{5}-1} + \frac{8}{\sqrt{5}+1}\right) \geq 100$$

$$\cancel{8}E\,\frac{\cancel{2}\sqrt{5}}{\cancel{4}} \geq 100$$

$$4E\sqrt{5} \geq 100$$

$$E \geq \frac{25}{\sqrt{5}} = 5\sqrt{5}$$

$$E_{MIN} = 5\sqrt{5} = \sqrt{125} \Rightarrow n = 125$$

RPP-1. Let $a, b, c, d \in R$; $a+b+c+d=10$. The minimum value of $a^2\cot 9° + b^2\cot 27° + c^2\cot 63° + d^2\cot 81°$ is $\sqrt{n}$; $n \in N$, then 'n' is —

$\Rightarrow a^2\cot 9° + b^2\cot 27° + c^2\cot(90°-27°) + d^2\cot(90°-9°)$

$= a^2\cot 9° + b^2\cot 27° + c^2\tan 27° + d^2\tan 9° = R.$

RPP 1

Now, $\dfrac{a^2\cot 9° + b^2\cot 27° + c^2\tan 27° + d^2\tan 9°}{4} \geq (a^2b^2c^2d^2)^{1/4}$

$\Rightarrow \dfrac{R}{4} \geq (abcd)^{1/2}$ —— (i)

$A \geq B$

Now, $\dfrac{a+b+c+d}{4} \geq (abcd)^{1/4}$

$\not\Rightarrow A \geq C.$

$\Rightarrow \dfrac{10}{4} \geq (abcd)^{1/4}$

$C \geq B$

$\Rightarrow \dfrac{100}{16} \geq (abcd)^{1/2}$ —— (ii)

from (i) & (ii) $\Rightarrow$

$\dfrac{R}{4} \geq \dfrac{100}{16}$

$\boxed{R \geq 25}$

Now, we need the min. value of $R = 25$.

$\sqrt{n} = 25$

$\boxed{\therefore n = 625}$

$\therefore$ n is odd & divisible by 5. (B) & (C).

sourik Maiti
West Bengal

Bajrangi singh 4 hours ago

ashish sir ek doubt tha aap hi clear kr sakte ho please class me aane se pehle check kr lijiyega 1;24;57 par agr hm T me counter example (2-√3) , (2+√3),(1-2√3) le to ye transitive nhi hoga sir please ek baar check kr lijiyega it's my humble request lot's of love sir ek baar check kr lijiyega please sir please 🥺

1

Shivam Gautam 5 hours ago

sorry sir,i was wrong NO TRUTI NO TRUTI NO TRUTI

0

Shivam Gautam 5 hours ago

sb loog please check this ki kya me glt hu, kyunki sir ne bhi toh kuchh socha hoga tabhi toh symmetric btaya hai

0

Shivam Gautam 5 hours ago

sir TRUTI TRUTI TRUTI at 1;37;39 becoz (1,6) belongs to R ,but (6,1) not belons to R , so it is not symmetric relation

0

QUESTION [JEE Mains 2023 (31 Jan)]

KCLS

Reflexive : ✓

Among the relations

$S = \{(a, b) : a, b \in \mathbb{R} - \{0\}, 2 + \frac{a}{b} > 0\}$ and $T = \{(a, b) : a, b \in \mathbb{R}, a^2 - b^2 \in \mathbb{Z}\}$

(A) S is transitive but T is not

(B) both S and T are symmetric

(C) neither S nor T is transitive

(D) T is symmetric but S is not

$(a,b) \in S \Leftrightarrow 2 + \frac{a}{b} > 0 \quad a, b \in R - \{0\}.$

① Symmt:

$(-2, 4) \in S$ But $(4, -2) \notin S$

$2 + \frac{-2}{4} = \frac{3}{2} > 0 \qquad 2 + \frac{4}{-2} = 0 \not> 0$

② Transitive: lie in S $\left(\frac{4}{}, \frac{3}{}\right) - 2 + \frac{4}{3} > 0$, $\left(\frac{3}{}, \frac{-2}{}\right) - 2 + \frac{3}{-2} > 0$

$(4,3)(3,-2) \in S$ But $(4,-2) \notin S \Rightarrow$ Not Transitive

$a, b \in R$

$(a,b) \in T \Rightarrow a^2 - b^2 \in Z$

① Symmt: If $(a,b) \in T$ then $a^2 - b^2 \in Z$

$-(b^2 - a^2) \in Z$

$b^2 - a^2 \in Z - (b,a) \in T$

② Transitive: let $(a,b)\ (b,c) \in T$

$a^2 - b^2 \in Z$, $b^2 - c^2 \in Z$ $\Rightarrow a^2 - b^2 + b^2 - c^2 \in Z$

$a^2 - c^2 \in Z$

$\Downarrow$

$(a,c) \in T$

$(2 - \sqrt{3}, 2 + \sqrt{3}) \quad (2 + \sqrt{3}, 1 - 2\sqrt{3})$

$(2 - \sqrt{3})^2 - (2 + \sqrt{3})^2$

$\|$

$-8\sqrt{3}.$

QUESTION [JEE Mains 2023 (15 April)] **(KTK 1)**

Let $A = \{1, 2, 3, 4\}$ and R be a relation on the set AA defined by
$R = \{((a, b), (c, d)) : 2a + 3b = 4c + 5d\}$.
Then the number of elements in R is ________

$((a,b),(c,d)) \in R \quad 2a+3b = 4c+5d$

$4c \in \{4, 8, 12, 16\}$

$5d \in \{5, 10, 15, 20\}$

$2a \in \{2, 4, 6, 8\}$

$3b \in \{3, 6, 9, 12\}$

$2a+3b \in \{5, 8, 11, 14, 7, 10, 13, 16, 9, 12, 15, 18, 11, \boxed{14}, 17, 20\}$

$4c+5d \in \{9, 14, 19, 24, 13, 18, 23, 28, 17, 22, 27, 32, 21, 26, 31, 36\}$

for 9

$a=3 \quad c=1$
$b=1 \quad d=1$

$((3,1)(1,1))$

Ans. 6

QUESTION [JEE Mains 2022 (28 June)]

(KTK 3)

Let R_1 and R_2 be relations on the set (1, 2, ..., 50) such that
$R_1 = \{(p, p^n) : p$ is a prime and $n \geq 0$ is an integer) and
$R_2 = \{(p, p^n) : p$ is a prime and $n = 0$ or 1). $R_2 = \{(2, 2^0)\ (2, 2^1)$
Then, the number of elements in $R_1 - R_2$ is ______ $(3, 3^0)\ (3, 3^1)$

$R_1 - R_2 = \{(2, 2^2)\ (2, 2^3) \ldots (2, 2^5)$
$(3, 3^2)\ (3, 3^3)$
$(5, 5^2), (7, 7^2)\}$

(8 elements)

Ans. 8

QUESTION [JEE Mains 2023 (11 April)]

KTK 5

Let A = {1,3, 4, 6, 9} and B = {2, 4, 5, 8, 10). Let R be a relation defined on A × B such that $R = \{((a_1, b_1), (a_2, b_2)) : a_1 \leq b_2 \text{ and } b_1 \leq a_2)$. Then the number of elements in the set R is :

(A) 180

(B) 26

(C) 52

(D) 160

$R \subseteq (A \times B) \times (A \times B)$

$((a_1, b_1), (a_2, b_2)) \Rightarrow a_1 \leq b_2,\ b_1 \leq a_2$

10 ways

16 ways.

$a_1 = 1$ b_2 has 5 choices.

$a_1 = 3$ b_2 has 4 choices.

$a_1 = 4$ b_2 has 4 choices.

$a_1 = 6$ b_2 has 2 choiches.

$a_1 = 9$ b_1 has 1 choice.

a_1, b_2 can be selected in 16 way

$a_2 = 3$ b_1 has 1 choice

$a_2 = 4$ b_1 has 2 choices

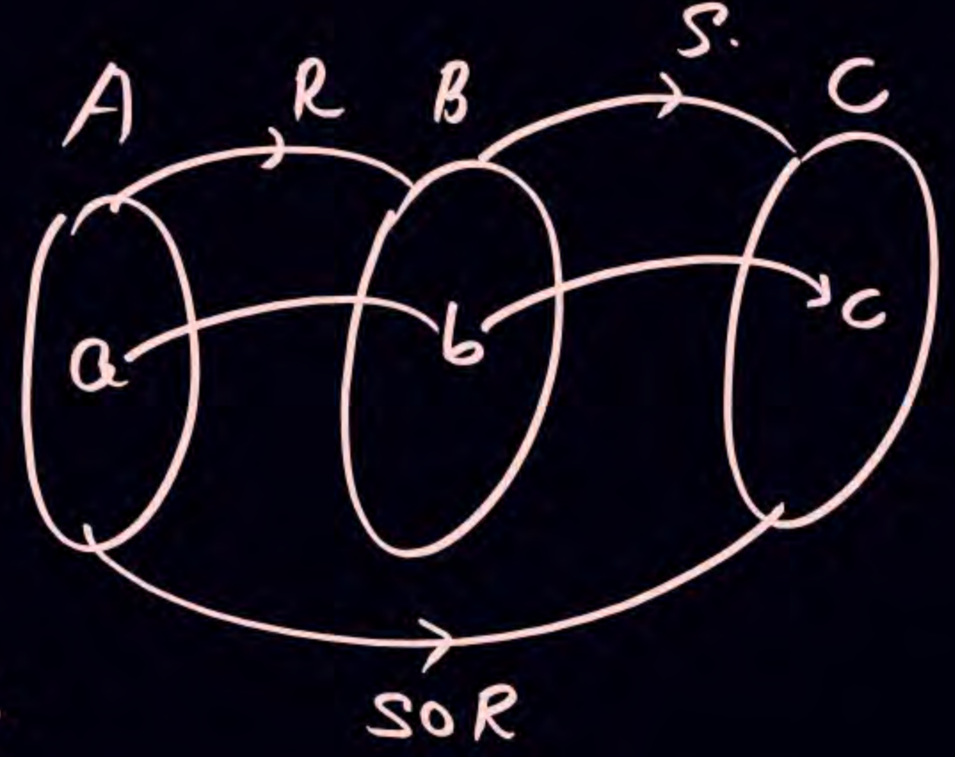
Ans. D

$R : A \to B$

$S : B \to C$

$\Downarrow$

$SoR : A \to C$



$$\left.\begin{array}{l}(a,b) \in R \\ (b,c) \in S\end{array}\right\rangle (a,c) \in SoR$$

$1 \to 2 \to 11$

$2 \to 5 \to 12, 11$

$1 \to 5 \to 12, 11$

$3 \to 8$

Ex: $A = \{1,2,3,4\}$, $B = \{2,5,6,8\}$, $C = \{10,11,12\}$

$R : A \to B \quad R = \{(1,2)\ (2,5)\ (1,5)\ (3,8)\}$.

$S : B \to C \quad S = \{(2,11)\ (5,12)\ (5,11)\}$

$SoR = \{(1,11)\ (2,11)\ (2,12), (1,12)\}$

Composite Relation

Let R and S be two relations from set A to B and B to C respectively. Then we can define a relation S o R from A to C such that $(a, c) \in S \circ R \Leftrightarrow \exists\, b \in B$ such that $(a, b) \in R$ and $(b, c) \in S$.

This relation is called the composition of R and S.

QUESTION

Let $R = \{(1,3),(2,2),(3,2)\}$ and $S = \{(2,1),(3,2),(2,3)\}$ be two relations on set $A = \{1,2,3\}$. Then $R \circ S =$

$R \circ S = \{(2,3)\ (3,2)\ (2,2)\}$.

A) $\{(1, 3), (2, 2), (3, 2), (2, 1), (2, 3)\}$

B) $\{(3, 2), (1, 3)\}$

C) $\{(2, 3), (3, 2), (2, 2)\}$

D) $\{(2, 3), (3, 2)\}$

$S^{-1} = \{(1,2)(2,3)(3,2)\}$

$R^{-1} = \{(3,1)\ (2,2)\ (2,3)\}$

$(R \circ S)^{-1} = \{(3,2)(2,3)\ (2,2)\}$

$S^{-1} \circ R^{-1} = \{(3,2)\ (2,3)\ (2,2)\}$

QUESTION

Consider three sets $A = \{1, 2, 3, 4\}$, $B = \{2, 3, 4, 5, 6\}$, $C = \{4, 5, 6, 7, 8, 9\}$ and R_1 is defined from A to B such that $R_1 = \{(x, y), 2x = y, x \in A, y \in B\}$. Similarly R_2 is defined from B to C such that

$R_2 = \{(x, y)$: 'x divides y' $x \in B$ and $y \in C\}$, then:

(i) $R_2 \text{ o } R_1$

(ii) $R_1^{-1} \text{ o } R_2^{-1}$

$R_1 = \{(1,2)\ (2,4)\ (3,6)\}$

$R_2 = \{(2,4)\ (2,6)\ (2,8)\ (3,6)\ (3,9)\ (4,4)\ (4,8)\ (5,5)\ (6,6)\}$

$\overleftarrow{R_2 \text{ o } R_1} = \{(1,4)\ (1,6)\ (1,8)\ (2,4)\ (2,8)\ (3,6)\}$

$R_1^{-1} \text{ o } R_2^{-1} = (R_2 \text{ o } R_1)^{-1} = \{(4,1)\ (6,1)\ (8,1)\ (4,2)\ (8,2)\ (6,3)\}$

Equivalence class

let R be an equivalence Relation on a non empty set A then equivalence class of $a \in A$ is denoted by $[a]$ and is the set of all $x \in A$ s.t $(x, a) \in R$.

$$[a] = \{x : (a, x) \in R\}$$

$x \in [a]$ if $(x, a) \in R$

OR

$x \in [a]$ if $(a, x) \in R$

QUESTION

$x \in [a]$ if $(x,a) \in R$

Consider the following equivalence relations defined on $A = \{1, 2, 3, 4, 5\}$.

$R_1 = \{(1,1), (2,2), (3,3), (4,4), (5,5)\}$ $[1]=\{1\}$, $[2]=\{2\}$, $[3]=\{3\}$, $[4]=\{4\}$ $[5]=\{5\}$

$R_2 = \{(1,1), (2,2), (3,3), (4,4), (5,5), (2,1), (1,2)\}$ $[1]=\{1,2\}$ $[2]=\{1,2\}$, $[3]=\{3\}$

$R_3 = \{(1,1), (2,2), (3,3), (4,4), (5,5), (1,3), (3,1), (2,4), (4,2)\}$ $[4]=\{4\}$, $[5]=\{5\}$.

$[1]=\{1,\boxed{3}\}$

$[2]=\{2,4\}$

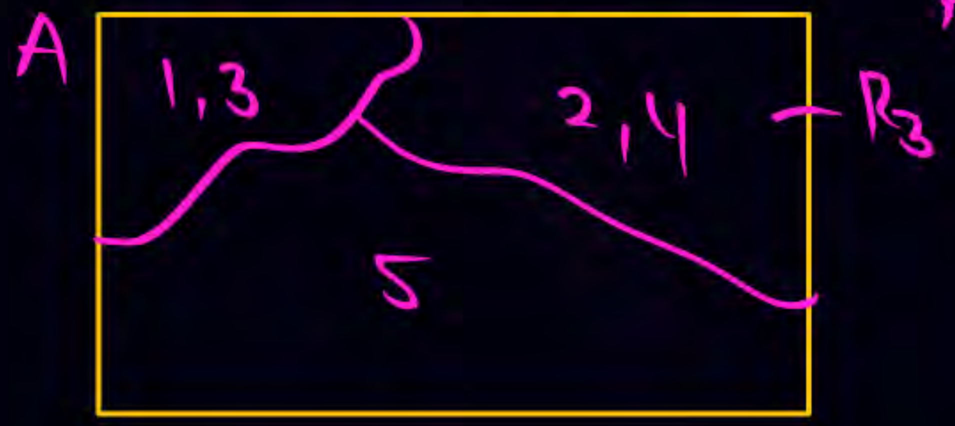
$[3]=\{3,\boxed{1}\}$.

$[4]=\{4,2\}$

$[5]=\{5\}$

★ if $a \in [b] \Rightarrow b \in [a]$ & $[a]=[b]$

★ Equivalence classes are either disjoint or identical.



Every Equivalence Relation on A gives a partition of A & vice versa

$\Downarrow$

No: of Equivalence Relations on A. = No: of possible partitions of A

Ex: $A = \{1, 2, 3\}$

Equivalence Classes of an Equivalence Relation

Let R be equivalence relation in $A(\neq \phi)$. Let $a \in A$. Then the equivalence class of a, denoted by $[a]$ or $\{\bar{a}\}$ is defined as the set of all those points of A which are related to a under the relation R. Thus $[a] = \{x \in A : x R a\}$.

It is easy to see that

(1) $b \in [a] \Rightarrow a \in [b]$

(2) $b \in [a] \Rightarrow [a] = [b]$ collection for which $(x, a) \in R$

(3) Two equivalence classes are either disjoint or identical.

QUESTION [JEE Mains 2021]

(R is equivalence Relation)

Let R = {(P, Q) | P and Q are at the same distance from the origin} be a relation, then the equivalence class of (1, –1) is the set:

(A) $S = \{(x, y) \mid x^2 + y^2 = 4\}$

(B) $S = \{(x, y) \mid x^2 + y^2 = 1\}$

(C) $S = \{(x, y) \mid x^2 + y^2 = \sqrt{2}\}$

(D) $S = \{(x, y) \mid x^2 + y^2 = 2\}$

$(P, Q) \in R \Rightarrow OP = OQ$

$P = (1, -1) \quad Q = (x, y)$

Equivalence class of $P = [P] = \{Q : (Q, P) \in R\}$

$\Downarrow$

$OQ = OP.$

$\sqrt{(x-0)^2 + (y-0)^2} = \sqrt{(0-1)^2 + (0+1)^2}$

$x^2 + y^2 = 2.$

$[P] = S = \{(x, y) : x^2 + y^2 = 2\}.$

QUESTION [JEE Mains 2021]

(R is equivalence Relation)

Let R = {(P, Q) | P and Q are at the same distance from the origin} be a relation, then the equivalence class of (1, –1) is the set:

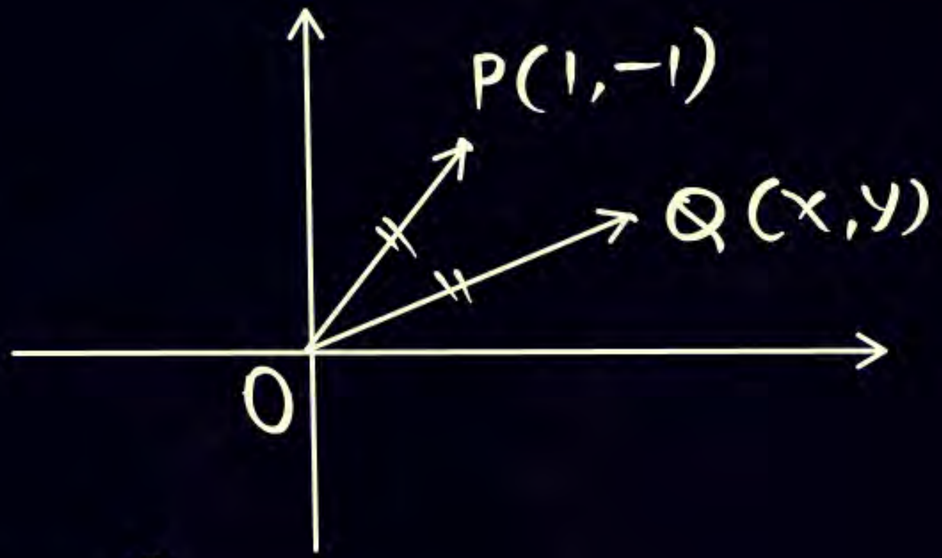
(A) $S = \{(x, y) \mid x^2 + y^2 = 4\}$

(B) $S = \{(x, y) \mid x^2 + y^2 = 1\}$

(C) $S = \{(x, y) \mid x^2 + y^2 = \sqrt{2}\}$

(D) $S = \{(x, y) \mid x^2 + y^2 = 2\}$

$(P, Q) \in R \Rightarrow OP = OQ$



$P = (1, -1)$

$Q = (x, y)$

$[P] = \{Q : (Q, P) \in R\}$

$\Downarrow$

$OQ = OP$

$\sqrt{(x-0)^2 + (y-0)^2} = \sqrt{(0-1)^2 + (0+1)^2}$

$x^2 + y^2 = 2$

QUESTION [JEE Mains 2021 (March)]

Let $A = \{2, 3, 4, 5, \ldots, 30\}$ and '$\simeq$' be an equivalence relation on $A \times A$, defined by $(a, b) \simeq (c, d)$, if and only if $ad = bc$. Then the number of ordered pairs which satisfy this equivalence relation with ordered pair $(4, 3)$ is equal to:

(A) 5

(B) 6

(C) 8

(D) 7

$((a,b),(c,d)) \in \simeq \Rightarrow ad = bc.$

$((4,3),(x,y)) \in \simeq \quad 4y = 3x.$

$x = \frac{4}{3}y.$

$2 \leq \frac{4}{3}y \leq 30$

$1.5 \leq y \leq 22.5$

$y = 3, 6, 9, 12, 15, 18, 21$

$x = 4, 8, 12, 16, 20, 24, 28$

FUNCTION :

A fn from A to B is a Relation from A to B s.t each & every element of A is uniquely associated to some element of B.

Every function is a Relation But not the converse

$A = \{1, 2, 3\}$

$B = \{4, 5, 6\}$

$R_1 = \{(1,4)(2,5)\}$ ✗ b'coz $3 \in A$ is not associated.

$R_2 = \{(1,4)(2,5)(3,6)(1,5)\}$ ✗ b'coz 1 is not uniquely associated.

$R_3 = \{(1,5)(2,5)(3,5)\}$ ✓

Function

What is a Function ?

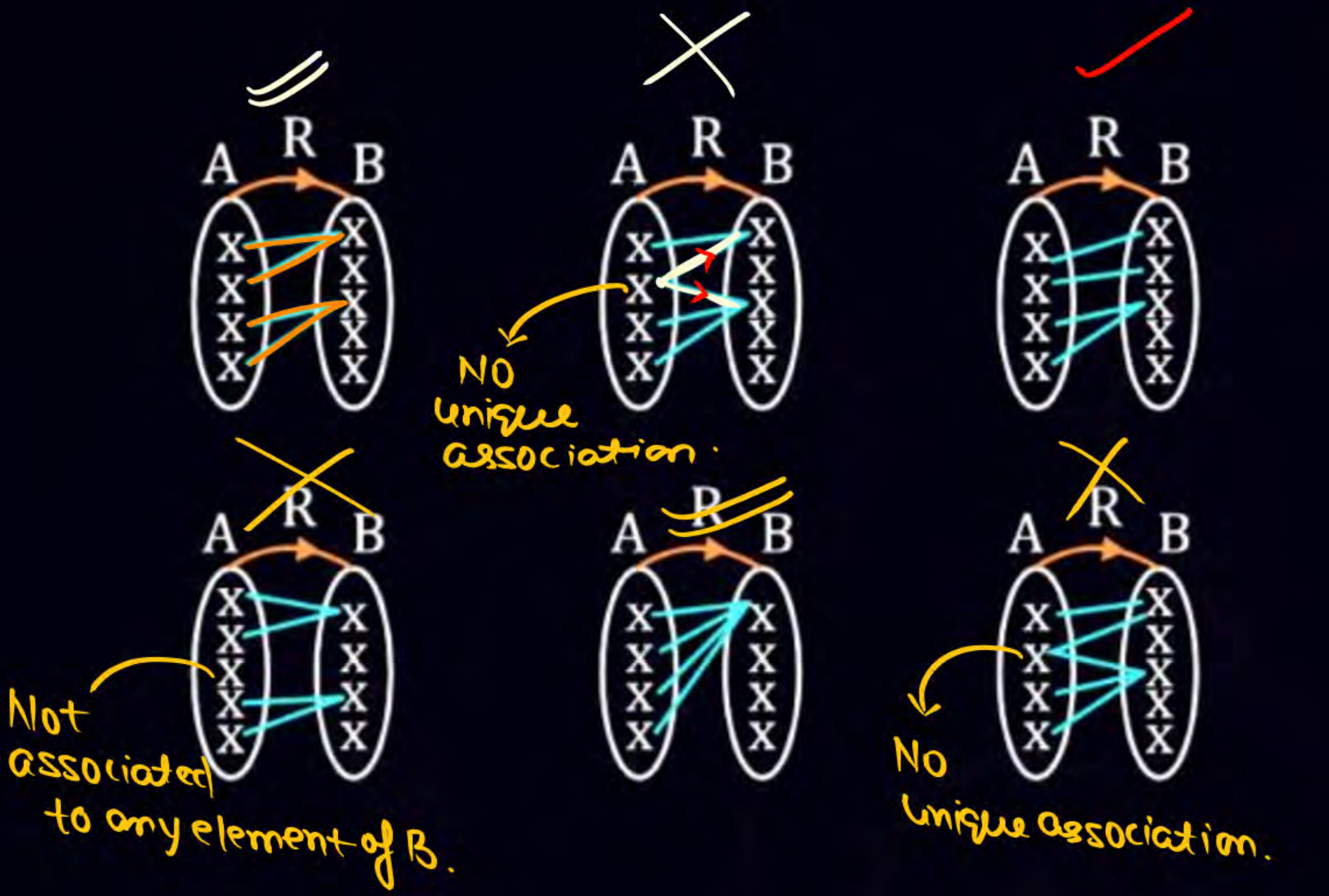
Definition-1

Let A and B be two sets and let there exist a rule or manner or correspondence 'f' which associates to each element of A to a unique element in B, then f is called a function or mapping from A to B. It is denoted by the symbol

$$f : A \to B \text{ or } A \xrightarrow{f} B$$

which reads 'f' is a function from A to B' or 'f maps A to B'.

Function

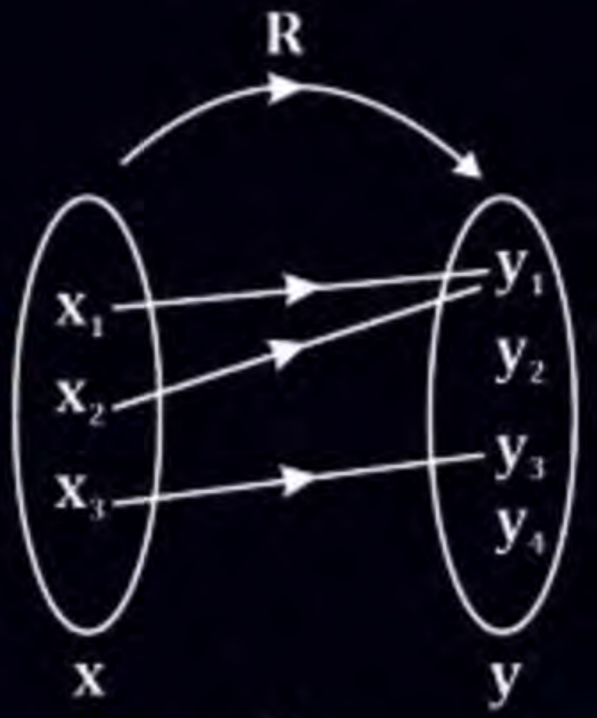


A
R
B
A
R
B
NO
unique
association.
A
R
B
A
R
B
Not
associated
to any element of B.
A
R
B
A
R
B
NO
unique association.

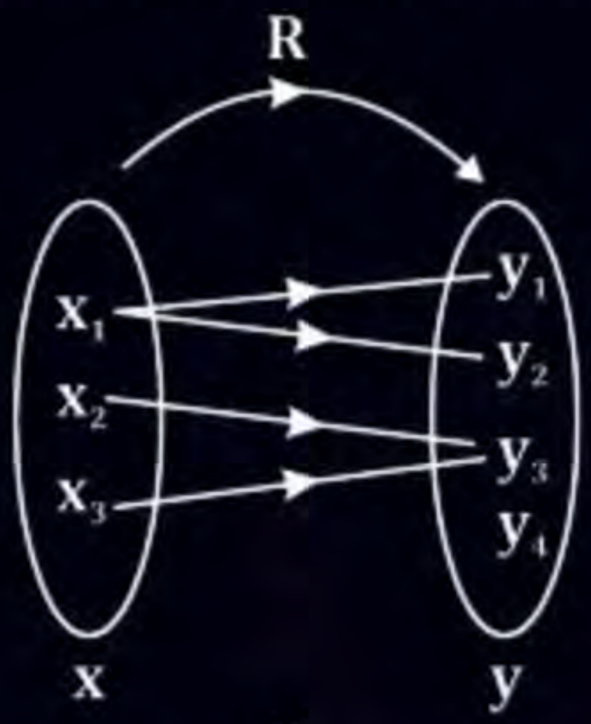
QUESTION

Identify which of the following relations is/are function(s) from $x \rightarrow y$?

A

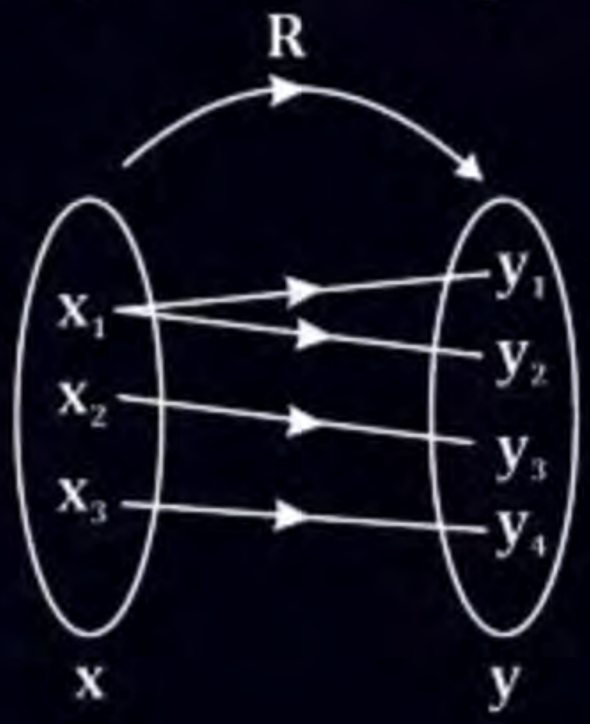


B



C

D



QUESTION

Which of the following relations are function from set X to Y; where $X = \{1, 3, 5, 7\}$ and set $Y = \{2, 4, 6, 8\}$?

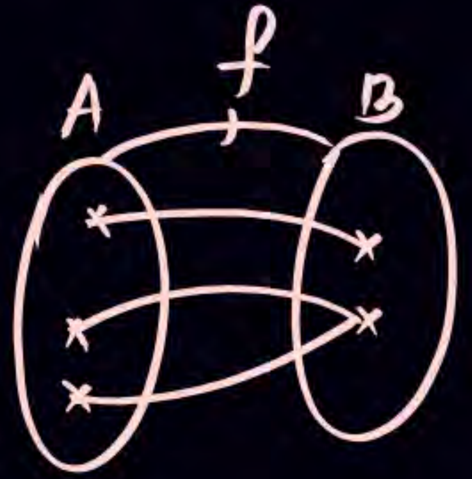
(A) $\{(3, 2), (3, 4), (5, 4), (7, 4), (1, 8)\}$ $\quad 3 \to 2,\ 3 \to 4$

(B) $\{(1, 2), (5, 8), (3, 6)\}$ No association for 7.

(C) $\{(1, 4), (3, 8), (5, 2), (7, 6)\} = f$

(D) $\{(1, 4), (3, 4), (5, 8), (7, 8)\}$

$f(1)=4$
$f(3)=8$
$f(5)=2$
$f(7)=6.$



⭐ If $(a,b) \in$ function f then it is written as $f(a)=b$. and is read as value of f at 'a' is b

⭐ b is called image of a under f

⭐ a is called preimage of b under f

In a function $f: A \to B$ an element of B may have more than one preimages

In a function $f: A \to B$ every element of A has a unique image.

Preimage & Image

If an element $a \in A$ is associated with an element $b \in B$ then b is called 'the f image of a' or 'image of a under f' or 'the value of the function f at a'. Also a is called the preimage of b or argument of b under the function f. We write it as $f(a) = b$.

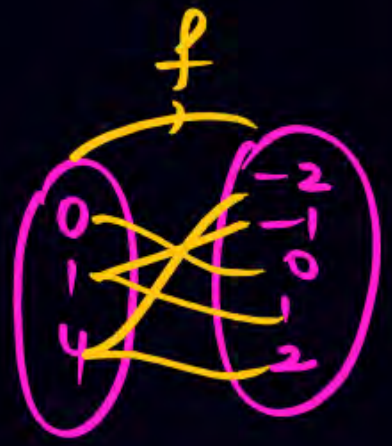
Note : Preimage = input
image = output

QUESTION

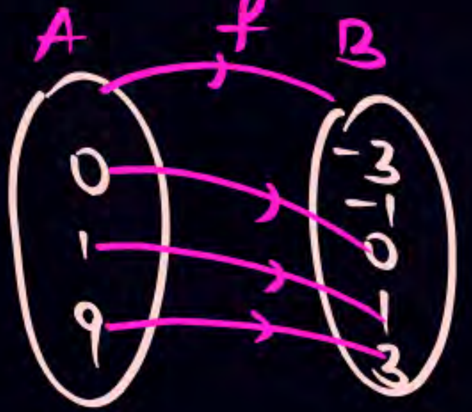
Which of the following correspondences can be called a function?

(A) $f:\{-2,0,2\}\to\{0,1,8,3\};\ f(x)=x^3$ ✗ -2 is not associated

(B) $f:\{0,1,4\}\to\{-2,-1,0,1,2\};\ f(x)=\pm\sqrt{x}$ ✗ $f(1)=\pm 1$ $\quad 1 < \begin{matrix} 1 \\ -1 \end{matrix}$



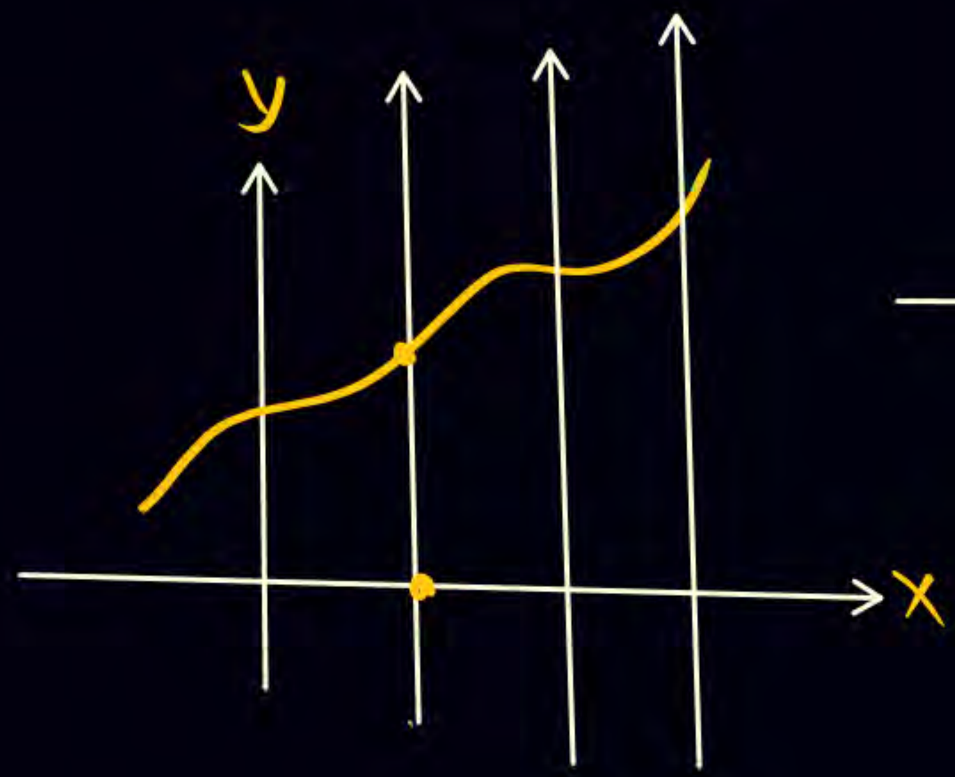
(C) $f:\{0,1,9\}\to\{-3,-1,0,1,3\};\ f(x)=\sqrt{x}$

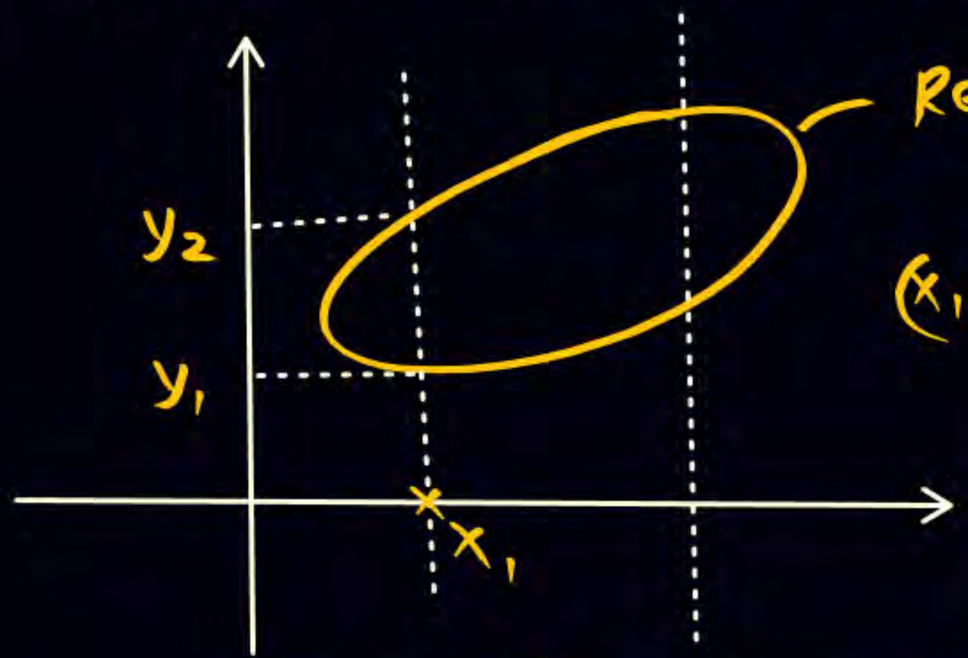


(D) $f:\{0,1,9\}\to\{-3,-1,0,1,3\};\ f(x)=-\sqrt{x}$

Vertical Line Test

If a vertical line intersects the graph of f in two or more points then it cannot be graph of a function





Relation R

$(x_1, y_1)\ (x_1, y_2) \in R$ — Not a fn

If every line parallel to y axis intersects the graph at atmost one point then it is graph of a fn.

$A = \{1, 2, 3\}$

$B = \{1, 2, 3, 4\}$

$R = \{(1,2)\,(1,3)\,(1,4)\,(2,3)\,(3,4)\}$

$A = \{1, 2, 3, 4\}$

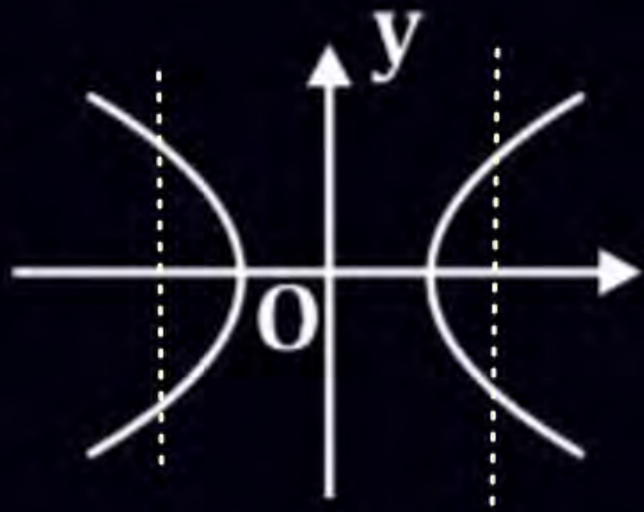
$B = \{1, 2, 3, 4, 5\}$

yes a fn

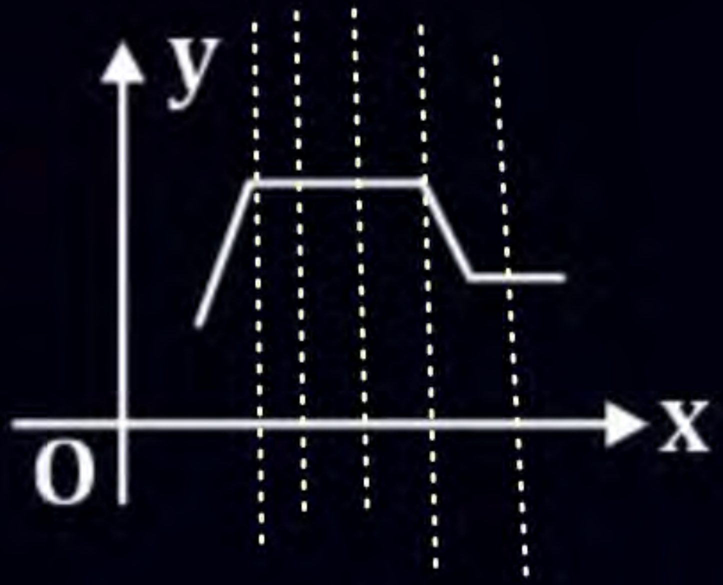
QUESTION

Which of the given graphs is/are Functions?

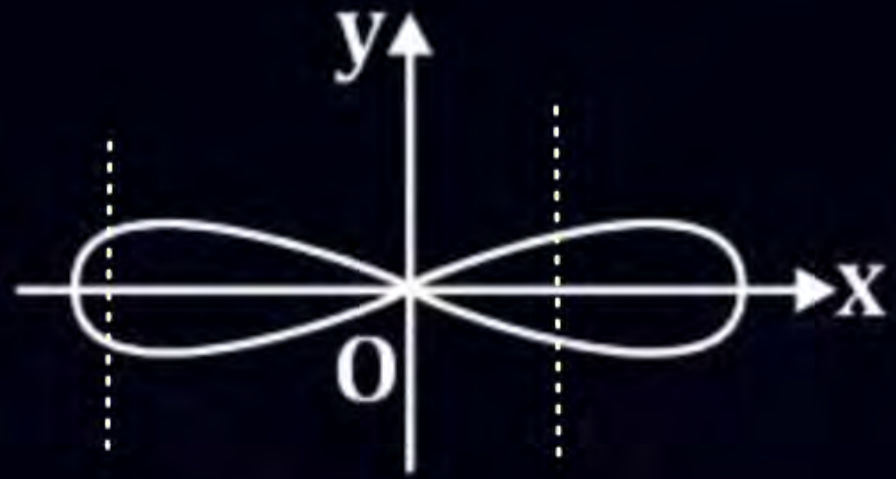
(a)



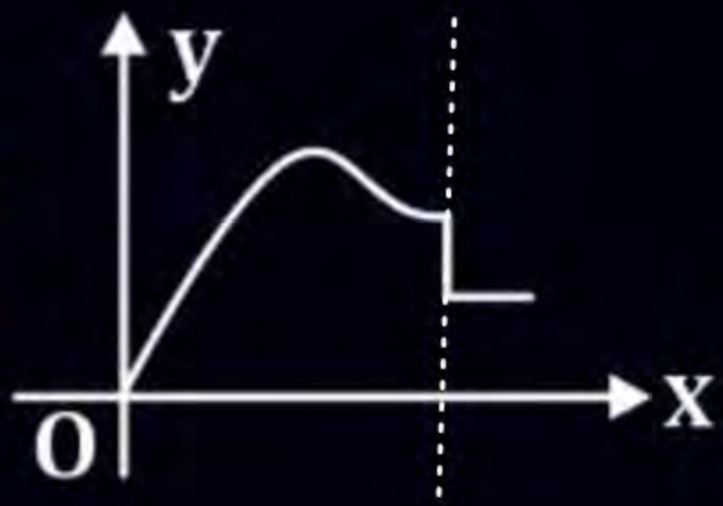
(c)



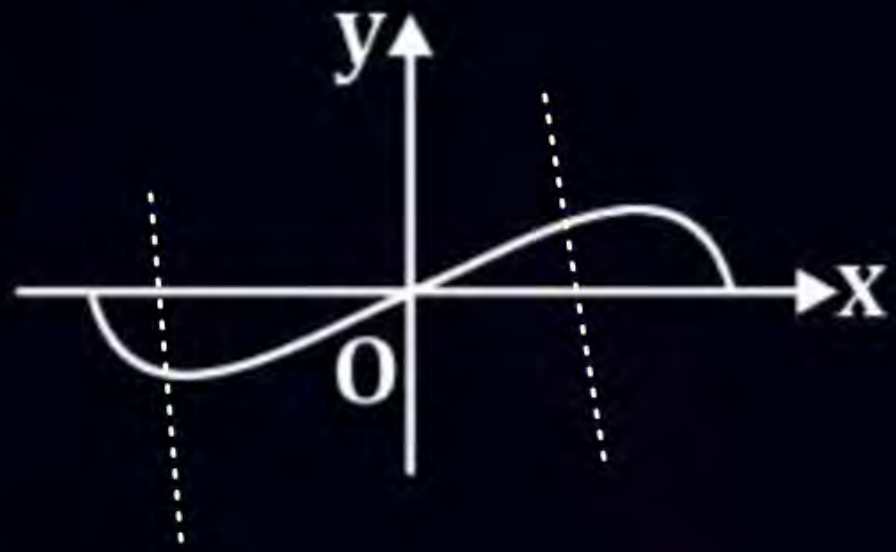
(e)



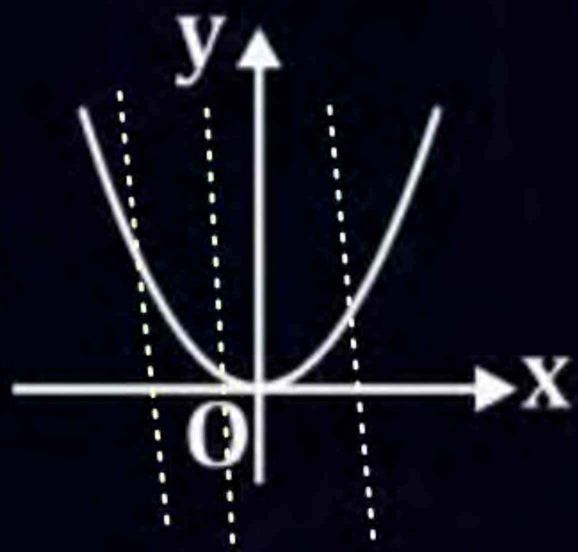
(b)



(d)



(f)



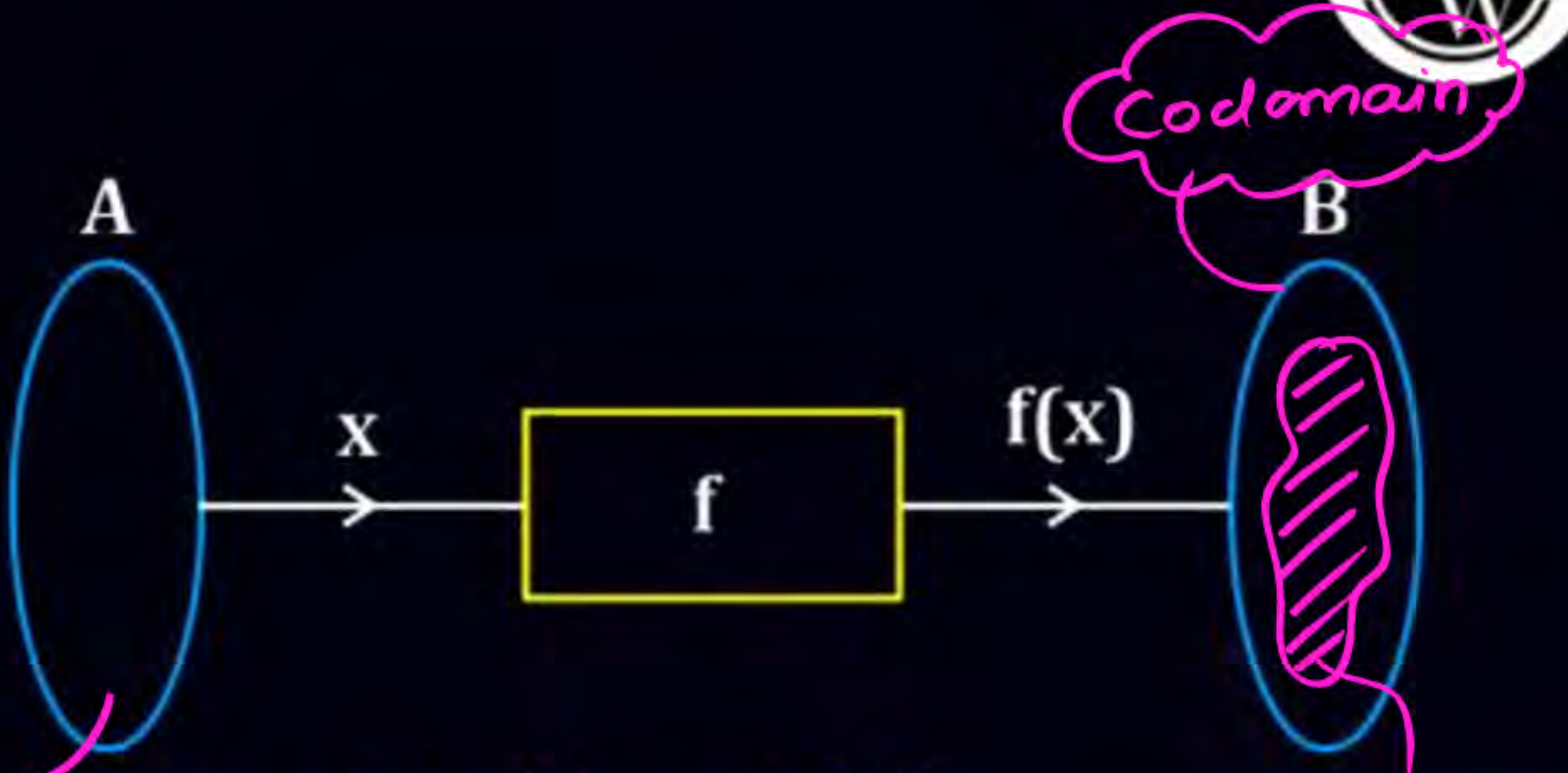
(g)

Function as a Machine

- $f(x) = \frac{1}{x}$ Domain $= R_0$

- $f(x) = \sqrt{x}$ Domain $= [0, \infty)$

- $f(x) = \frac{1}{\sqrt{x}}$. Domain: $(0, \infty)$



Domain. : set of Inputs (A)

Range : set of outputs.

codomain : B.

Range $\subseteq$ Codomain

Domain of Function

- Domain is the set of all inputs of the function.
- Range is the set of all outputs or in other words it is set of all values which a function can take.

Four Important Points to Note

#1. Range is always a subset of codomain.

#2. If only rule of function is given then domain of function is set of real numbers where function is defined.

$$f(x) = \sqrt{x^2 - 5x + 6}$$

Domain: $x^2 - 5x + 6 \geq 0$

$(x-3)(x-2) \geq 0$

$x \in (-\infty, 2] \cup [3, \infty)$

$y = f(x)$

- Domain hoti hai x ki values isiliyay x axis pe milaygi

- Range hoti hai $f(x) = y$ ki value isiliyay y axis pe milaygi

#3. **For a continuous function interval from minimum to maximum value gives its range.**

$$Ex: y = \sin x \begin{cases} \min = -1 \\ \max = 1 \end{cases}$$

$\downarrow$

continuous

$\Downarrow$

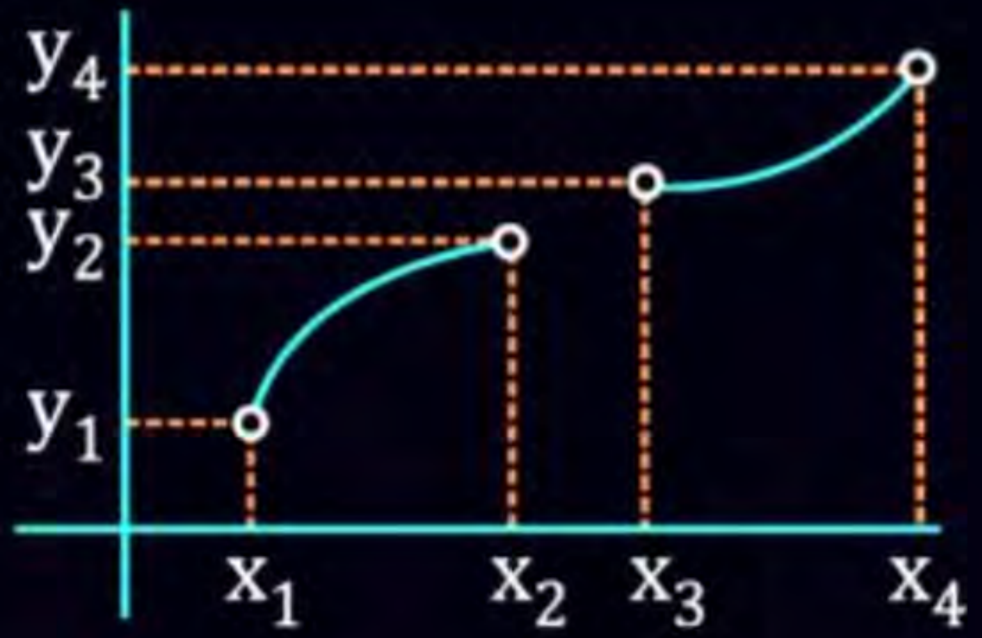
Range $= [-1, 1]$.

Domain : (x_1, x_4)

Range : $[y_4, y_2]$

All polynomial fns are continuous.

#4. For discontinuous function



Domain : $(x_1, x_2) \cup (x_3, x_4)$

Range : $(y_1, y_2) \cup (y_3, y_4)$

3 Golden Points

$f(x) = \frac{1}{x^2-6x+8}$ Domain $= R - \{2,4\}$

$x^2-6x+8 \neq 0$

$x \neq 2, 4$

1. If the rule of function is given then the domain of function is the set of real values of x for which it is defined and does not become infinite undefined or imaginary.

2. For a continuous function the range is the interval from minimum to maximum value.

3. In case the codomain of the function is not given then it is taken to be R.

Sabse Important Baat Yaad Rahe

Sabhi Class Illustrations Retry Karnay hai...

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with good Rank

Class illustrations

Module,DPP

KTK,TAH
CHALLENGER

QUESTION [JEE Mains 2023 (13 April)] KTK 1

Let $A = \{-4, -3, -2, 0, 1, 3, 4\}$ and $R = \{(a, b) \in A \times A : b = |a| \text{ or } b^2 = a + 1\}$ be a relation on A. Then the minimum number of elements, that must be added to the relation R so that it becomes reflexive and symmetric, is

Ans. 7

QUESTION [JEE Mains 2021 (31 Aug)]

KTK 2

Which of the following is not correct for relation R on the set of real numbers?

(A) $(x, y) \in R \Leftrightarrow 0 < |x| - |y| \leq 1$ is neither transitive nor symmetric.

(B) $(x, y) \in R \Leftrightarrow 0 < |x - y| \leq 1$ is symmetric and transitive.

(C) $(x, y) \in R \Leftrightarrow |x| - |y| \leq 1$ is reflexive but not symmetric.

(D) $(x, y) \in R \Leftrightarrow |x - y| \leq 1$ is reflexive and symmetric.

Ans. B

QUESTION [JEE Mains 2024 (6 April)]

KTK 3

Let the relations R_1 and R_2 on the set $X = \{1, 2, 3, \ldots, 20\}$ be given by $R_1 = \{(x, y) : 2x - 3y = 2\}$ and $R_2 = \{(x, y) : -5x + 4y = 0\}$. If M and N be the minimum number of elements required to be added in R_1 and R_2, respectively, in order to make the relations symmetric, then $M + N$ equals

(A) 16

(B) 12

(C) 8

(D) 10

Ans. D

QUESTION [JEE Mains 2023 (10 April)]

KTK 4

Let $A = \{2, 3, 4\}$ and $B = \{8, 9, 12\}$. Then the number of elements in the relation $R = \{((a_1, b_1), (a_2, b_2)) \in (A \times B, A \times B) : a_1 \text{ divides } b_2 \text{ and } a_2 \text{ divides } b_1\}$ is:

(A) 18

(B) 24

(C) 36

(D) 12

Ans. C

QUESTION [JEE Mains 2023 (30 Jan)]

KTK 5

The minimum number of elements that must be added to the relation $R = \{(a, b), (b, c)\}$ on the set $\{a, b, c\}$ so that it becomes symmetric and transitive is:

(A) 7

(B) 3

(C) 4

(D) 5

Ans. A

Homework from Module

Chapter: SETS

Prarambh: COMPLETE

Prabal : COMPLETE

(Revision Practice Problems)

QUESTION [MHT CET 2023 (14 May)]

RPP 1

Let $A = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$. If $B = I - {}^3C_1(\text{adj } A) + {}^3C_2(\text{adj } A)^2 - {}^3C_3(\text{adj } A)^3$, then the sum of all elements of the matrix B is

(A) -1

(B) -3

(C) -4

(D) -5

Ans. D

QUESTION

RPP 2

The complete set of values of 'b' for which the equation $2\log_{1/25}(bx+28) = -\log_5(12-4x-x^2)$ has only one solution.

(A) $(-\infty, -14) \cup \left[\frac{14}{3}, \infty\right) \cup \{4\}$

(B) $(-\infty, -14) \cup \{4\} \cup \left[\frac{14}{3}, \infty\right) \cup \{-12\}$

(C) $(-\infty, -14) \cup \left[\frac{14}{3}, \infty\right) \cup \{-12\}$

(D) $(-\infty, -14) \cup \left[\frac{14}{3}, \infty\right)$

Ans. A

QUESTION

RPP 3

Values of k for which the inequality $k\sin^2 x - k\sin x + 1 \geq 0$ is true $\forall\, x \in R$ is

(A) $k > -\frac{1}{2}$

(B) $k > 4$

(C) $-\frac{1}{2} \leq k \leq 4$

(D) $\frac{1}{2} \leq k \leq 5$

Ans. C

Previous TAH

Solutions

QUESTION [JEE Mains 2022 (27 July)]

Let R_1 and R_2 be two relations defined on $\mathbb{R}$ by $a\,R_1\,b \Leftrightarrow ab \geq 0$ and $a\,R_2\,b \Leftrightarrow a \geq b$. Then

(A) R_1 is an equivalence relation but not R_2

(B) R_2 is an equivalence relation but not R_1

(C) both R_1 and R_2 are equivalence relations

(D) neither R_1 nor R_2 is an equivalence relation

[Jee main 2022] $\boxed{\text{TAH-1}}$ Let R_1 & R_2 be Two Relation defined on R By $aR_1b \iff ab \geq 0$ & aR_2b $a \geq b$ Then

$aR_1b \iff a.b \geq 0$

For Reflexive
(a,a)
$a.a \geq 0$ $a^2 \geq 0$
Always True

* Reflexive ✓

Chetan Jodhav MH

$a.b \geq 0$ $b.a \geq 0$
Correct (True)

* $\boxed{\text{Symmetric}}$

Transitive
$a.b \geq 0$
Counter example :-

$(-3, 0)(0, 3)$

$-3.0 \geq 0$ ✓ $(0.3) \geq 0$ ✓

$(-3.3) \not\geq 0$

Not Transitive

$\boxed{R_1 \text{ is Not equivalence Relation}}$

aR_2b $a \geq b$

Reflexive
$a \geq a$ ✓
$b \geq b$ ✓
$\boxed{\text{yes, it is Reflexive}}$

$\boxed{\text{Symmetric}}$
$a \geq b$
$4 \geq 3$ $4 \not\leq 3$
Not Symmetric
$(4,3)(3,4)$

Transitive
aR_2b $a \geq b$

$a \geq b$ $b \geq c$

$a \geq c$ $a > c$

yes, Transitive

Neither R_1 & R_2 is an equivalence Relation

QUESTION [JEE Mains 2023 (8 April)]

Let $A = \{0, 3, 4, 6, 7, 8, 9, 10\}$ and R be the relation defined on A such that $R = \{(x, y) \in A \times A : x - y$ is odd positive integer or $x - y = 2\}$. The minimum number of elements that must be added to the relation R, so that it is a symmetric relation, is equal to ________

Ans. 19

TAH-2

Sol^n: $A = \{0,3,4,6,7,8,9,10\}$

$(x,y) \in R$ if $x-y$ = odd (+)ve integer

$x-y=2$

$R = \{(3,0)(7,0)(9,0)(4,3)(6,3)(8,3),(10,3)$
$(6,4)(7,4)(9,4)(7,6)(8,6)(9,6)(8,7)$
$(9,7)(10,7)(9,8)(10,8)(10,9)(0,3)(0,7)(0,9)$
$(3,4)(3,6)(3,8)(3,10)(4,6)(4,7)(4,9)(6,7)$
$(6,8)(6,9)(7,8)(7,9)(7,10)(8,9)(8,10)(9,10)\}$

$\therefore$ The min. no. of elements that must be added $= 19$.

Name- Bhumika Sharma
From- Sri Ganganagar, Rajasthan

QAH-7 Let $A=\{0,3,4,6,7,8,9,10\}$ and R be the relation defined on such that $R=\{(x,y)\in A\times A : x-y$ is odd positive integer or $x-y=2\}$. The min number of elements that must be added to the relation R, so that it is a symmetric relation is = ?

$R=\{(x,y)\in A\times A : x-y$ is odd (+) integer or $(x-y)=2\}$

$A=\{0,3,4,6,7,8,9,10\}$

$R=\{(3,0),(7,0),(9,0),(4,3)\ (6,4)\ (7,6)\ (8,7),(10,9)$
$(6,3)\ (7,4)\ (8,6)\ (9,7)\ (10,8)\ (9,6)\ (9,4)\ (8,3)$
$(10,3),(10,7)\ (9,8)\}$ $\hookrightarrow (a,b)$

for symm relation we need elements type $\rightarrow (b,a)$

So we need 19 more elements.

Shivanshi Mishra
Mr. Anoop Mishra (UP)
Mrs. Smita mishra

QUESTION [JEE Mains 2024 (31 Jan)]

Let $A = \{1, 2, 3, 4\}$ and $R = \{(1,2), (2, 3), (1, 4)\}$ be a relation on A. Let S be the equivalence relation on A such that $R \subset S$ and the number of elements in S is n. Then, the minimum value of n is

Ans. 16

TAH-3

Sol^n: $A = \{1,2,3,4\}$

$R = \{(1,2),(2,3),(1,4)\}$ $\quad R \subset S$

No. of elements in $S = n$

Min. value of $n = ?$

$S = \{(1,2)(2,3)(1,4)(1,1)(2,2)(3,3)(4,4)$
$(2,1)(3,2)(4,1)(1,3)(2,4)(3,1)$
$(4,2)(4,3)(3,4)\}$

$\therefore$ Min. value of $n = 16$.

Name-Bhumika Sharma
From- Sri Ganganagar, Rajasthan

$\boxed{\text{JAH-3}}$ Main - 24

$A = \{1,2,3,4\}$ $\quad R = \{(1,2),(2,3),(1,4)\}$ be a relation on A

$R \subset S$, $n(S)/_{min} = ?$

Souvik Mondal
Form WestBengal
Murshidabad.

$R = \{(1,2),(2,3),(1,4)\}$

$S = \{(1,2)(2,3)(1,4)(1,1)(2,2)(3,3)(4,4)(2,1)(3,2)(4,1)$
$(1,3)(3,1)(4,3)(3,4)(2,4)(4,2)\}$

$\downarrow$

equivalence relation

$n(S) = 16$

QUESTION [JEE Mains 2020]

Let R_1 and R_2 be two relations defined as follows:

$R_1 = \{(ab) \in R^2 : a^2 + b^2 \in Q\}$ and $R_2 = \{(ab) \in R^2 : a^2 + b^2 \notin Q\}$

where Q is the set of all rational numbers. Then :

(A) R_1 is transitive but R_2 is not transitive.

(B) R_1 and R_2 are both transitive.

(C) R_2 is transitive but R_1 is not transitive.

(D) Neither R_1 nor R_2 is transitive.

TAH.4 $R_1 \Rightarrow \{(a,b) \in R^2 \quad a^2+b^2 \in Q\}$

$R_2 \Rightarrow \{(a,b) \in R^2 \quad a^2+b^2 \in Q\}$

Let $a \Rightarrow 2+\sqrt{3} \qquad b = 2-\sqrt{3} \qquad c = 48^{1/4}$

Now

$a^2+b^2 = 7+4\sqrt{3}+7-4\sqrt{3} = 14 \in Q$

$b^2+c^2 = 7-4\sqrt{3}+48^{1/2}$

$\Rightarrow 7-4\sqrt{3}+4\sqrt{3} = 7 \in Q$

But

$a^2+c^2 = 7+4\sqrt{3}+4\sqrt{3} \notin Q$

So R_1 is not Transitive,

TAH-4 Main-20

$R_1 = \{(a,b) \in R^2 : a^2+b^2 \in Q\}$ & $R_2 = \{(a,b) \in R^2 : a^2+b^2 \notin Q\}$

① Transitive ×

$(\sqrt{2}+\sqrt{3}, \sqrt{2}-\sqrt{3})$
$(\sqrt{2}-\sqrt{3}, \sqrt{6}+1)$ lies in R.

But $(\sqrt{2}+\sqrt{3}, \sqrt{6}+1) \notin R$.

① Transitive ×

$(1+\sqrt{2}, \sqrt{2})$
$(\sqrt{2}, 1-\sqrt{2})$ lies in R

But $(1+\sqrt{2}, 1-\sqrt{2}) \notin R$

Neither R_1 nor R_2 is transitive.

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From West Bengal
Murshidabad.

QUESTION [JEE Mains 2024 (29 Jan)]

Let R be the relation on $Z \times Z$ defined by $(a, b)\ R\ (c, d)$ if and only if $ad - bc$ is divisible by 5. Then R is

(A) Reflexive and transitive but not symmetric

(B) Reflexive and symmetric but not transitive

(C) Reflexive but neither symmetric nor transitive

(D) Reflexive, symmetric and transitive

Ans. B

JAHS $(a,b) R (c,d)$ if & only if $ad - bc = 5\lambda$

* For Reflexive

$ac - ac = 0 = 5\lambda$

is divisible by 5

Yes, It is Reflexive

Now

$ad - bc \Rightarrow 5\lambda$

$bc - ad \Rightarrow k$

$0 = 5\lambda + k$

$k \rightarrow -5\lambda$

* Yes, It is Symmetric.

* For Transitive

$(a,b), (c,d)$ & (e,f)

$ad - bc = k_1 \times 5$

& $cf - ed = k_2 \times 5$

Let $(3,1) R (10,5)$

$ad - bc = 15 - 10 = 5$

So $(10,5) R (1,1)$

$cf - ed \Rightarrow 10 - 5 = 5$

Now $(3,1), (1,1)$

$af - be \Rightarrow 3 - 1 = 2 \neq 5\lambda$

So It is not transitive Ans

QUESTION [JEE Mains 2024 (1 Feb)]

Consider the relations R_1 and R_2 defined as $a\,R_1\,b \Leftrightarrow a^2 + b^2 = 1$ for all $a, b \in R$ and $(a, b)\,R_2\,(c, d) \Leftrightarrow a + d = b + c$ for all $(a, b), (c, d) \in N \times N$. Then:

(A) R_1 and R_2 both are equivalence relations

(B) Only R_1 is an equivalence relation

(C) Only R_2 is an equivalence relation

(D) Neither R_1 nor R_2 is an equivalence relation

Ans. C

TAMG $a R_1 b \iff a^2 + b^2 = 1$,

$(a,b) R_2 (c,d) \iff a + d = b + c$,

R_1 $\quad a^2 + b^2 = 1$

$(1,1)$ $\quad 1 + 1 \neq 1$ $\quad$ It is not Reflexive.

$(a,b) R_2 (c,d)$

Now for R_2

$a + d = b + c$

$1 + 1 = 1 + 1$ $\quad$ Reflexive

$(1,2) R (2,1)$

$3 = 3 \in R_2$ $\quad$ So $(2,1) R (1,2)$ are also satisfied, $\quad$ Symmetric

Now for Transitive.

$(a,b), (c,d), (e,f) \in N \times N$

$a + d = c + b$ — (i), $\quad c + f = d + e$ — (ii)

Adding eq (i) & (ii)

$a + f + \cancel{c + d} = b + e + \cancel{c + d}$

$\boxed{a + f = b + e}$ $\quad (a,b) R_2 (e,f)$

So R_2 is Transitive.

So R_2 is an equivalence Relation Ans.

(Solution to KTK)

QUESTION [JEE Mains 2023 (15 April)]

(KTK 1)

Let $A = \{1, 2, 3, 4\}$ and R be a relation on the set AA defined by

$R = \{((a, b), (c, d)) : 2a + 3b = 4c + 5d\}$.

Then the number of elements in R is ________

Ans. 6

Que – let $A = \{1, 2, 3, 4\}$ and R relation on AA
$R = \{((a, b), (c, d)) : 2a + 3b = 4c + 5d\}$
no. of elements in R ____ ?

$AA = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3)$
$(2,4), (3,1), (3,2), (3,3), (3,4), (4,1), (4,2)$
$(4,3), (4,4)\}$

$R = \{(a, b) \cdot (c, d) : 2a + 3b = 4c + 5d\}$

after check –

$R = \{((1,4), (1,2)), ((2,3)(2,1)), ((3,1)(1,1)),$
$((3,4), (2,2)) ((4,2), (1,2)) \cdot ((4,3), (3,1))\}$

$\boxed{\text{no. of elements} = 6}$

Shivanshi Mishra
Mr Anoop Mishra (UP)
Mrs. smita mishra

KEK-2

QUESTION [JEE Mains 2018 (15 April)]

(KTK 2)

Consider the following two binary relations on the set $A = \{a, b, c\}$:

$R_1 = \{(c, a), (b, b), (a, c), (c, c), (b, c), (a, a)\}$ and

$R_2 = \{(a, b), (b, a), (c, c), (c, a), (a, a), (b, b), (a, c))$.

Then:

(A) both R_1 and R_2 are not symmetric.

(B) R_1 is not symmetric but it is transitive.

(C) R_2 is symmetric but it is not transitive.

(D) both R_1 and R_2 are transitive.

Ans. C

KTK 02.

Shivani
From bihar

Set $A = \{a, b, c\}$

$R_1 = \{ (c,a) (b,b) (a,c) (c,c) (b,c) (a,a) \}$

$R_2 = \{(a,b) (b,a) (c,c) (c,a) (a,a) (b,b) (a,c)\}$

R_1 is not Symmt

$\Rightarrow$ b'coz $(b,c) \in R_1$ but $(c,b) \notin R_1$

Transitive: R_1 is not transitive

$\Rightarrow$ b'coz $(b,c)\ (c,a) \in R_1$ But $(b,a) \notin R_1$

R_2 is Symmt.

Transitive: R_2 is not Transitive

$\Rightarrow$ b'coz $(c,a)\ (a,b) \in R_2$ but $(c,b) \notin R_2$

Jee main 2018 (KTK-2)

Consider the Following Binary Relation on the set $A = (a,b,c)$.

$R_1 = \{(c,a)(b,b)(a,c)(c,c)(b,c)(a,a)\}$

$R_2 = \{(a,b)(b,a)(c,c)(c,a)(a,a)(b,b)(a,c)\}$

$\Rightarrow$

R_1 is Not symmetric B'coz $\{(a,a)(b,b)(c,c)(a,c)(c,a)(b,c)\}$

$(b,c) \in R_1$ $(c,b) \notin R_1$

R_1 'is' Not Transitive b'coz $(b,c) \in R_1$ $(c,a) \in R_1$ but $(b,a) \notin R_1$

$R_2 = \{(a,a)(b,b)(c,c)(a,b)(b,a)(c,a)(a,c)\}$

Clearly R_2 is symmetric

$(b,a) \in R_2$ $(a,c) \in R_2$

$(b,c) \notin R_2$

Then R_2 is Not Transitive

R_2 is Symmetric but it is Not Transitive

Chetan
Jadhav
MH

KTK-2

Consider the following two binary relation on the set $A = \{a, b, c\}$:

$R_1 = \{(c,a), (b,b), (a,c), (c,c), (b,c), (a,a)\}$ and

$R_2 = \{(a,b), (b,a), (c,c), (c,a), (a,a), (b,b), (a,c)\}$

then

Sol $R_1 = \{(a,a), (b,b), (c,c), (a,c), (c,a), (b,c)\}$

reflex ✓ symm ✗ ↳ Becaue (c,b) missing

trans ✗ ↳ Becaue $(b,c) \in R_1$, $(c,a) \in R_1$ But $(b,a) \notin R_1$

$R_2 = \{(a,a), (b,b), (c,c), (a,c), (c,a), (a,b), (b,a)\}$

reflex ✓ symm ✓ trans ✗ ↳ Becau $(b,a) \in R_2$, $(a,c) \in R_2$ But $(b,c) \notin R_2$

Ans (C) R_2 is symm but it is not transitive.

QUESTION [JEE Mains 2022 (28 June)]

(KTK 3)

Let R_1 and R_2 be relations on the set (1, 2, ..., 50) such that
$R_1 = \{(p, p^n) : p$ is a prime and $n \geq 0$ is an integer) and
$R_2 = \{(p, p^n) : p$ is a prime and $n = 0$ or 1).
Then, the number of elements in $R_1 - R_2$ is ______

Ans. 8

KTK03.

Set $\{1, 2, \cdots, 50\}$

$R_1 = \{(P, P^n) : P \text{ is a prime } \& \ n \geq 0\}$

$R_2 = \{(P, P^n) : P \text{ is a Prime and } n = 0 \text{ or } 1\}$

$\therefore$ no. of elements in $R_1 - R_2 = ?$

$R_1 : \{ (2, 2^0)\ (2, 2^1) \cdots\cdots (2, 2^5)$

$(3, 3^0)\ (3, 3^1) \cdots\cdots (3, 3^3)$

$(5, 5^0)\ (5, 5^1)\ (5, 5^2)$

$(7, 7^0)\ (7, 7^1)\ (7, 7^2)$

$(11, 11^0)\ (11, 11^1)\ (13, 13^0)\ (13, 13^1) \cdots\cdots (17, 17^0)$

$(17, 17)\ (19, 19^0)\ (19, 19^1)\ (23, 23^0)\ (23, 23)\ (29, 29^0)\ (29, 29)$

$(31, 31^0)\ (31, 31)\ (37, 37^0)\ (37, 37)\ (41, 41^0)\ (41, 41)$

$(43, 43)\ (43, 43)\ (47, 47^0)\ (47, 47)$

$R_2 : \{ (2, 2^0)\ (2, 2^1)\ (3, 3^0)\ (3, 3^1)\ (5, 5^0)\ (5, 5^1) \cdots$

$\cdots\cdots (47, 47^0)\ (47, 47)$

$\therefore R_2 - R_1 = \{((2, 2^2)\ (2, 2^3)\ (2, 2^4)\ (2, 2^5)\ (3, 3^2)$

$(3, 3^3)\ (5, 5^2)\ (7, 7^2) \}$

$\therefore$ no. of elements $= 8$

Shivani
From bihar

KTK3

Set $(1, 2 \cdots\cdots 50)$.

$R_1 \Rightarrow (P, P^n) : P$ is a prime and $n \geq 0$ is an Integer.

$R_2 \Rightarrow (P, P^n) : P$ is a prime and $n = 0$ or 1.

$R_1 \Rightarrow (2, 2^0), (2, 2^1), \cdots\cdots (2, 2^5)$.

$(3, 3^0), (3, 3^1) \cdots\cdots (3, 3^3)$.

$(5, 5^0), (5, 5^1), (5, 5^2)$.

$(6, 6^0), (6, 6^1), (6, 6^2), (7, 7^0), (7, 7^1), (7, 7^2)$,

$\cdots (11, 11^0), (11, 11^1)$.

$R_2 \Rightarrow (2, 2^0), (2, 2^1)$

$(3, 3^0), (3, 3^1) \cdots\cdots (47, 47^0), (47, 47^1)$.

$n(R_1) \Rightarrow 6 + 4 + 3 + 3 + 2 \times 10 = 36$

$n(R_2) \Rightarrow 14 \times 2 = 28$

$n(R_1) - n(R_2) = 36 - 28 = 8$ Ans

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Muzaffarpur

KTK - 3

[Jee main 2022]

Let R_1 & R_2 Be Relation's on the set $\{1, 2, \ldots 50\}$ such that

→ $R_1 = (p, p^n)$ p is prime and $n \geq 0$ is an Integer

$R_1 = \{(2,2)(2,4)(2,8)(2,16)(2,32),(3,3)(3,9)$
$(3,27)(5,5)(5,25)(7,7)(7,49)$
$(11,11)(13,13)(17,17)(19,19)(23,23)(29,29)$
$(31,31)(37,37)(41,41)(43,43)(47,47)\}$

$n(R_1) = 23$

$n(R_1 \cap R_2) = 15$

$R_2 = \{(p, p^n)$ p is prime $n = 0$ or $1)\}$

$= \{(2,1)(2,2)(3,1)(3,3)(5,1)(5,5)(7,7)(7,1)$
$(11,11)(13,13)\ldots\ldots(47,1)(47,47\}$

$R_1 - R_2 = R_1 - (R_1 \cap R_2)$

$= 23 - (15)$

$= 8$

$\boxed{R_1 - R_2 = 8}$

Chetan
Jadhav
MH

QUESTION [JEE Mains 2020 (2 Sept)]

(KTK 4)

If $R = \{(x, y) : x, y \in Z, x^2 + 3y^2 \leq 8\}$ is a relation on the set of integers Z, then the domain of R^{-1} is:

(A) $\{0, 1\}$

(B) $\{-2, -1, 1, 2\}$

(C) $\{-1, 0, 1\}$

(D) $\{-2, -1, 0, 1, 2\}$

Ans. C

$\boxed{\text{KTK-4}}$

IF $R = \{(x, y)$ $x, y \in Z$ $x^2+3y^2 \le 8$ is a Relation on the set of Integer's Z, then The Domain R^{-1} is.

$\Rightarrow$ $x^2+3y^2 \le 8.$

$$\boxed{\text{Domain of } R^{-1} = \text{Range of } R}$$

R :- $(1, 0)\ (1, 1)\ (0, 1)\ (1, -1)\ (0, -1)\ (-1, 0) \ldots\ldots)$

Domain of R^{-1} = Range of R

$$\boxed{(0, 1, -1)}$$

Chetan
Jadhav MH

KTK4 $R = \{(x,y) : x, y \in Z \quad x^2 + 3y^2 \le 8\}$

$R \to (1,1), (-1,-1), (1,-1), (-1,1).$

$(2,1), \cancel{(1,2)}, \cancel{(-1,2)}, (-2,1), \cancel{(1,-2)}, (2,-1).$

$(0,0), (0,1), (1,0), (-1,0), (0,-1).$

$\because$ Range of R = Domain of R^{-1}

$= \{-1, 0, 1\}.$ Ans

QUESTION [JEE Mains 2023 (11 April)]

KTK 5

Let $A = \{1,3, 4, 6, 9\}$ and $B = \{2, 4, 5, 8, 10)$. Let R be a relation defined on $A \times B$ such that $R = \{((a_1, b_1), (a_2, b_2)) : a_1 \leq b_2$ and $b_1 \leq a_2)$. Then the number of elements in the set R is :

(A) 180

(B) 26

(C) 52

(D) 160

Ans. D

KTK5 $A \Rightarrow \{1,3,4,6,9\}$ & $B \Rightarrow \{2,4,5,8,10\}$.

$R = \{(a_1,b_1),(a_2,b_2) : (a_1 \le b_2$ & $b_1 \le a_2)$

$R \Rightarrow (1,2),(1,4),(1,5),(1,8),(1,10)$.

$(3,2),(3,4),(3,5),(3,8),(3,10)$.

$(4,2),(4,4),(4,5),(4,8)(4,10)$.

$(6,2)(6,4)(6,5)(6,8)(6,10)$

$(9,2),(9,4),(9,5),(9,8),(9,10)$

$a_1 = 1$,	5 choice of b_2	$b_1 \Rightarrow 2$,	4 Coice of a_2
$a_1 = 3$,	4 " " "	$b_1 \Rightarrow 4$,	3 " " "
$a_1 = 4$,	4 " " "	$b_1 \Rightarrow 5$,	2 " " "
$a_1 = 6$,	2 " " "	$b_1 \Rightarrow 8$,	1 " " "
$a_1 = 9$	1 " " "	$b_1 \Rightarrow 10$,	0 " " "

For (a_1, b_2) 16 ways. For (a_2, b_1) 10 ways

$\Rightarrow$ Total Elements $\Rightarrow 16 \times 10$

$\Rightarrow 160$ Ans

KTK-5 Let $A = \{1,3,4,6,9\}$ and $B = \{2,4,5,8,10\}$. Let R be a Relation on $A\times B$ such that $R = \{((a_1,b_1),(a_2,b_2)) : a_1 \le b_2$ and $b_1 \le a_2\}$. Then the number of elements in the set R is:

KTK 5

$\Rightarrow$ $R = \{((a_1,b_1),(a_2,b_2)) : a_1 \le b_2$ & $b_1 \le a_2\}$

$\therefore R \subseteq A\times B$, $A = \{1,3,4,6,9\}$, $B = \{2,4,5,8,10\}$

* $a_1 \le b_2$

$1 \le 2,4,5,8,10 \longrightarrow$ No: of cases = 5.

$3 \le 4,5,8,10 \longrightarrow$ No: of cases = 4

$4 \le 4,5,8,10 \longrightarrow$ No: of cases = 4

$6 \le 8,10 \longrightarrow$ No: of cases = 2

$9 \le 10 \longrightarrow$ No: of cases = 1

Total no: of cases = 16

* $b_1 \le a_2$

$2 \le 3,4,6,9 \longrightarrow$ No: of cases = 4

$4 \le 4,6,9 \longrightarrow$ No: of cases = 3

$5 \le 6,9 \longrightarrow$ No: of cases = 2.

$8 \le 9 \longrightarrow$ No: of cases = 1

Total no: of cases = 10.

sourik Maiti
West Bengal

$\therefore$ The numbers of elements in the set 'R' $= 16\times 10 = (160)$ Ans.

(Solution to RPP)

QUESTION [WB JEE 2022]

(RPP 1)

If a, b are odd integers, then the roots of the equation $2ax^2 + (2a + b)x + b = 0, a \neq 0$ are

(A) rational

(B) irrational

(C) non-real

(D) equal

Ans. A

RPP-1

if a, b are odd integers

$$2ax^2+(2a+b)x+b=0$$

$$2ax^2+2ax+bx+b=0$$

$$2ax(x+1)+b(x+1)=0$$

$$(x+1)(2ax+b)=0$$

$$\Rightarrow \boxed{x=-1}, \quad 2ax=-b$$

$$\Rightarrow \boxed{x=\frac{-b}{2a}}$$

ROHINI SOLANKI

then the roots are rational.

QUESTION [BITSAT 2023]

(RPP 2)

If A, B, C ⍰ $[0, \pi]$ and A, B, C are in A.P., then $\frac{\sin A+\sin C}{\cos A+\cos C}$ is equal to

(A) $\sin B$

(B) $\cos B$

(C) $\cot B$

(D) $\tan B$

Ans. D

Rpp2

~~Rpp2~~

if $A, B, C \in [0, \pi]$, A, B, C are in A.P

$$2B = A + C$$

$$\Rightarrow \boxed{B = \frac{A+C}{2}}$$

ROHINI SOLANKI

Now-

$$\frac{\sin A + \sin C}{\cos A + \cos C}$$

$$= \frac{\cancel{2}\sin\left(\frac{A+C}{2}\right)\cdot\cancel{\cos\left(\frac{A-C}{2}\right)}}{\cancel{2}\cos\left(\frac{A+C}{2}\right)\cdot\cancel{\cos\left(\frac{A-C}{2}\right)}}$$

$$\Rightarrow \frac{\sin B}{\cos B}$$

$\Rightarrow$ $\tan B$ Answer.

QUESTION [VITEEE 2022]

(RPP 3)

The roots of the equation $\cos x + \sqrt{3}\sin x = 2\cos 2x$, are

(A) $-2n\pi + \frac{\pi}{3}, n \in Z$

(B) $\frac{2n\pi}{3} + \frac{\pi}{9}, n \in Z$

(C) $2n\pi - \frac{\pi}{3}, n \in Z$

(D) $\frac{2n\pi}{3} - \frac{\pi}{9}, n \in Z$

Que The roots of the equation $\cos x + \sqrt{3}\sin x = 2\cos 2x$, ...

(A) $-2n\pi + \frac{\pi}{3}$, $n \in Z$

(C) $2n\pi - \frac{\pi}{3}$, $n \in Z$

(B) $\frac{2n\pi}{3} + \frac{\pi}{9}$, $n \in Z$

(D) $\frac{2n\pi}{3} - \frac{\pi}{9}$, $n \in Z$

RPP 3
MOHIT SINGH
GHAZIABAD, U. P.

Ans $2\cos 2x = \cos x + \sqrt{3}\sin x$

multiply & divide by 2

$\cancel{2}\cos 2x = \cancel{2}\left(\frac{1}{2}\cos x + \frac{\sqrt{3}}{2}\sin x\right)$

$\cos x = \cos\alpha$
$x = 2n\pi \pm \alpha$

$\cos 2x = \cos\left(\frac{\pi}{3} - x\right)$

$2x = 2n\pi \pm \left(\frac{\pi}{3} - x\right)$

$x = -(\pi/3 + 2n\pi)$

$x = -2n\pi - \pi/3$

$2x = 2n\pi + \frac{\pi}{3} - x$

$2x = 2n\pi - \frac{\pi}{3} + x$

$x = 2(-n)\pi - \pi/3$

$x = 2N\pi - \pi/3$

$x = \frac{2n\pi}{3} + \frac{\pi}{9}$, $n \in Z$

$x = 2n\pi - \frac{\pi}{3}$, $n \in Z$

Mathematical Gyaan

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (ERT) Row operation:

1. Interchanging of Rows
2. multiplying a Row by some non zero number
3. Adding multiple of one Row to another Row.

$$\downarrow R_1 \leftrightarrow R_2$$

Ex: $E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ (Elementary matrix)

matrix obtained by applying a single ERT on a Identity Matrix is called Elementary MAtrix

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} c & d \\ a & b \end{bmatrix}$$

Ex: $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \xrightarrow{R_1 \to 2R_1} E = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

$$EA = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 2a & 2b \\ c & d \end{bmatrix}$$

EA gives a matrix Obtained by applying the same ERT on A as we applied on I to get E

ECT on $I \longrightarrow E$

AE given same ECT applied on A

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Big\downarrow C_2 \to 2C_2$$

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$$

$$AE = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} a & 2b \\ c & 2d \end{bmatrix}$$


THANK
YOU

PRAYAS

JEE 2025

Lecture- 06 | **Mathematics**

Relation & Functions

By- Ashish Agarwal Sir (IIT Kanpur)

Topics *to be covered*

1. Important Functions
2. Domain & Range Problems

Recap of previous lecture

1. Range is the set of outputs of functions and it is always a subset of codomain.

2. Two functions f & g are identical if $D_f = D_g$ & $R_f = R_g$ & $f(x) = g(x)$ $\forall x \in$ common Domain.
or they are identical if they have same graphs.

3. Range of odd degree polynomial defined over R is also R or $(-\infty, \infty)$

4. An even degree polynomial can never have range equal to R it is always a proper subset of R.

Recap of previous lecture

5. If for a polynomial function f, we have $f(x) + f(1/x) = f(x) \cdot f(1/x)$ then $f(x)$ can be $1+x^n$ or $1-x^n$ or 0 or 2

6. $\sqrt{\log_{g(x)} f(x)}$ is defined if $\log_{g(x)} f(x) \geq 0$, $f(x) > 0$, $g(x) > 0$, $g(x) \neq 1$

7. $\dfrac{1}{\sqrt{\log f(x)}}$ is defined if $\log f(x) > 0$, $f(x) > 0$.

8. $\dfrac{1}{f(x)}$ is defined if $f(x) \neq 0$.

Recap of previous lecture

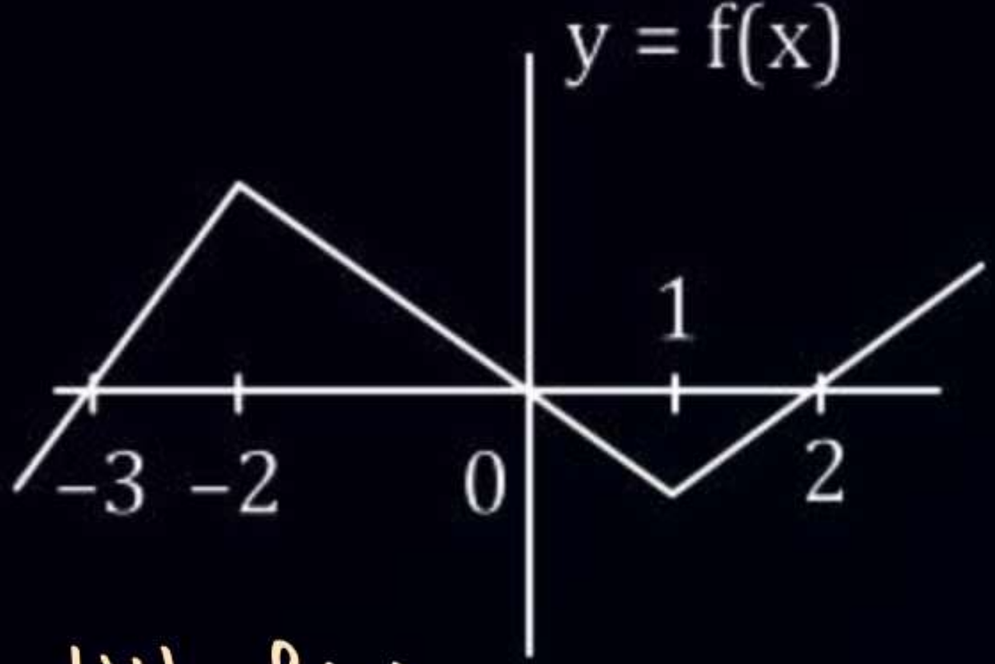
9. $\frac{1}{\sqrt{f(x)}}$ is defined if $f(x) > 0$

10. $\sqrt{f(x)}$ is defined if $f(x) \geq 0$

11. $\frac{1}{\log f(x)}$ is defined if $\log f(x) \neq 0,\ f(x) > 0$.

Recap *of previous lecture*

12.

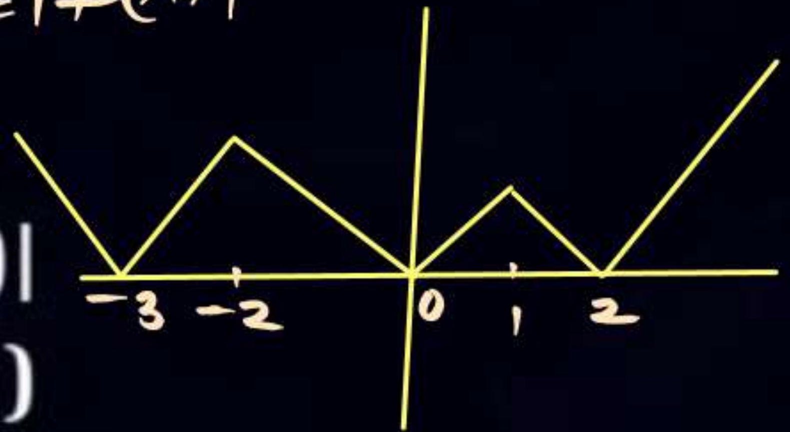


then draw $y = |f(x)|$

then draw $y = f(|x|)$

then draw $|y| = f(x)$

$y = |f(x)|$



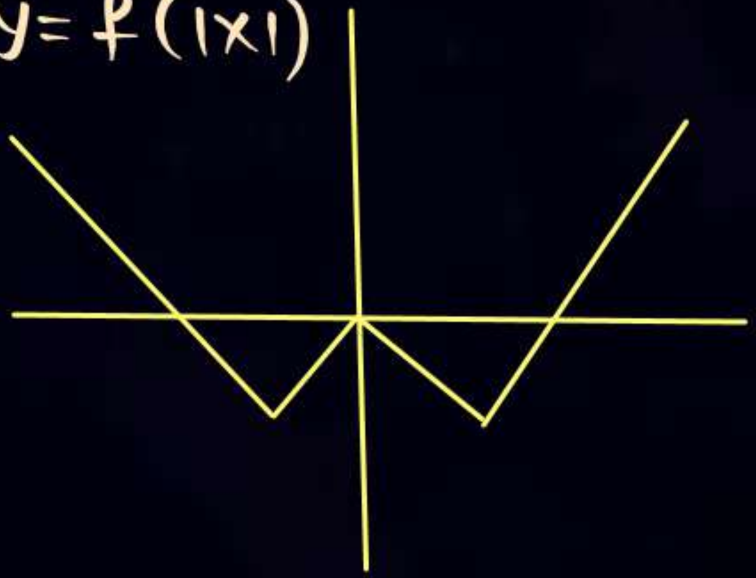
$|y| = f(x) \longrightarrow f(x) \geq 0$

$|y| = f(1)$

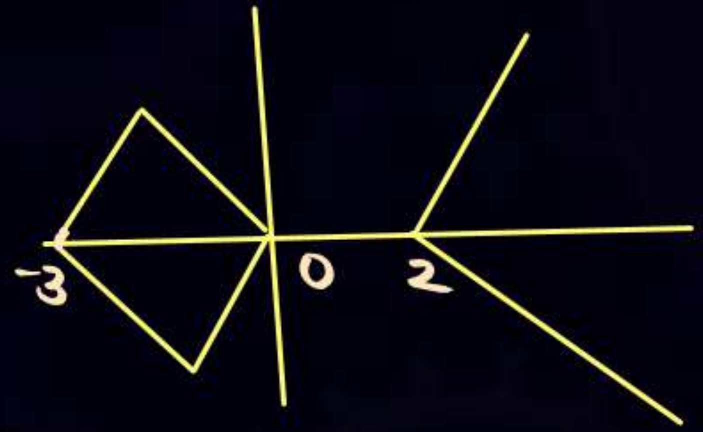
$y = f(1), -f(1)$

$|y| = f(2) \Rightarrow y = f(2), -f(2)$

$y = f(|x|)$



$|y| = f(x)$



Discussion of Homework of Previous Class

QUESTION

RPP 3

Values of k for which the inequality $k\sin^2 x - k\sin x + 1 \geq 0$ is true $\forall\, x \in R$ is

(A) $k > -\frac{1}{2}$

(B) $k > 4$

(C) $-\frac{1}{2} \leq k \leq 4$

(D) $\frac{1}{2} \leq k \leq 5$

M(1) $k\left(\sin^2 x - \sin x + \frac{1}{4} - \frac{1}{4}\right) + 1 \geq 0 \;\forall\, x \in R$

$k\left(\left(\sin x - \frac{1}{2}\right)^2 - \frac{1}{4}\right) + 1 \geq 0 \;\forall\, x \in R$

$k(\sin x - 1/2)^2 + 1 - \frac{k}{4} \geq 0 \;\forall\, x \in R.$

$k(\sin x - 1/2)^2 \geq k/4 - 1. \;\forall\, x \in R$

$[-1, 1]$

$[-3/2, 1/2] = [-3/2, 0] \cup [0, 1/2]$

$[0, 9/4]$

Ans. C

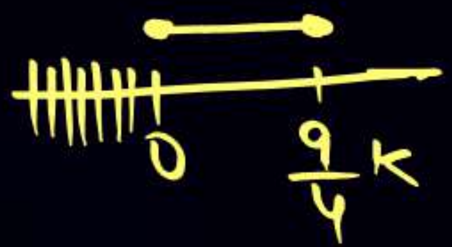
$$k\left(\sin x-\frac{1}{2}\right)^2 \geq k/4-1 \quad \forall x \in R$$

$\left(\sin x-\frac{1}{2}\right)^2 \in [0, 9/4]$

if $k>0$

$$k\left(\sin x-\frac{1}{2}\right)^2 \geq k/4-1$$

$k\left(\sin x-\frac{1}{2}\right)^2 \in \left[0, \frac{9}{4}k\right]$



$$\frac{k}{4}-1 \leq 0$$

$$k \leq 4$$

$$\Downarrow$$

$$k \in (0, 4]$$

if $k=0$

$$0 \geq -1$$

$$k=0$$

if $k<0$

$$k\left(\sin x-1/2\right)^2 \geq k/4-1$$

$k\left(\sin x-1/2\right)^2 \in \left[\frac{9}{4}k, 0\right]$

$$\frac{k}{4}-1 \leq \frac{9}{4}k.$$

$$-1 \leq 8k/4 = 2k$$

$$k \geq -1/2 \Rightarrow k \in [-1/2, 0)$$

$$k \in [-1/2, 4]$$

QUESTION

RPP 3

Values of k for which the inequality $k\sin^2 x - k\sin x + 1 \geq 0$ is true $\forall\, x \in R$ is

(A) $k > -\frac{1}{2}$

(B) $k > 4$

(C) $-\frac{1}{2} \leq k \leq 4$

(D) $\frac{1}{2} \leq k \leq 5$

M(2) put $\sin x = t \in [-1, 1]$

$kt^2 - kt + 1 \geq 0 \quad \forall\, t \in [-1, 1]$

$k > 0$

$kt^2 - kt + 1 \geq 0 \; \forall\, t \in [-1, 1]$

$y = kt^2 - kt + 1$

$\downarrow$

vertex at $t = 1/2$

$\longrightarrow D \leq 0 \quad k^2 - 4k \leq 0$

$k \in (0, 4]$

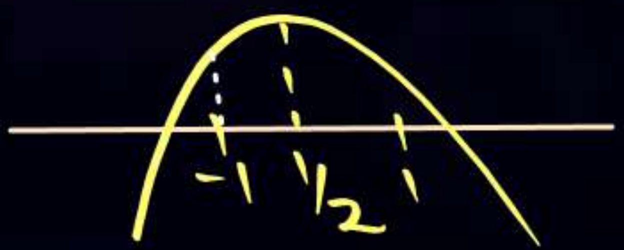
$k = 0$

$\Downarrow$

$1 \geq 0$

$k < 0$

$kt^2 - kt + 1 \geq 0 \quad t \in [-1, 1]$



$f(-1) \geq 0$ & $f(1) \geq 0$

$2k + 1 \geq 0$ $\quad$ $1 \geq 0$ true.

$k \geq -1/2$ But $k < 0$

$k \in [-1/2, 0)$

Ans. C

Ans $\underbrace{K \in [-\tfrac{1}{2}, 4]}$.

Some Important Functions

Algebraic functions: functions whose building blocks are polynomials, using $+, -, \times, \div$ & taking roots.

Ex: $\sqrt{x^2+x+1}$, $\dfrac{2x^2+3x+6}{x^2+x+1}$, $\sqrt[3]{x^2-x+1}+\dfrac{1}{\sqrt{x^2+x+1}}$ etc.

Functions which are not Algebraic are called Transcedental functions Ex: $\ln x, e^x, \sin x, \tan x$ ---

ILATE

Alg.

Transcedental fns.

Rational functions / fractional functions :

functions of type

$$\phi(x) = \frac{f(x)}{g(x)}$$ is called Rational fn

where $f(x), g(x)$ are polynomial fns where $g(x)$ is not the zero polynomial.

① $f(x) = \dfrac{ax+b}{cx+d}$ → Range: $R - \left\{\frac{a}{c}\right\}$.

② $f(x) = \dfrac{\cancel{(ax+b)}(cx+d)}{(ex+f)\cancel{(ax+b)}}$ — Range: $R - \left\{\frac{c}{e}, f\left(-\frac{b}{a}\right)\right\}$.

$f(x) = \dfrac{cx+d}{ex+f}, \; x \neq -b/a$

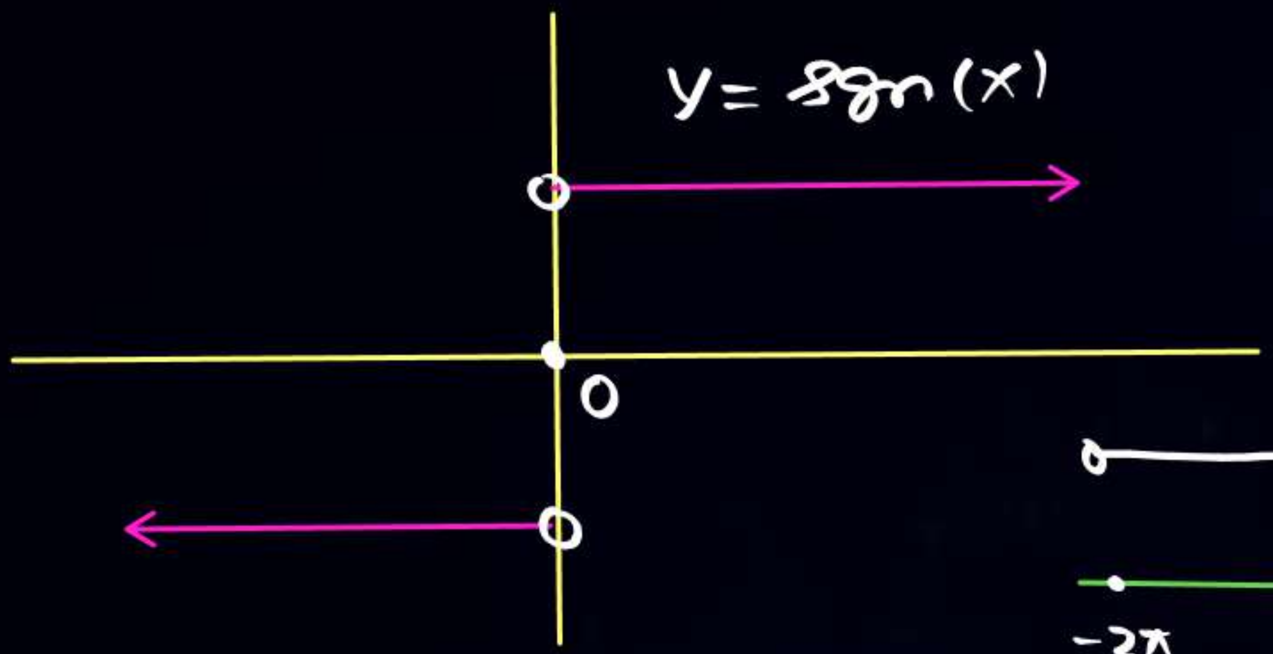
Every Rational fn is Algebraic but not the converse

Signum function / Sign function

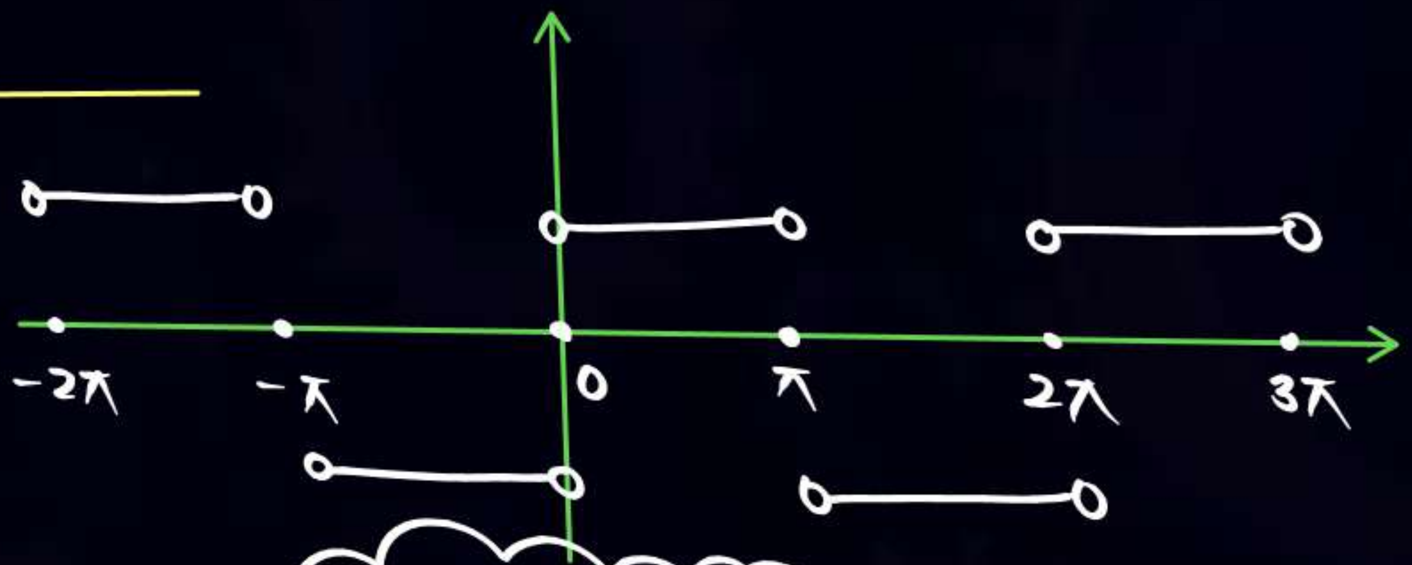
$$\operatorname{sgn}(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

Domain : R

Range : $\{-1, 0, 1\}$



Ex: $f(x) = \operatorname{sgn}(\sin x)$



periodic Yes.

$T = 2\pi$

$$\operatorname{sgn}(\operatorname{sgn}(x)) = \begin{cases} \operatorname{sgn}(1) & x>0 \\ \operatorname{sgn}(0) & x=0 \\ \operatorname{sgn}(-1) & x<0 \end{cases} = \begin{cases} 1 & x>0 \\ 0 & x=0 \\ -1 & x<0 \end{cases} = \operatorname{sgn} x$$

$$\operatorname{sgn}(\operatorname{sgn}(x)) = \operatorname{sgn}(x)$$

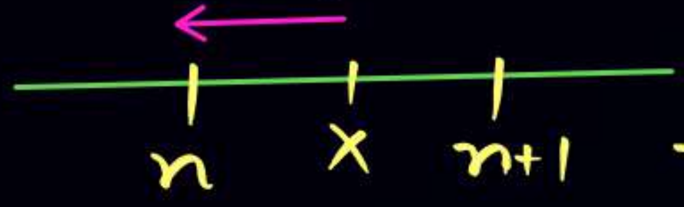
$$\operatorname{sgn}(\operatorname{sgn}(\operatorname{sgn}\ldots(\operatorname{sgn}(x)))\ldots) = \operatorname{sgn}(x).$$

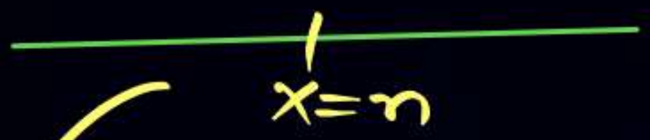
Yaad Rakho

GREATEST INTEGER FUNCTION (GIF) / staircase funct / stepup fn.

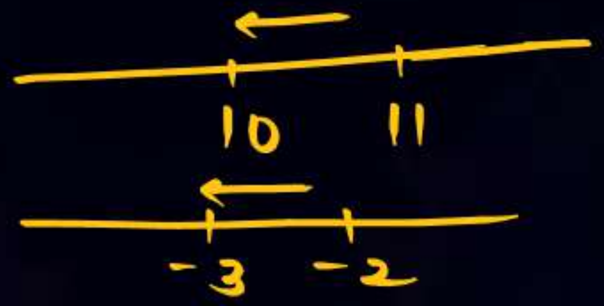
$f(x) = [x]$

$x \notin I$ $\qquad$ $n \in I$ $\qquad$ $x \in I$





$$[x] = n$$



Ex: $[2.68] = 2$

Ex: $[3.35] = 3$

Ex: $[-1.64] = -2$

Ex: $[\pi] = 3$

Ex: $[e] = 2$

Ex: $[9] = 9$

Ex: $[-6.35] = -7$

Ex: $[x] = 10 \Rightarrow x \in [10, 11)$

Ex: $[x] = -3 \Rightarrow x \in [-3, -2)$

Ex: $[x] = -4 \Rightarrow x \in [-4, -3)$

Ex: $[x] = 2.5$ — (No soln)

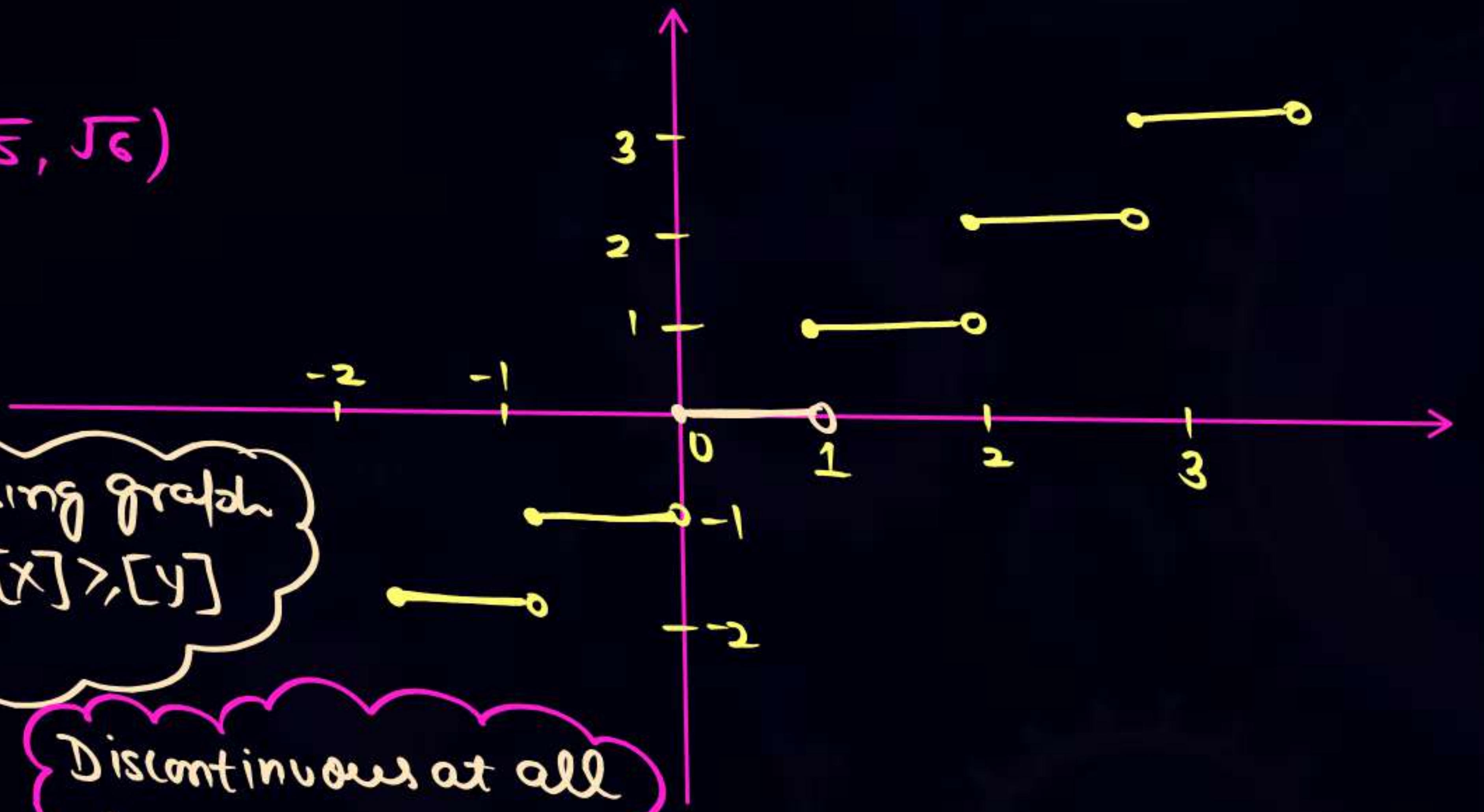
$[x]$ is always an Integer

Ex: $[x^2] = 5$

$x^2 \in [5, 6)$

$x \in (-\sqrt{6}, -\sqrt{5}] \cup [\sqrt{5}, \sqrt{6})$

Graph of $y = [x]$



Non decreasing graph
$\because$ if $x > y \Rightarrow [x] \geq [y]$

Discontinuous at all Integers

Properties.

P ① $[x]+[-x]=\begin{cases}0 & x\in I\\ -1 & x\notin I\end{cases}$

proof: Case ① if $x \in I$

say $x = n \in I \Rightarrow [x]=[n]=n.$

$\Rightarrow -x=-n\in I \Rightarrow [-x]=[-n]=-n$

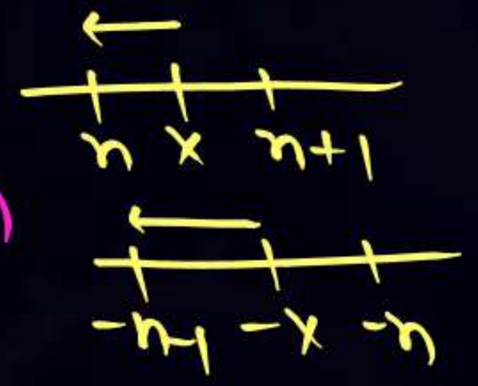
$[x]+[-x]=0.$

Case ⑪ if $x \notin I$

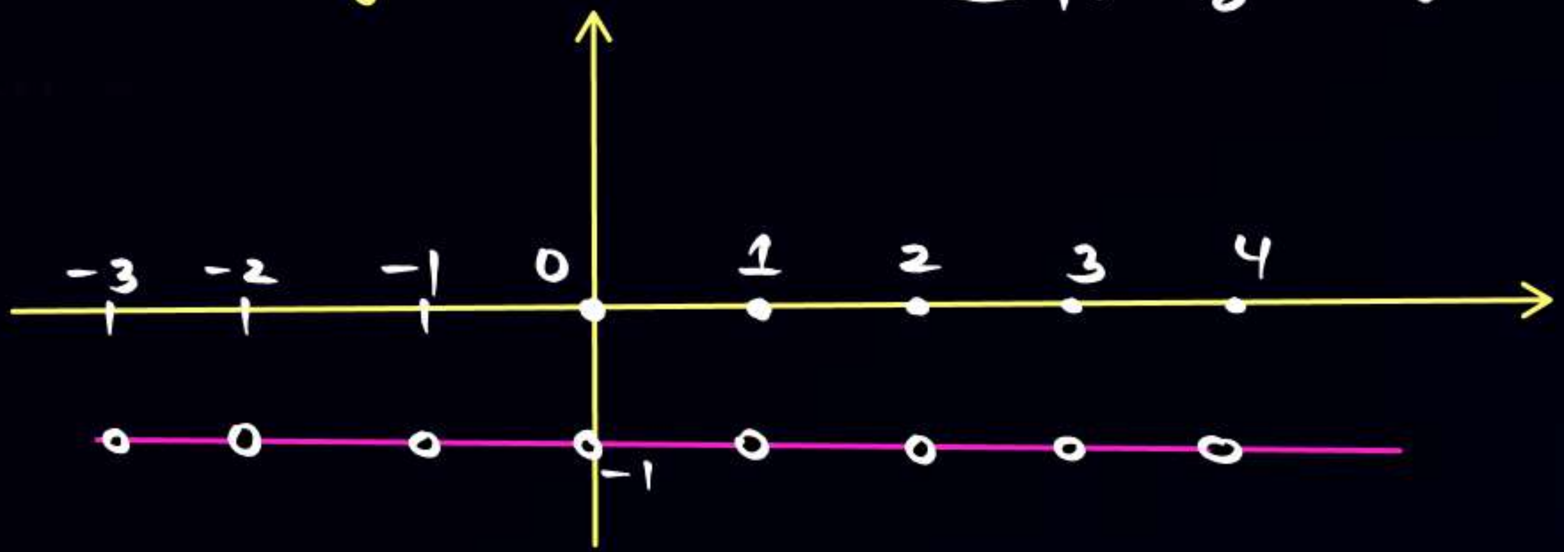
then $n<x<n+1$ $\quad -n-1<-x<-n$

$[x]=n \quad [-x]=-n-1$

$[x]+[-x]=-1$



$$y = [x] + [-x] \begin{cases} \text{Domain} = R \\ \text{Range} = \{0, -1\} \end{cases}$$



P② $[x+m] = [x] + m, \ m \in I, \ x \in R$

proof: Case① $x \in I$

say $x = n \in I$ — $x + m = n + m \in I$

$\Downarrow$ $[x] = n$ $\qquad$ $\Downarrow$ $[x+m] = [n+m] = n+m = [x] + m.$

Case (ii) If $x \notin I$

$n < x < n+1$ $\qquad$ $n+m < x+m < n+m+1$

$[x] = n$ $\qquad$ $\Downarrow$ $[x+m] = n+m$

$[x+m] = [x]+m.$

Ex: $[x-2] = [x+(-2)]$

$= [x]+(-2)$

$= [x]-2.$

Gadho | Gadhiyaa Aisaa naa karo

Ex: $[2x+3] = [2x]+3 = 2[x]+3$

Ex: $\left[\frac{3x}{5}\right] = \frac{3}{5}[x]$

Ex: $\left[\frac{x}{5}\right] = \frac{[x]}{5}.$

P③ Mostly used Sandwich Theorem

left side move karne pe no:s kum hotay hai $\longleftarrow$ 0 $\longrightarrow$ Right side move karne pe numbers bhadte hai

$$[x] \le x < [x]+1.$$

Case ① if $x \in I$

say $x = n,\ n \in I \Rightarrow [x] = n = x < n+1$

$$\Downarrow$$

$$[x] = x < [x]+1.$$

Case ⑪ $x \notin I \Rightarrow$  $\Rightarrow [x] = n$

$[x] = n < x < n+1$

$[x] < x < [x]+1$

$$[x] \le x < [x]+1$$

P(4) Naye Packet mai bechaay Tumhay cheez puraani

$$[x] \le x < [x]+1$$

$$[x] \le x \qquad [x] > x-1$$

$$x-1 < [x] \le x$$

Yaad Rakhe!!

★ $[x] = x \iff x \in I$

★ $[x] = 0 \iff x \in [0,1)$

★ $[x]$ — Domain : R; Range : I

Properties of Greatest Integer Function

(i) $[x] \leq x < [x] + 1$ and $x - 1 < [x] \leq x, 0 \leq x - [x] < 1$

(ii) $[x + m] = [x] + m$, if m is an integer.

(iii) $[x] + [-x] = \begin{cases} 0, & x \in I \\ -1, & x \notin I \end{cases}$

Least Integer function

The function f(x)= ⌈x⌉ is called least Integer function It represents the least integer greater than or equal to x.

$$f(x) = \lceil x \rceil$$

$x \notin I$ move Right $\longrightarrow$

$n \quad x \quad n+1$

$\lceil x \rceil = n+1$

$x = n+1$

Ex: $\lceil 2.5 \rceil = 3$

Ex: $\lceil -9.87 \rceil = -9$

Ex: $\lceil 5 \rceil = 5$

Ex $\lceil -4 \rceil = -4$.

Relation b/w $[x]$ & $\lceil x \rceil$

$x \notin I$

$n \quad x \quad n+1$

$[x] = n \quad \lceil x \rceil = n+1$

$\lceil x \rceil = [x] + 1$

$x \in I$

$x = n$

$\lceil x \rceil = [x] = n.$

$$\lceil x \rceil = \begin{cases} [x]+1 & \text{if } x \notin I \\ [x] & \text{if } x \in I \end{cases}$$

Name kaa Karan !!

$\lfloor 9.5 \rfloor = 9$, $\lceil 9.5 \rceil = 10$

Fractional Part Function

It is defined as : $g(x) = \{x\} = x - [x]$.

$x = I + f$, $f \in [0,1)$

I → Greatest Integer

f → fractional part

Ex: $-4.86 = -4 - 0.86$

$= -4 \underbrace{-1+1}- 0.86$

$= -5 + 0.14$

$[-4.86] = -5$

$\{-4.86\} = -4.86 - [-4.86] = -4.86 + 5 = 0.14$

Ex: $7.98 = 7 + 0.98$

$7 = [7.98]$

$0.98 = 7.98 - [7.98] = 7.98 - 7 = 0.98$

Ex: $5.67 = 5 + 0.67$

$5 = [5.67]$, $0.67 = \{5.67\} = 5.67 - [5.67] = 0.67$.

Properties:

P① $0 \le \{x\} < 1$

fractional part of x always has value in $[0,1)$.

proof:

we know $\{x\} = x - [x]$

also we know

$[x] \le x < [x]+1$

$0 \le x - [x] < 1$

$0 \le \{x\} < 1$

$\{x\} \in [0,1)$ H.P

$[x] = x \Leftrightarrow x \in I$

$\{x\} = 0 \Leftrightarrow x \in I$

$\{x\} = x - [x]$ if $x \in I$ then $[x] = x$

$\{x\} = x - x = 0.$

P② $[\{x\}] = 0 \quad \{x\} \in [0,1) \Rightarrow [\{x\}] = 0.$

$\{[x]\} = 0 \quad [x]$ is always an integer $\Rightarrow [x] \in I$

$$\Rightarrow \{[x]\} = 0.$$

P③ $\{x+m\} = \{x\}, \; m \in I$

$$\boxed{[x+m] = [x] + m, \; m \in I}$$

<u>proof</u>: $\{x+m\} = x+m-[x+m]$ (by defn)

$$= x + \cancel{m} - ([x] + \cancel{m})$$

$$= x - [x] = \{x\}$$

P④ $\{x\}+\{-x\}=\begin{cases}0 & x\in I\\ 1 & x\notin I\end{cases}$ $\rightarrow$ $[x]+[-x]=\begin{cases}0 & x\in I\\ -1 & x\notin I\end{cases}$

proof:

$$\{x\}+\{-x\}=\cancel{x}-[x]+(-\cancel{x}-[-x])$$

$$=-([x]+[-x])$$

$x\in I$

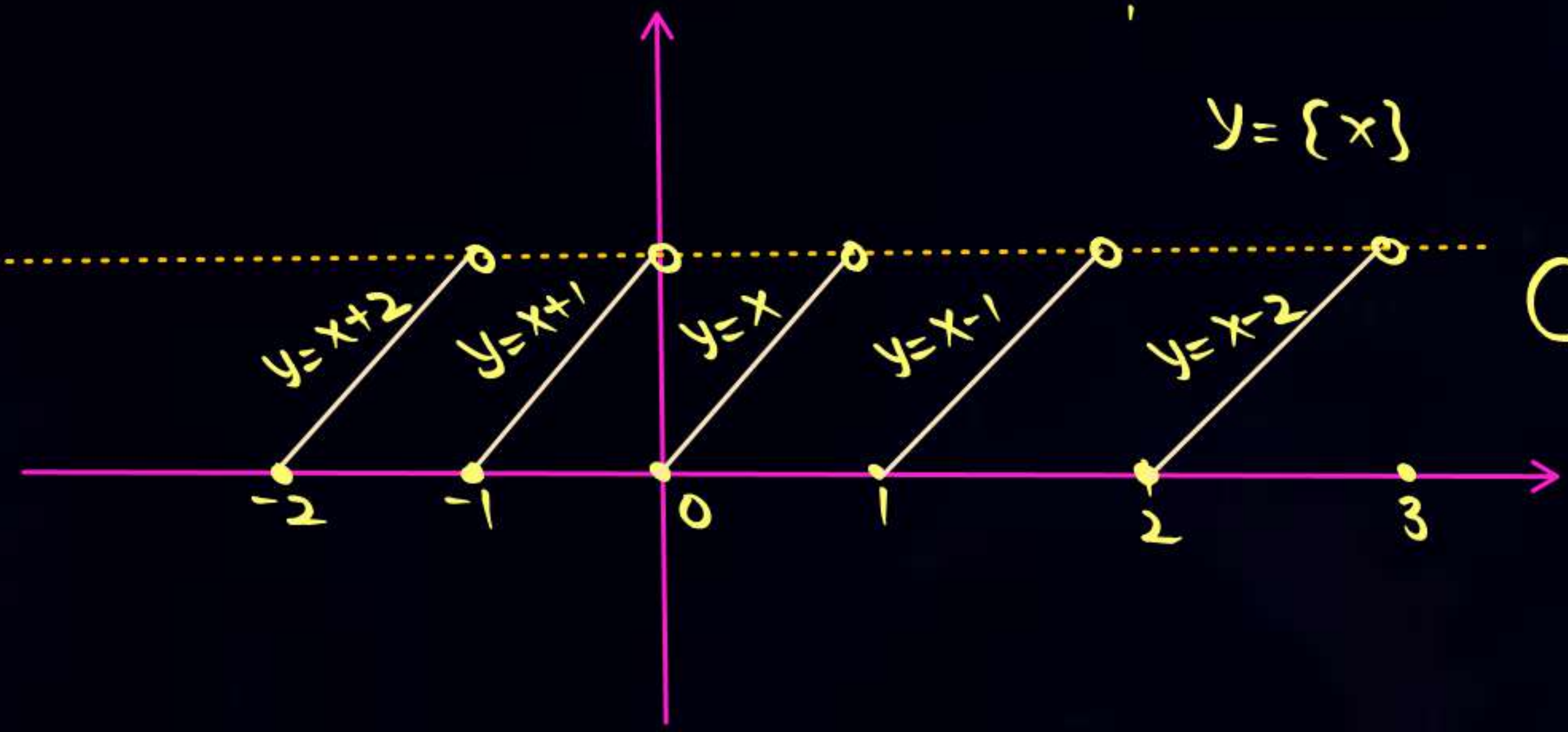
$\downarrow$

$\{x\}+\{-x\}=0$

$x\notin I$

$\{x\}+\{-x\}=-([x]+[-x])=-(-1)=1$

<u>Graph</u> $y = \{x\} = x - [x] = \begin{cases} x+2 & -2 \le x < -1 \\ x+1 & -1 \le x < 0 \quad [x] = -1 \\ x & 0 \le x < 1 \quad [x] = 0 \\ x-1 & 1 \le x < 2 \quad [x] = 1 \\ x-2 & 2 \le x < 3 \quad [x] = 2 \end{cases}$



periodic : T = 1

Tah(1) Draw graph of $y = \lceil x \rceil$

$$[x + \underbrace{[x]}_{\text{Integer}}] = [x] + [x] = 2[x]$$

$$[x + \underbrace{[x + [x]]}] = [x + 2\underbrace{[x]}_{\text{Integer}}] = 2[x] + [x] = 3[x]$$

$$\vdots \qquad \qquad \vdots$$

$$\underbrace{[x + [x + [x \ldots + [x + [x]]] \ldots]}_{n \text{ times}} = n[x]$$

Properties of Fractional Part Function

(i) $0 \le \{x\} < 1$

(ii) $\{[x]\} = [\{x\}] = 0$

(iii) $\{\{x\}\} = \{x\}$

(iv) $\{x + m\} = \{x\}, m \in I$

(v) $\{x\} + \{-x\} = \begin{cases} 1, & x \notin I \\ 0, & x \in I \end{cases}$

Problems on Domain of Functions

QUESTION

DMS

The domain of the function $f(x) = \sqrt{\frac{(x+1)(x-3)}{(x-2)}}$ is

(A) $[-1, 2) \cup [3, \infty)$

(B) $(-1, 2) \cup [3, \infty)$

(C) $[-1, 2] \cup [3, \infty)$

(D) None of these

$$\frac{(x+1)(x-3)}{(x-2)} \geq 0$$

$$\underset{-1}{\overset{-\quad +}{|}}\quad\underset{2}{\overset{-}{|}}\quad\underset{3}{\overset{+}{|}}$$

$x \in [-1, 2) \cup [3, \infty)$

QUESTION

DMS

The domain of function $\frac{1}{\sqrt[3]{(x-1)(x-2)(x-4)}}$

A $(1, 2) \cup (4, \infty)$

B $R - \{1, 2, 4\}$

C $\mathbb{R}$

D None of these

$(x-1)(x-2)(x-4) \neq 0$

$x \neq 1, 2, 4$

QUESTION

DMS

The domain of function $\dfrac{1}{\sqrt{x(x-2)(x-3)}}$

(A) $(0, 2)$

(B) $\mathbb{R} \sim \{0, 2, 3\}$

(C) $(0, 2) \cup (3, \infty)$

(D) None of these

$x(x-2)(x-3) > 0$

$-$ 0 $+$ 2 $-$ 3 $+$

$x \in (0,2) \cup (3, \infty)$

QUESTION

DMS

Find the domain of following functions :

(i) $y = \sqrt{5 - 2x}$

$5 - 2x \geq 0$

$x \leq 5/2$

$\downarrow$

Domain: $(-\infty, 5/2]$

(ii) $y = \frac{1}{\sqrt{x - |x|}}$

$x - |x| > 0$

$|x| < x$

$\Downarrow$

$x \in \phi$

$|-2| \geq -2$

$|3| \geq 3$

$|x| \geq x$

$|x| = x \iff x \in [0, \infty)$

$|x| > x \iff x \in (-\infty, 0)$

QUESTION

Find Domain of $f(x) = \sqrt{x^2 - 3x + 2} + \frac{1}{\sqrt{x^2-3x-4}}$

$f(x) + g(x)$

Domain $= D_f \cap D_g$

$x^2 - 3x + 2 \geq 0$

$(x-1)(x-2) \geq 0$

$x \in (-\infty, 1] \cup [2, \infty)$

$x^2 - 3x - 4 > 0$

$(x-4)(x+1) > 0$

$x \in (-\infty, -1) \cup (4, \infty)$

$\cap$

$x \in (-\infty, -1) \cup (4, \infty)$

QUESTION [JEE Mains 2021]

If the functions are defined as $f(x)=\sqrt{x}$ and $g(x)=\sqrt{1-x}$, then what is the common domain of the following functions: $f+g, f-g, f/g, g/f, g-f$ where $(f \pm g)(x)=f(x) \pm g(x), (f/g)(x)=\frac{f(x)}{g(x)}$.

(A) $0 \leq x < 1$

(B) $0 < x < 1$

(C) $0 < x \leq 1$

(D) $0 \leq x \leq 1$

$f(x)=\sqrt{x}$, $g(x)=\sqrt{1-x}$ — $D_f : [0,\infty)$, $D_g : (-\infty,1]$

$(f+g)(x)=f(x)+g(x)=\sqrt{x}+\sqrt{1-x}$

$(f-g)(x)=f(x)-g(x)=\sqrt{x}-\sqrt{1-x}$

$D_f \cap D_g = [0,1]$

$\left(\frac{f}{g}\right)(x)=\frac{f(x)}{g(x)}=\frac{\sqrt{x}}{\sqrt{1-x}}$ — $D_f \cap D_g - \{x \mid g(x)=0\}$

$= [0,1]-\{1\}=[0,1)$

$\left(\frac{g}{f}\right)(x)=\frac{g(x)}{f(x)}=\frac{\sqrt{1-x}}{\sqrt{x}}$ — $D_f \cap D_g - \{x \mid f(x)=0\} = [0,1]-\{0\}=(0,1]$

$(g-f)(x)=g(x)-f(x)$ — $D_f \cap D_g = [0,1]$

QUESTION

ASRQ

Given $f(x)$ is a polynomial function of x, $f(x)\cdot f(y)=f(x)+f(y)+f(xy)-2$ for all $x, y \in R$ and that $f(2)=5$ Then $f(3)$ is equal to

(A) 10

(B) 24

(C) 15

(D) none

$f(x)\cdot f(y)=f(x)+f(y)+f(xy)-2 \quad \forall\ x,y\in R \quad \text{— (I)}, \quad f(2)=5$

put $y=\frac{1}{x}$

$f(x)\cdot f\left(\frac{1}{x}\right)=f(x)+f(1/x)+f(1)-2. \quad \text{— (II)}$

put $x=1, y=2$ in (I)

$f(1)\cdot f(2)=f(1)+f(2)+f(2)-2.$

$5f(1)=f(1)+10-2.$

$4f(1)=8$

$f(1)=2$

$f\left(\frac{1}{x}\right)\cdot f(x)=f(1/x)+f(x)$

$\Downarrow$

$f(x)=1\pm x^n$

$f(2)=1\pm 2^n=5.$

$1\pm 2^n=5$

Taking +ve sign

$$2^n = 4$$

$$n = 2$$

$$\Downarrow$$

$$f(x) = 1 + x^2$$

$$f(3) = 10.$$

QUESTION

Tan 2

ASRQ

Let f be a polynomial function which satisfies the relation

$f(x)+f\left(\frac{x}{y^2}\right)+f\left(\frac{x}{y}\right)=f(x)\cdot f\left(\frac{1}{y}\right)-\frac{1}{y^3}+\frac{x^3}{y^6}+2\ \forall\ x\in R-\{0\}, f(1)\neq 1$ and $f(2)=9$.

The value of $\sum_{r=1}^{100} f(r)$ equals

(A) 5050

(B) $(5050)^2$

(C) $100+(5050)^2$

(D) $100+(5050)^3$

QUESTION

$$f(x) = \sqrt{\log_2\left(\frac{5x-x^2}{4}\right)} \text{ or } \sqrt{\log_{\frac{1}{2}}\frac{5x-x^2}{4}}$$

find Domain

$\log_{\frac{1}{2}}\left(\frac{5x-x^2}{4}\right) \geq 0$ & $\frac{5x-x^2}{4} > 0$.

base $= 1/2 \in (0,1)$

$\Downarrow$

log is dec.

$\frac{5x-x^2}{4} \leq (1/2)^0$

$5x-x^2 \leq 4$.

$x^2-5x+4 \geq 0$

$(x-1)(x-4) \geq 0$

$x \in (-\infty, 1] \cup [4, \infty)$

$5x-x^2 > 0$

$x(x-5) < 0$

$x \in (0,5)$

$\cap$

$x \in (0,1] \cup [4,5) =$ Domain

QUESTION

Find Domain of following functions

(i) $f(x) = \sqrt{\cos 2x} + \sqrt{16 - x^2}$

(ii) $f(x) = \log_7 \log_5 \log_3 \log_2\left(2x^3 + 5x^2 - 14x\right)$

(iii) $f(x) = \log_{100x}\left(\frac{2\log_{10} x+1}{-x}\right)$

$A \cap B \subseteq A, B$

(1) $f(x) = \sqrt{\cos 2x} + \sqrt{16 - x^2}$

$\cos 2x \geq 0$

$y = \cos\theta$

$\theta = 2x$

$2x \in \left[2n\pi - \pi/2, 2n\pi + \pi/2\right]$

$16 - x^2 \geq 0$

$x^2 - 16 \leq 0$

$(x-4)(x+4) \leq 0$

$+ \quad - \quad +$

$-4 \quad 4$

$x \in [-4, 4]$ — (1)

$$2x \in \left[2n\pi - \frac{\pi}{2},\ 2n\pi + \frac{\pi}{2}\right] \quad \& \quad x \in [-4, 4]$$

$$x \in [n\pi - \pi/4,\ n\pi + \pi/4] \ \cap$$

$\underline{n=0}$ $\quad [-\pi/4, \pi/4]$

$\underline{n=1}$ $\quad [3\pi/4, 5\pi/4]$

$\underline{n=-1}$ $\quad [-5\pi/4, -3\pi/4]$

$$\left[-\frac{5\pi}{4}, -\frac{3\pi}{4}\right] \cup [-\pi/4, \pi/4] \cup \left[3\pi/4, \frac{5\pi}{4}\right]$$

QUESTION

Find Domain of following functions

(i) $f(x) = \sqrt{\cos 2x} + \sqrt{16 - x^2}$

(ii) $f(x) = \log_7 \log_5 \log_3 \log_2 \left(2x^3 + 5x^2 - 14x\right)$ Tan3

(iii) $f(x) = \log_{100x}\left(\frac{2\log_{10} x+1}{-x}\right)$

(iii) $f(x) = \log_{100x}\left(\frac{2\log_{10} x + 1}{-x}\right)$ Clearly $x>0$

$100x>0$ & $100x \neq 1$, $\frac{2\log_{10} x + 1}{-x} > 0$

$x>0$ & $x \neq \frac{1}{100}$ & $2\log_{10} x + 1 < 0.$

$\log_{10} x < -1/2 \Rightarrow x < 10^{-1/2} \Rightarrow x < \frac{1}{\sqrt{10}}$

$x > 0,\ x \neq \frac{1}{100}\ \&\ x < \frac{1}{\sqrt{10}}$

$x \in \left(0, \frac{1}{\sqrt{10}}\right) - \left\{\frac{1}{100}\right\}.$

Sabse Important Baat Yaad Rahe

Sabhi Class Illustrations Retry Karnay hai...

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with good Rank

Class illustrations

Module,DPP

KTK,TAH CHALLENGER

QUESTION [JEE Mains 2024 (9 April)]

(KTK 1)

Let the range of the function $f(x) = \frac{1}{2+\sin 3x+\cos 3x}, x \in \mathbb{R}$ be $[a, b]$. If α and β are respectively the A.M. and the G.M. of a and b, then $\frac{\alpha}{\beta}$ is equal to

(A) π

(B) $\sqrt{\pi}$

(C) $\sqrt{2}$

(D) 2

Ans. C

QUESTION [JEE Mains 2024 (1 Feb)]

(KTK 2)

If the domain of the function

$f(x) = \frac{\sqrt{x^2-25}}{(4-x^2)} + \log_{10}(x^2 + 2x - 15)$ is $(-\infty, \alpha) \cup [\beta, \infty)$, then $\alpha^2 + \beta^3$ is equal to

(A) 140

(B) 175

(C) 125

(D) 150

Ans. D

QUESTION [JEE Mains 2024 (30 Jan)]

(KTK 3)

If the domain of the function $f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right) + \{\log_e(3-x)\}^{-1}$ is $[-\alpha, \beta) - \{\gamma\}$, then $\alpha + \beta + \gamma$ is equal to :

(A) 11

(B) 12

(C) 9

(D) 8

Ans. A

QUESTION [JEE Mains 2021 (1 Sep)]

(KTK 4)

The range of the function,

$$f(x) = \log_{\sqrt{5}}\left(3 + \cos\left(\frac{3x}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{3\pi}{4} - x\right)\right) \text{ is}$$

(A) $(0, \sqrt{5})$

(B) $[-2, 2]$

(C) $\left[\frac{1}{\sqrt{5}}, \sqrt{5}\right]$

(D) $[0, 2]$

Ans. D

QUESTION [JEE Mains 2020 (8 Jan)]

(KTK 5)

Let $f:(1,3) \to R$ be a function defined by $f(x) = \frac{x[x]}{1+x^2}$, where $[x]$ denotes the greatest integer $\leq x$. Then the range of f is

A $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{4}, \frac{4}{5}\right]$

B $\left(\frac{3}{5}, \frac{4}{5}\right)$

C $\left(\frac{2}{5}, \frac{4}{5}\right]$

D $\left(\frac{2}{5}, \frac{3}{5}\right] \cup \left(\frac{3}{4}, \frac{4}{5}\right)$

Ans. A

(Revision Practice Problems)

QUESTION [JEE Mains 2024 (8 April)]

(RPP 1)

The sum of all the solutions of the equation $(8)^{2x} - 16 \cdot (8)^{x} + 48 = 0$ is :

(A) $1 + \log_8(6)$

(B) $1 + \log_6(8)$

(C) $\log_8(6)$

(D) $\log_8(4)$

Ans. A

QUESTION [JEE Mains 2024 (8 April)]

(RPP 2)

Let α, β be the roots of the equation $x^2 + 2\sqrt{2}x - 1 = 0$. The quadratic equation, whose roots are $\alpha^4 + \beta^4$ and $\frac{1}{10}(\alpha^6 + \beta^6)$, is:

(A) $x^2 - 180x + 9506 = 0$

(B) $x^2 - 195x + 9506 = 0$

(C) $x^2 - 190x + 9466 = 0$

(D) $x^2 - 195x + 9466 = 0$

Ans. B

QUESTION [JEE Mains 2024 (1 Feb)]

(RPP 3)

If $\tan A = \frac{1}{\sqrt{x(x^2+x+1)}}$, $\tan B = \frac{\sqrt{x}}{\sqrt{x^2+x+1}}$ and $\tan C = \left(x^{-3}+x^{-2}+x^{-1}\right)^{1/2}$,

$0 < A, B, C < \frac{\pi}{2}$, then $A + B$ is equal to :

(A) C

(B) $\pi - C$

(C) $2\pi - C$

(D) $\frac{\pi}{2} - C$

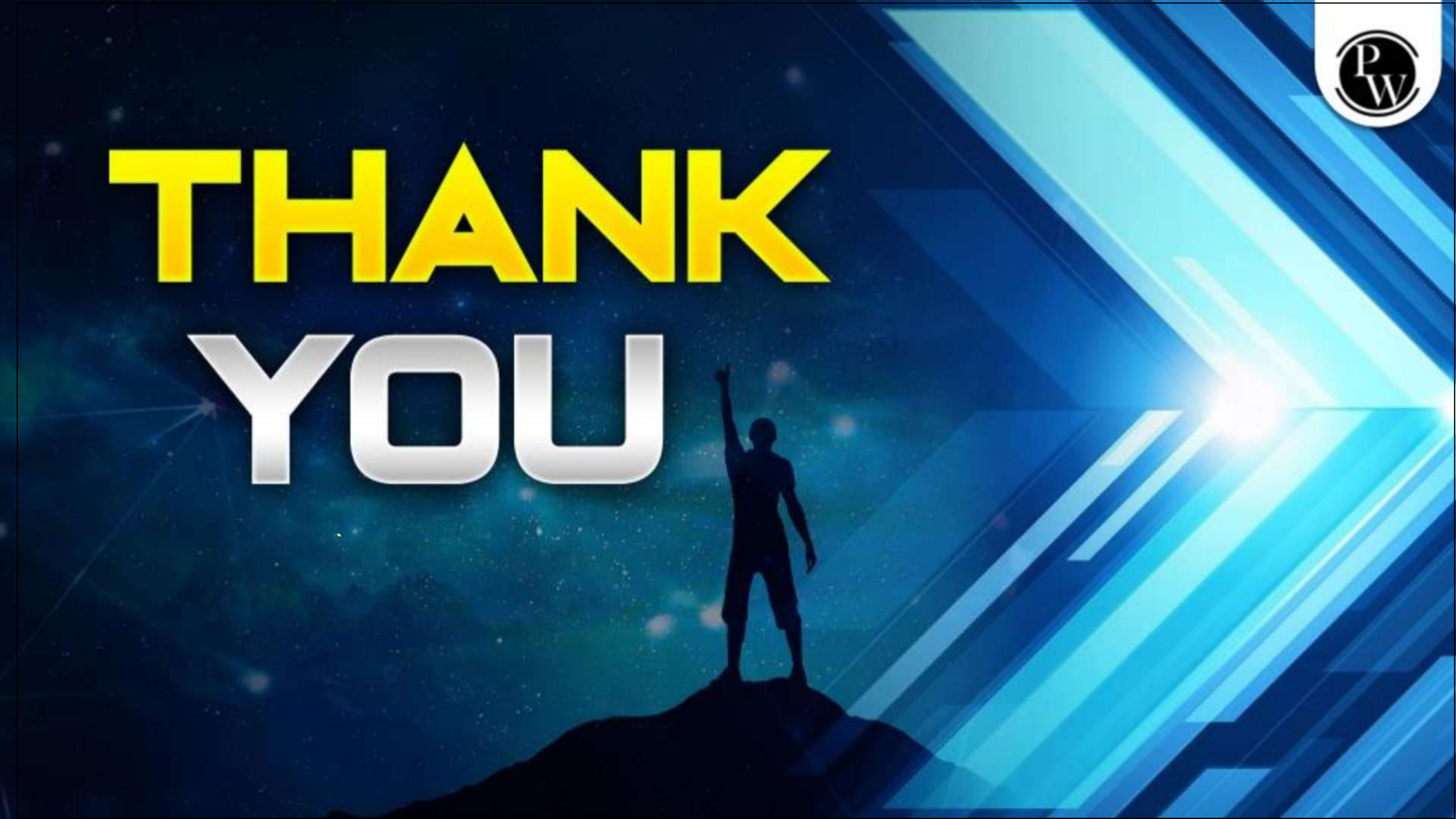
Ans. C

Homework from Module

Chapter: SETS

Prarambh: COMPLETE

Prabal : COMPLETE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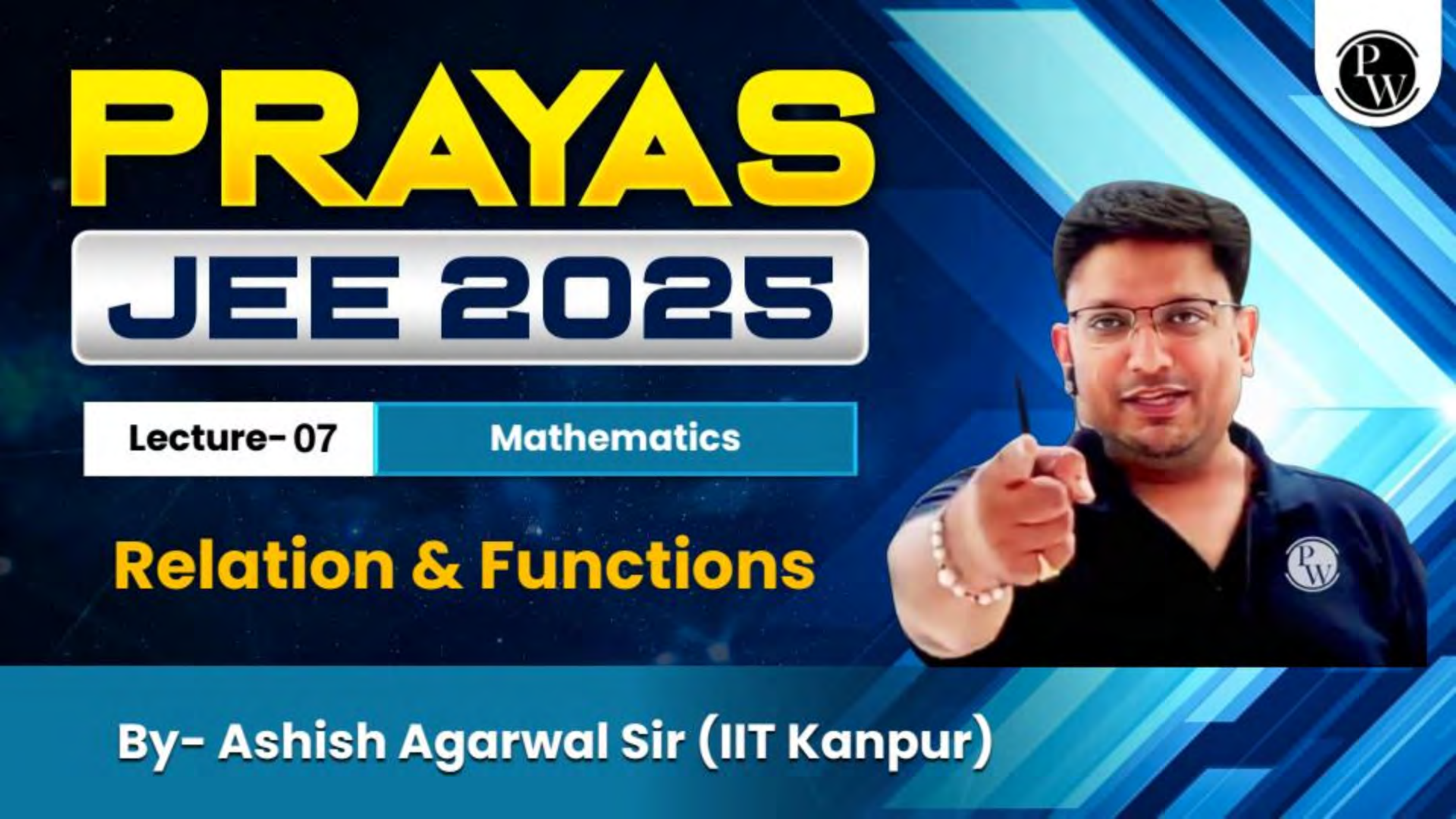
YOU


PRAYAS
JEE 2025
Lecture- 07
Mathematics
Relation & Functions
By- Ashish Agarwal Sir (IIT Kanpur)

Topics *to be covered*

1. Domain & Range Problems
2. Classification of Functions

Problems on Domain of Functions

QUESTION

ASRQ

$f(x) = \ln\left(\sqrt{x^2 - 5x - 24} - x - 2\right)$, find Domain of f.

$\sqrt{x^2-5x-24} - (x+2) > 0$ & $x^2-5x-24 \geq 0$

$(x-8)(x+3) \geq 0$

$\sqrt{x^2-5x-24} > x+2$

$x \in (-\infty, -3] \cup [8, \infty)$ — 1

Case 1 if $x+2 \geq 0 \Rightarrow x \geq -2$

$\sqrt{x^2-5x-24} > x+2$

S.B.S

$x^2 - 5x - 24 > x^2 + 4 + 4x$

$9x < -28$

$x < -28/9$

$x \in \phi$

Case 11 if $x+2 < 0 \Rightarrow x < -2$

$\sqrt{x^2-5x-24} > x+2$

always true

$\Downarrow$

$x \in (-\infty, -2)$.

$\cup$

$x \in (-\infty, -2)$ — 11

$\underline{(1) \cap (11)}$

$x \in (-\infty, -3]$

QUESTION

ASRQ + KCLS

$$f(x) = \sqrt{(x^2 - 3x - 10) \cdot \ln^2(x-3)}$$

$(x^2-3x-10)\cdot \underbrace{\ln^2(x-3)}_{\geq 0} \geq 0$ & $x-3>0$

$x>3$ — (I)

$(x-5)(x+2) \geq 0$, $\ln(x-3)=0$ is also possible

$x \in (-\infty,-2] \cup [5,\infty)$, $x-3=1$ is also possible

$x=4$ is also possible.

$x \in (-\infty,-2] \cup [5,\infty) \cup \{4\}$ — (II)

$\cap$

$x \in [5,\infty) \cup \{4\}$.

QUESTION

ASRQ

If the domain of $g(x)$ is $[3, 4]$, then the domain of $g\left(\log_2\left(x^2+3x-2\right)\right)$ is

(A) $[-4,-1] \cup [2,7]$

(B) $[-3,2]$

(C) $[-6,-5] \cup [2,3]$

(D) $\left[\frac{3}{2}, 5\right]$

$y = g(\underset{[3,4]}{x})$

$y = g(\underbrace{\log_2(x^2+3x-2)}_{[3,4]})$

$3 \le \log_2(x^2+3x-2) \le 4$

$2^3 \le x^2+3x-2 \le 2^4$

$x^2+3x-10 \ge 0$

$(x+5)(x-2) \ge 0$

$x \in (-\infty,-5] \cup [2,\infty)$

$x^2+3x-18 \le 0$

$(x+6)(x-3) \le 0$

$x \in [-6,3]$

$\cap$

$[-6,-5] \cup [2,3]$

Ans.

QUESTION

Given that y = f(x) is a function whose domain is [4, 7] and range is [–1, 9]. Find the range and domain of

a. $g(x) = \frac{1}{3}f(x)$

$\Downarrow$

$g(x) = \frac{f(x)}{3}$

[4,7]

[-1,9]

[-1/3, 3]

$D_g = [4,7]$

$R_g = [-1/3, 3]$

b. $h(x) = f(x - 7)$

[4,7]

[-1,9]

$4 \le x-7 \le 7$

$11 \le x \le 14$

$\downarrow$

$D_h = [11,14]$

$R_h = [-1,9]$

QUESTION [JEE Mains 2024 (30 Jan)]

Tah!

If the domain of the function $f(x) = \log_e\left(\frac{2x+3}{4x^2+x-3}\right) + \cos^{-1}\left(\frac{2x-1}{x+2}\right)$ is $(\alpha, \beta]$, then the value of $5\beta - 4\alpha$ is equal to

(A) 9

(B) 12

(C) 11

(D) 10

$\frac{2x+3}{4x^2+x-3} > 0$ & $-1 \le \frac{2x-1}{x+2} \le 1$

Ans. B

QUESTION [JEE Mains 2021]

$|x| \geq a, a \in R^+$
$x \leq -a$ or $x \geq a$

$|x| \leq a, a \in R^+$
$-a \leq x \leq a$

Let $[x]$ denote the greatest integer $\leq x$, where $x \in R$. If the domain of the real valued function $f(x) = \sqrt{\frac{|[x]|-2}{|[x]|-3}}$ is $(-\infty, a) \cup [b, c) \cup [4, \infty), a < b < c$, then the value of $a + b + c$ is :

(A) 8

(B) 1

(C) -2

(D) -3

Let $|[x]| = t$

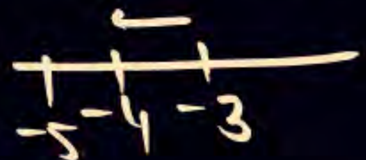
$\frac{|[x]|-2}{|[x]|-3} \geq 0 \longrightarrow \frac{t-2}{t-3} \geq 0 \Longrightarrow t \in (-\infty, 2] \cup (3, \infty)$

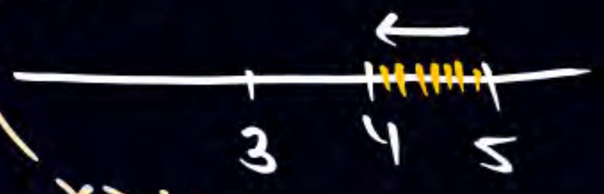
$\Downarrow$

$t \leq 2$ or $t > 3$

$|[x]| \leq 2$ or $|[x]| > 3$

$-2 \leq [x] \leq 2$ or $[x] < -3$ or $[x] > 3$





$-2 \leq x < 3$

$x < -3$ $\cup$ $x \geq 4$

$x \in [-2, 3)$ $\cup$ $x \in (-\infty, -3) \cup [4, \infty)$

$x \in (-\infty, -3) \cup [-2, 3) \cup [4, \infty)$

$a = -3, b = -2, c = 3$

$a + b + c = -2$.

QUESTION [JEE Mains 2023 (29 Jan)]

Tanz

The domain of $f(x) = \frac{\log_{(x+1)}(x-2)}{e^{2\log_e x} - (2x+3)}, x \in \mathbb{R}$ is

(A) $(-1, \infty) - \{3\}$

(B) $\mathbb{R} - \{-1, 3\}$

(C) $(2, \infty) - \{3\}$

(D) $\mathbb{R} - \{3\}$

Ans. C

QUESTION

Domain of the function $f(x) = \sqrt{1 - \sqrt{2 - \sqrt{3 - x}}}$ is

(A) [0, 2]

(B) [-1, 1]

(C) [-1, 2]

(D) [1, 2]

$1 - \sqrt{2 - \sqrt{3-x}} \geq 0$ & $2 - \sqrt{3-x} \geq 0$ & $3 - x \geq 0$

$1 \geq \sqrt{2 - \sqrt{3-x}}$ s.b.s

$1 \geq 2 - \sqrt{3-x}$

$\sqrt{3-x} \geq 1$

$3 - x \geq 1$

$x \leq 2$

$\sqrt{3-x} \leq 2$

$\Downarrow$

$3 - x \leq 4$

$x \geq -1$

$x \leq 3$

$\cap$

$x \in [-1, 2]$

Ans. C

Graphical Transformation

ⓐ $y = f(x) + a,\ a \in R^+$

① Draw $y = f(x)$

② Shift up the graph by 'a' units

OR

pull down x axis by 'a' units

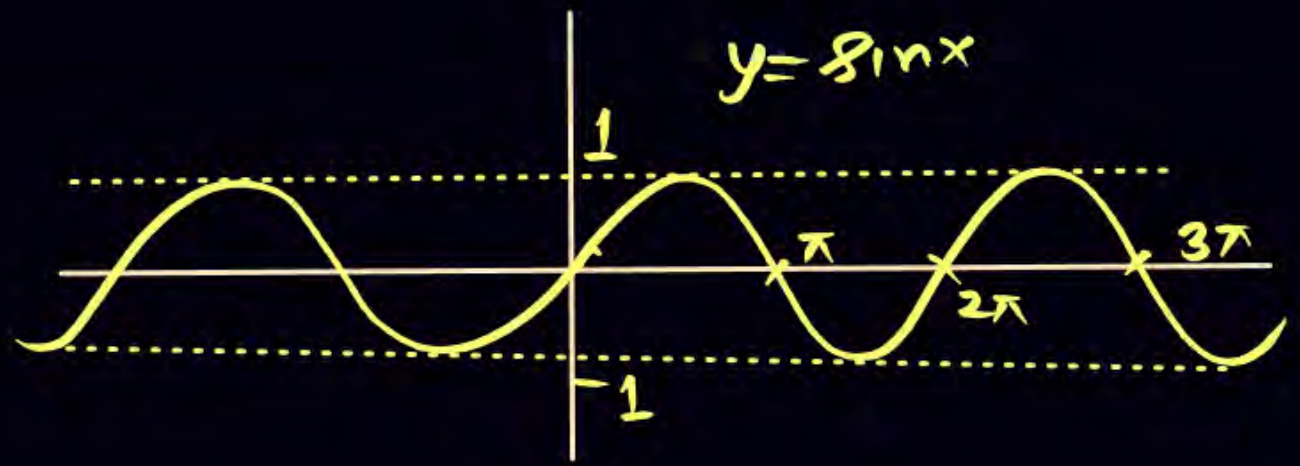
ⓑ $y = f(x) - a,\ a \in R^+$

① Draw $y = f(x)$

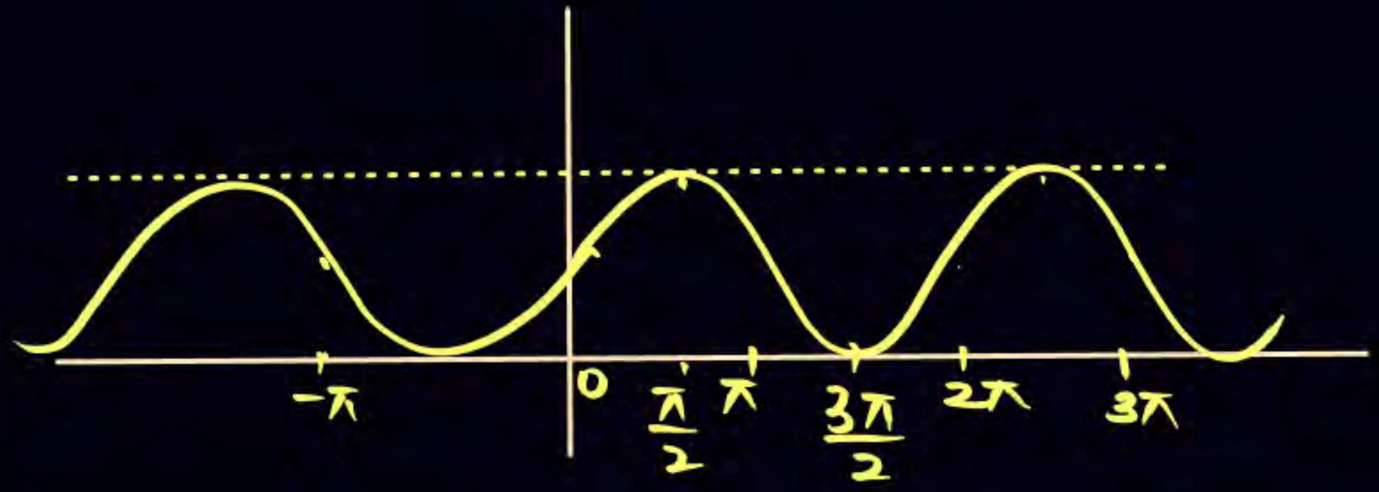
② Shift down the graph by 'a' units

OR

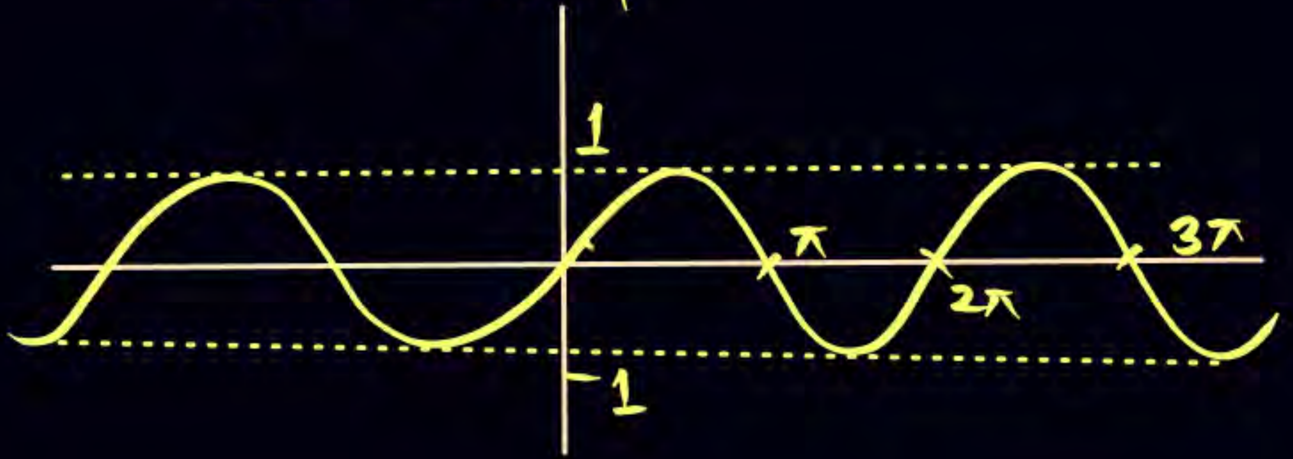
pull up. x axis by 'a' units

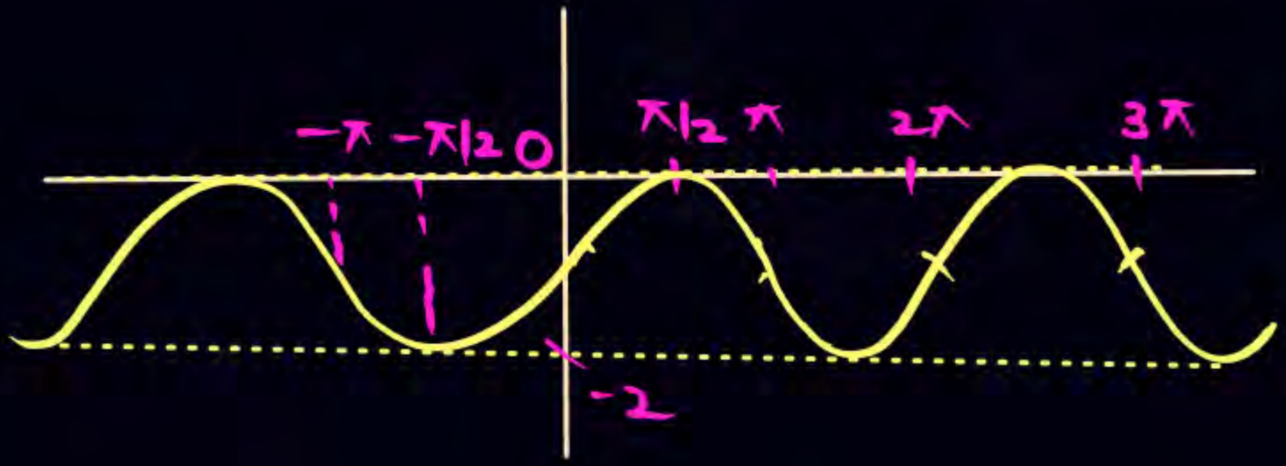


$y = 1 + \sin x$



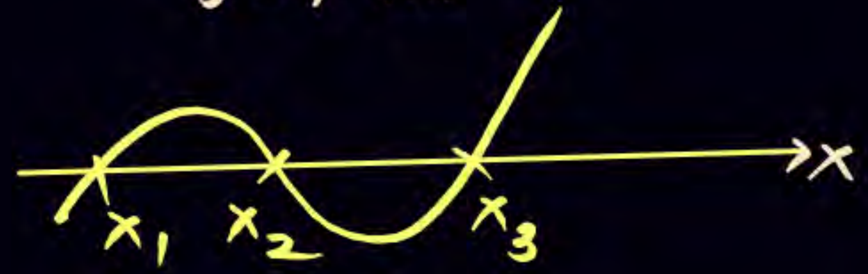
$y = \sin x - 1$





(c) $y = -f(x)$ — Draw : $y = f(x)$
— Reflect the entire graph about x axis

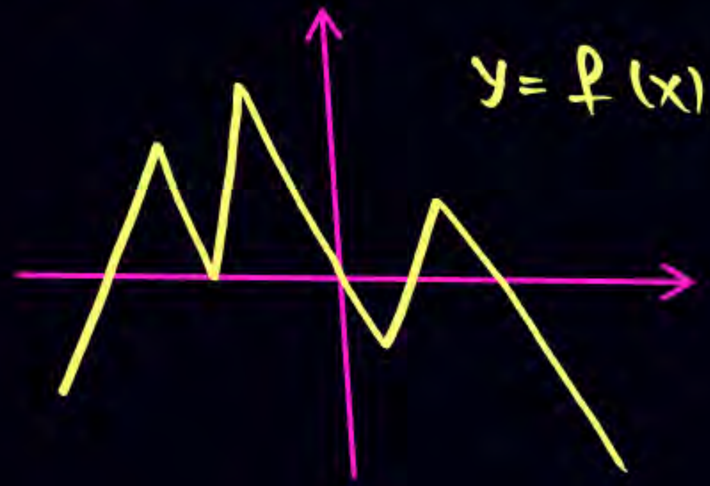
$y = f(x)$



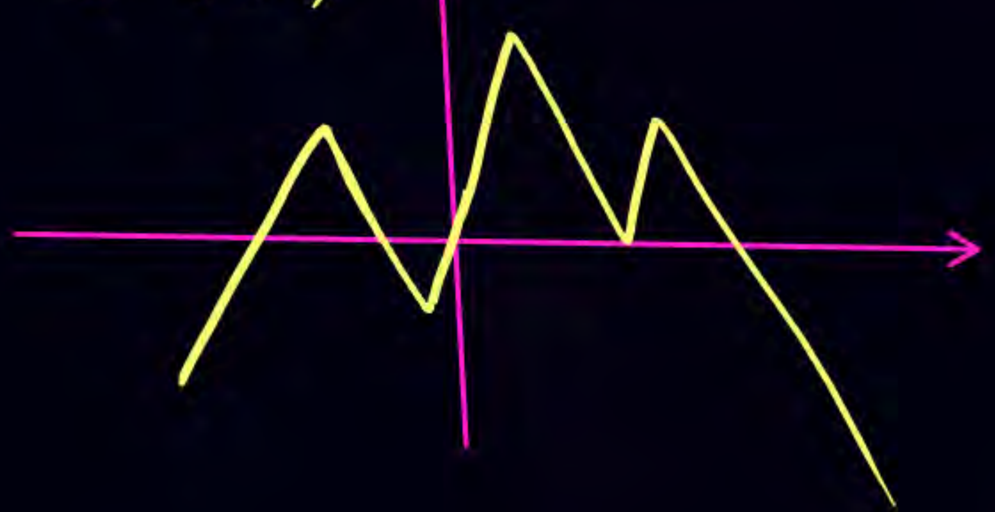
$y = -f(x)$

(d) $y = f(-x)$ — Draw : $y = f(x)$
— Reflect entire graph about y axis.

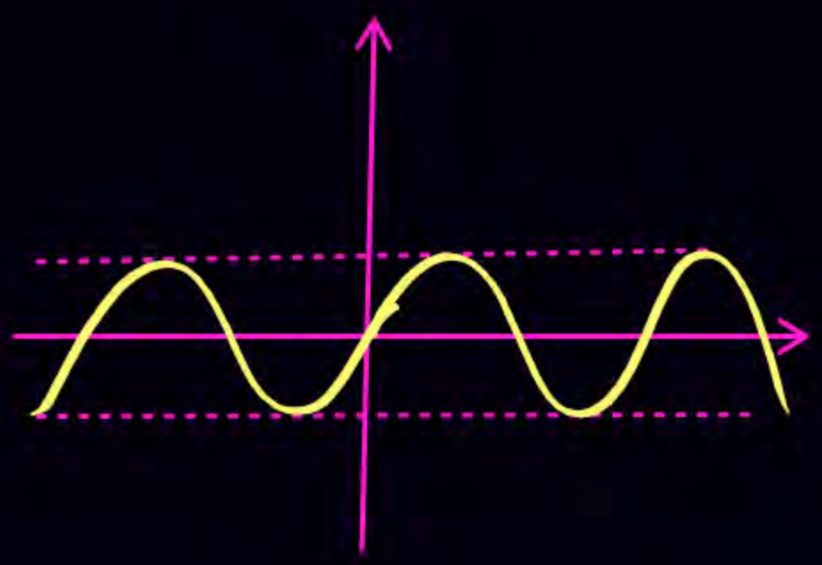
$y = f(x)$



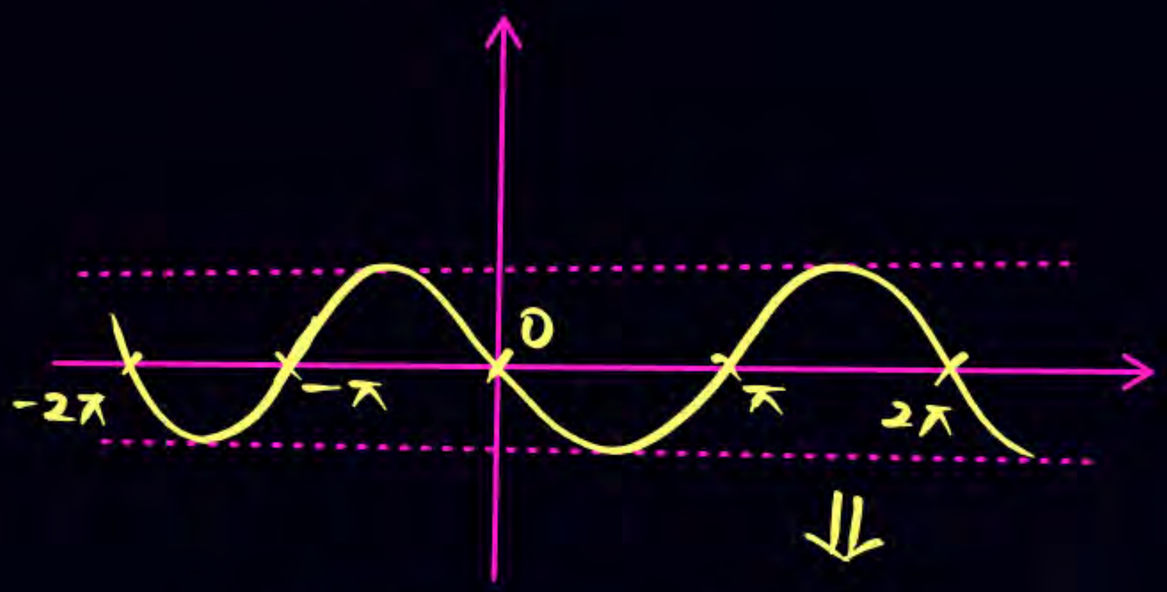
$y = f(-x)$



$y = \sin x$



$y = -\sin x$



$\Downarrow$

$y = 1 - \sin x$

$y = -\sin x + 1$

QUESTION

ASRQ

Let $f(x) = \left| |x^2 - 4x + 3| - 2 \right|$. Which of the following is/are correct?

(A) $f(x) = m$ has exactly two real solutions of different sign $\forall\ m > 2$.

(B) $f(x) = m$ has exactly two real solutions $\forall\ m \in (2, \infty) \cup \{0\}$.

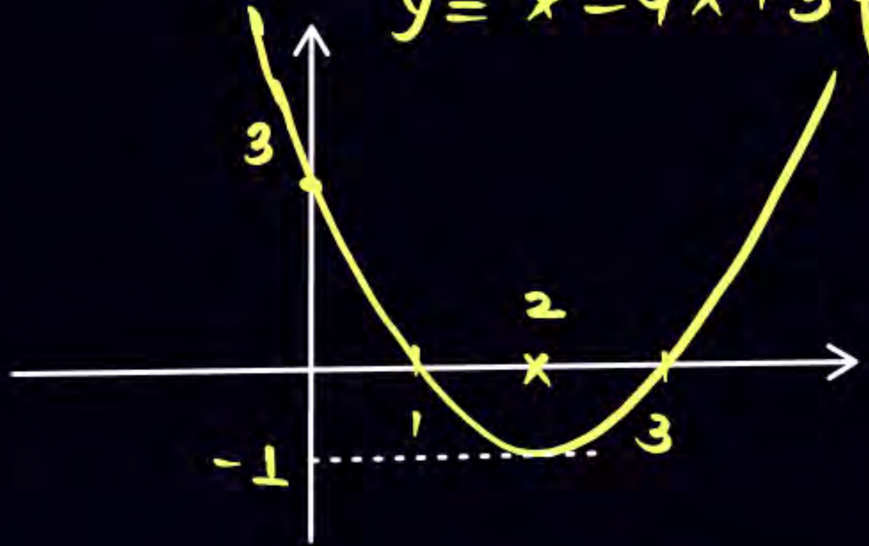
(C) $f(x) = m$ has no solutions $\forall\ m < 0$.

(D) $f(x) = m$ has four distinct real solutions $\forall\ m \in (0, 1)$.

Ans. A, B, C, D

$$f(x) = \big|\,|x^2 - 4x + 3| - 2\,\big|$$

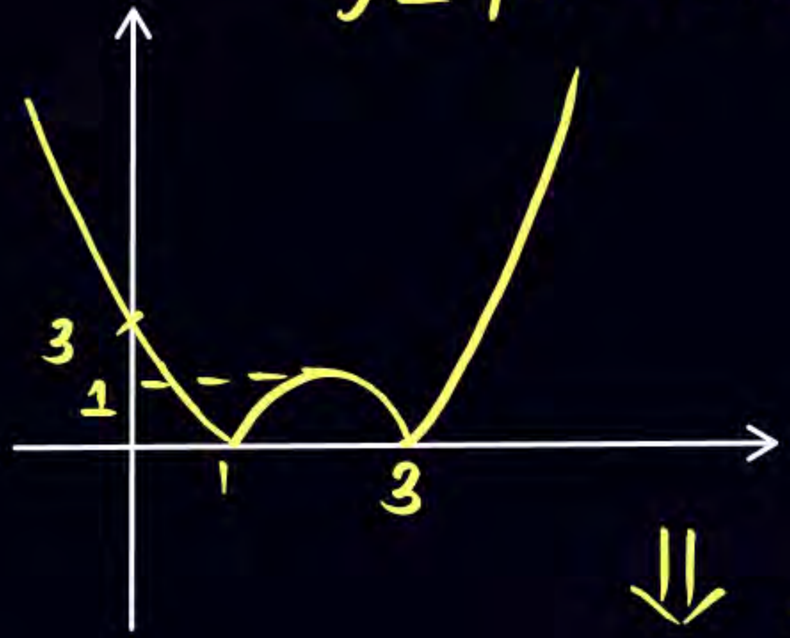
$$y = x^2 - 4x + 3 \begin{cases} x = 1 \\ x = 3 \\ x = 2,\ y = -1 \end{cases}$$



$\Rightarrow$

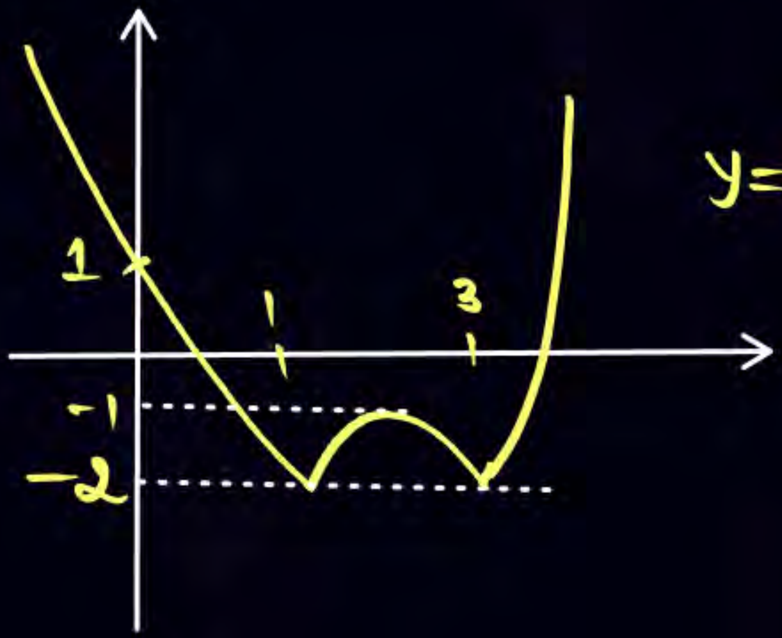
$$\boxed{y = ax^2 + bx + c \quad \text{vertex}\left(\frac{-b}{2a}, \frac{-D}{4a}\right)}$$

$$y = |x^2 - 4x + 3|$$



$\Downarrow$

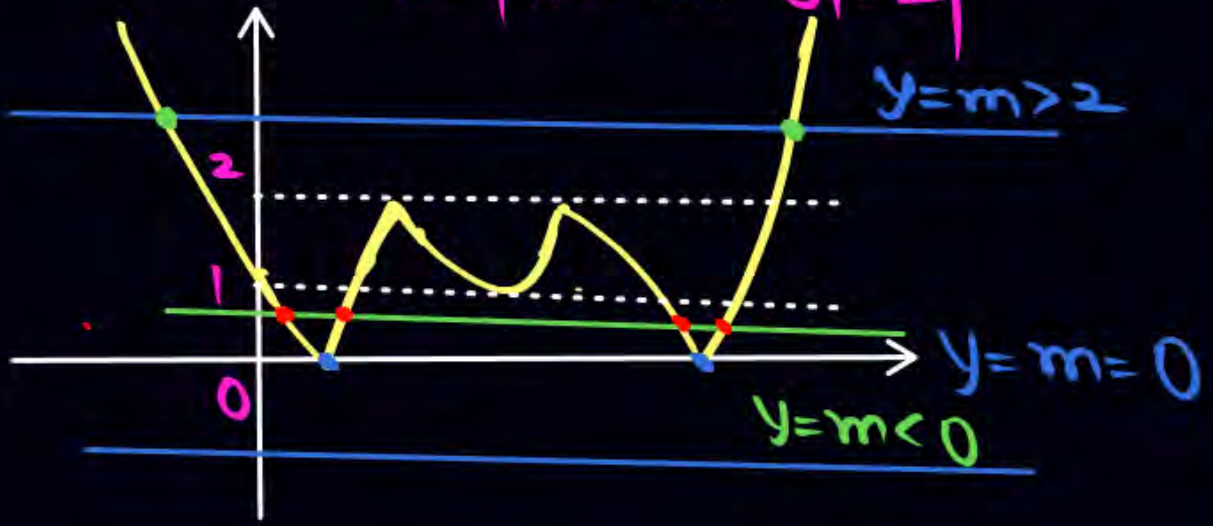
Shift up x axis by 2 units.



$$y = |x^2 - 4x + 3| - 2.$$

$\Leftarrow$

$$y = \big|\,|x^2 - 4x + 3| - 2\,\big|$$



QUESTION

Identify the equal function

(i) $f(x) = \log_x e; g(x) = \frac{1}{\log_e x}$ (I) $D_f : R^+ - \{1\}$ & $g(x) = \frac{1}{\log_e x} = \log_x e = f(x)$

$D_g : R^+ - \{1\}$

(ii) $f(x) = \log_e x; g(x) = \frac{1}{\log_x e}$ (N.I) $1 \in D_f$ but $1 \notin D_g \implies D_f \neq D_g$

(iii) $f(x) = \sqrt{x^2 - 1}; g(x) = \sqrt{x-1}\sqrt{x+1}$ (N.I) $-2 \in D_f$ but $-2 \notin D_g$

$D_f \neq D_g$

(iv) $f(x) = \log(x+2) + \log(x-3); g(x) = \log(x^2 - x - 6)$ (N.I)

$-3 \notin D_f$ but $-3 \in D_g$

$\Downarrow$

$D_f \neq D_g$

(v) $f(x) = x|x|; g(x) = x^2 \operatorname{sgn} x$

(vi) $f(x) = \frac{1}{1+\frac{1}{x}}; g(x) = \frac{x}{1+x}$

(vii) $f(x) = [\{x\}]; g(x) = \{[x]\}$

Tah 3

QUESTION

find Domain

$$f(x) = \frac{1}{[x]} + \log_{1-\{x\}}(x^2 - 3x + 10) + \frac{1}{\sqrt{2-|x|}} + \frac{1}{\sqrt{\sec(\sin x)}}$$

$[x] \neq 0$

$x \notin [0,1)$

$R - [0,1)$

$1-\{x\} > 0$, $1-\{x\} \neq 1$

& $x^2 - 3x + 10 > 0$ ⟨ $D < 0$, $a > 0$

⇓

$\{x\} < 1$, $\{x\} \neq 0$. $x \in R$

$x \in R$ $\quad$ $x \notin I$

$x \notin I$

$\cap$

$2 - |x| > 0$

$|x| < 2$

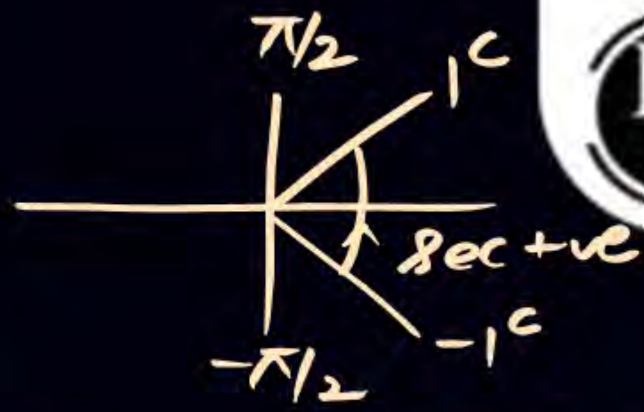
$x \in (-2, 2)$

$x \in R$

$[-1, 1]$

+ve

$x \in R$.



$\sec\theta \in (-\infty, -1] \cup [1, \infty)$

$\text{cosec}\,\theta \in (-\infty, -1] \cup [1, \infty)$

$x \in (-2,-1) \cup (-1,0) \cup (1,2)$

QUESTION

Tany

$$f(x) = \begin{cases} x+1 & x<2 \\ x+3 & x\geq 2 \end{cases} \ \& \ g(x) = \begin{cases} x^2+2x+7 & x<1 \\ x^2+5x+7 & x\geq 1 \end{cases}$$

Find $f(x) \pm g(x)$ and $\frac{f(x)}{g(x)}$.

$$(f+g)(x) = f(x) + g(x) = \begin{cases} x+1+x^2+2x+7 & x<1 \\ x+1+x^2+5x+7 & 1\leq x<2 \\ x+3+x^2+5x+7 & x\geq 2 \end{cases} = \begin{cases} x^2+3x+8 & x<1 \\ x^2+6x+8 & 1\leq x<2 \\ x^2+6x+10 & x\geq 2 \end{cases}$$

QUESTION

TAH 5

Find the domain of the following function :

(i) $y = \log_{(x-4)}(x^2 - 11x + 24)$

(ii) $f(x) = \log_2\left(-\log_{\frac{1}{2}}\left(1 + \frac{1}{\sqrt[4]{x}}\right) - 1\right)$

Problems on Range of Functions

Methods To Range

★ **M①** put $y = f(x)$ & find x in terms of y & use the condition $x \in R$

★ **M②** for a continuous fn interval from min to max is Range.

★ **M③**: find Domain & try to find Range using Domain.

★ **M④**: Gola Method.

★ **M⑤**: Draw Graph.

Range Finding Method

M1: Put $y = f(x)$ and then & solve x in terms of y and then use the condition $x \in R$.

M2: For continuous function interval from minimum to maximum value gives range.

M3: Find Domain & try to find outputs as per domain.

M4: Draw graph..

M5: Use Gola Method

QUESTION

Find Range of

$$f(x) = \frac{2e^x}{3e^x + 5}$$

M 1

$$y = \frac{2e^x}{3e^x+5}$$

$$3ye^x + 5y = 2e^x$$

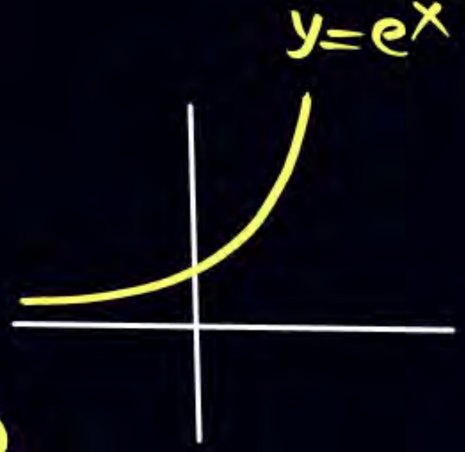
$$5y = (2-3y)e^x$$

$$e^x = \frac{5y}{2-3y} > 0$$

$$\frac{5y}{2-3y} > 0$$

$$\frac{y}{3y-2} < 0 \quad \overset{+}{}\ \underset{0}{|}\ \overset{-}{}\ \underset{2/3}{|}\ \overset{+}{}$$

$$y \in (0, 2/3)$$



Tan 6

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x}-1}{e^{2x}+1}$$

M(2)

$$f(x)=\frac{2e^x}{3e^x+5}=\frac{\frac{2}{3}\cdot(3e^x+5-5)}{3e^x+5}=\frac{\frac{2}{3}(3e^x+5)-5\cdot\frac{2}{3}}{3e^x+5}=\frac{2}{3}-\frac{10/3}{3e^x+5}$$

$$e^x \in (0,\infty)$$

$$(5,\infty)$$

$$\frac{10}{3}\cdot\left(0,\frac{1}{5}\right)$$

$$\left(0,\frac{2}{3}\right)$$

$$R_f=(0,2/3)$$

M(3)

$$f(x)=\frac{2e^x}{3e^x+5}\Rightarrow f(x)=\frac{2}{3+5\cdot e^{-x}}$$

$$5\cdot e^{-x} \in (0,\infty)$$

$$(3,\infty)$$

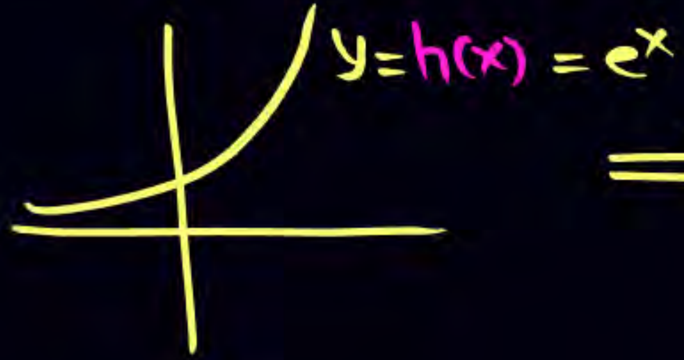
$$2(0,1/3)$$

$$\parallel$$

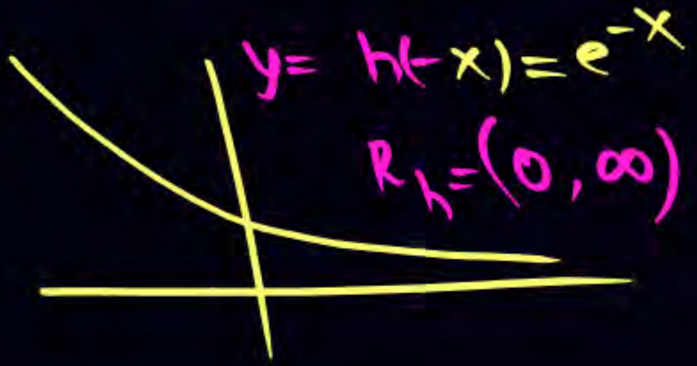
$$(0,2/3)$$

$$\parallel$$

$$R_f.$$



$\Longrightarrow$



QUESTION

ASRQ

$y = 2x^2 - 10x = 2x(x-5)$

$y = x^2 - 5$ $x = \pm\sqrt{5}$

Let $f(x) = \begin{cases} 2x^2 - 10x, & -\infty < x \le -5 \\ x^2 - 5, & -5 < x < 3 \\ x^2 + 1, & 3 \le x < \infty \end{cases}$.

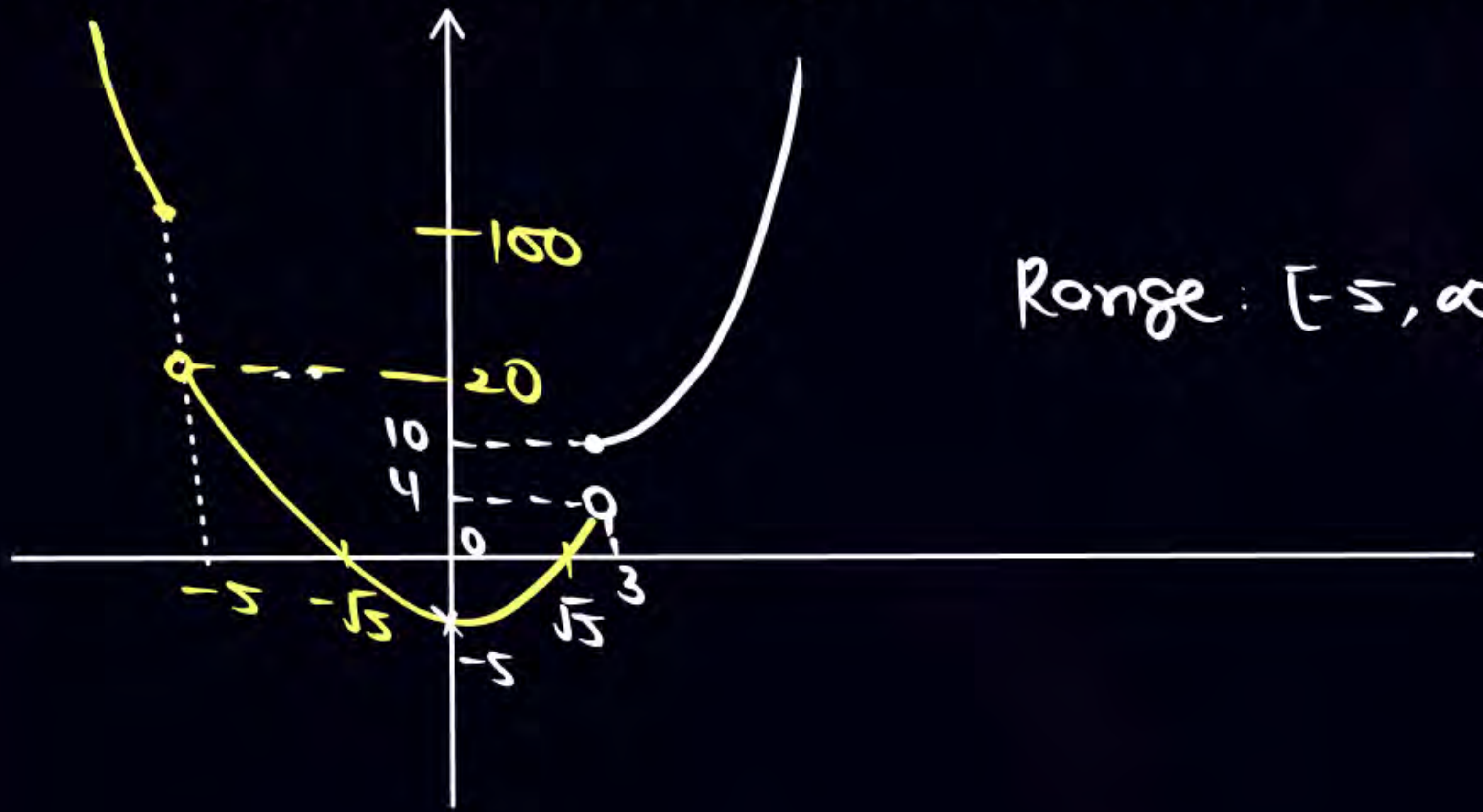
Number of negative integers in the range of the function $f(x)$ is

(A) 6

(B) 5

(C) 4

(D) 3



Range: $[-5, \infty)$

QUESTION

Tan 7

(a) Let $f(x) = \begin{cases} x, & -2 \le x \le -1 \\ x^2 + 2x, & -1 < x \le 0 \\ 2x - x^2, & 0 < x \le 1 \\ 2 - x, & 1 < x \le 2 \end{cases}$

Find the number of integers in the range of $f(x)$.

Homework from Module

Chapter: SETS

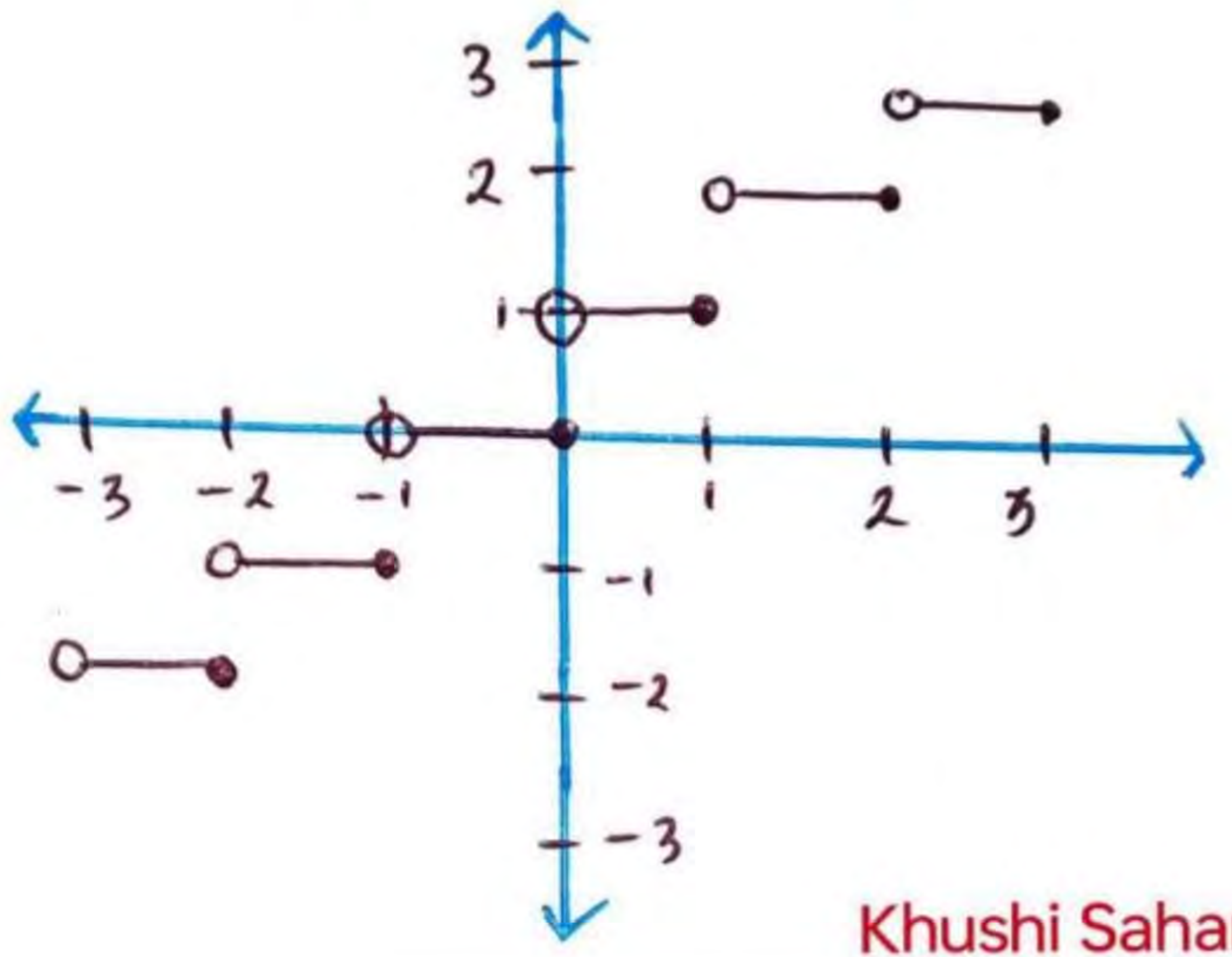
Prarambh: COMPLETE

Prabal : COMPLETE

Previous TAH

Solutions

TAH - (1) Draw graph of $y = \lceil x \rceil$



Khushi Sahani, UP

QUESTION

ASRQ

Let f be a polynomial function which satisfies the relation $f(x) + f\left(\frac{x}{y^2}\right) + f\left(\frac{x}{y}\right) = f(x) \cdot f\left(\frac{1}{y}\right) - \frac{1}{y^3} + \frac{x^3}{y^6} + 2 \; \forall \, x \in R - \{0\}, f(1) \neq 1$ and $f(2) = 9$.

The value of $\sum_{r=1}^{100} f(r)$ equals

(A) 5050

(B) $(5050)^2$

(C) $100 + (5050)^2$

(D) $100 + (5050)^3$

Tah ① $f(x)+f\left(\frac{x}{y^2}\right)+f\left(\frac{x}{y}\right)=f(x)\cdot f\left(\frac{1}{y}\right)-\frac{1}{y^3}+\frac{x^3}{x^6}+2 \quad \forall\, x \in R-\{0\}$

$f(1)\neq 1 \qquad f(2)=9 \qquad \sum_{r=1}^{100} f(r)$

Ans Replace $y \to x$

$f(x)+f\left(\frac{1}{x}\right)+f(1)=f(x)\cdot f\left(\frac{1}{x}\right)-\cancel{\frac{1}{x^3}}+\cancel{\frac{1}{x^3}}+2$

$f(x)+f\left(\frac{1}{x}\right)+f(1)=f(x)\cdot f\left(\frac{1}{x}\right)+2 \quad \text{-- Eq}^n$

Put $x=1$ in Eq^n

$f(1)+f(1)+f(1)=f^2(1)+2$

$3f(1)=f^2(1)+2$

$f^2(1)-3f(1)+2=0$

$(f(1)-2)(f(1)-1)=0$

$f(1)=2 \ , \ f(1)=1$ → Rejecte $f(1)\neq 1$

$f(1)=2$

Put $y=1$ in Given Relation.

$f(x)+f(x)+f(x)=f(x)f(1)-1+x^3+2 \quad \because (f(1)=2)$

$3f(x)=2f(x)+1+x^3$

$f(x)=1+x^3$ ------ fxn Come.

It also satisfy $f(2)=9$

$f(r)=1+r^3 \qquad \sum_{r=1}^{100}(1+r^3)=\sum_{r=1}^{100}1+\sum_{r=1}^{100}r^3=100+(5050)^2$

Boby hr
Tah 2

TAH – ② $f(x)+f\left(\frac{x}{y^2}\right)+f\left(\frac{x}{y}\right)=f(x)f\left(\frac{1}{y}\right)-\frac{1}{y^3}+\frac{x^3}{y^6}-2$ — ⓘ

put $y=x$, $f(x)+f\left(\frac{1}{x}\right)+f(1)=f(x)f\left(\frac{1}{x}\right)-\cancel{\frac{1}{x^3}}+\cancel{\frac{1}{x^3}}+2$

$f(x)+f(1/x)+f(1)=f(x)f(1/x)+2$ — ⓘⓘ

Puting $y=1, x=2$ in ⓘ

$f(2)+f(2)+f(2)=f(2)f(1)-1+8+2.$

$3f(2)=f(2)f(1)+9.$

$\Rightarrow 27=9f(1)+9$

$\Rightarrow 3=f(1)+1, \ f(1)=2.$

Putting $f(1)=2$ in eq^n ⓘⓘ,

$f(x)+f(1/x)+\cancel{2}=f(x)f(1/x)+\cancel{2}$

$\Rightarrow f(x)=1+x^n.$

$f(2)=1+2^n=9 \Rightarrow 1+2^3$

$\Rightarrow 2^n=8, \ n=3$

$f(x)=1+x^3.$

$\sum_{r=1}^{100}(1+r^3)=\sum_{r=1}^{100}+\sum_{r=1}^{100}r^3=100+\left(\frac{100\times 101}{2}\right)^2.$

$=100+\frac{\cancel{100}^{50}\times\cancel{100}^{50}\times 101\times 101}{\cancel{2}\times\cancel{2}}$

Khushi Sahani $=100+2500\times 10201$

From UP. $=100+50^2\cdot 101^2=100+(5050)^2$ Ans.

QUESTION

Find Domain of following functions

(i) $f(x) = \sqrt{\cos 2x} + \sqrt{16 - x^2}$

(ii) $f(x) = \log_7 \log_5 \log_3 \log_2 (2x^3 + 5x^2 - 14x)$

(iii) $f(x) = \log_{100x}\left(\frac{2 \log_{10} x + 1}{-x}\right)$

TAH 2 RAHUL DHAKAD

FROM AGRA UP

9 July Homework

TAH-3

Q $f(x) = \log_7 \log_5 \log_3 \log_2 (2x^3 + 5x^2 - 14x)$

Ans. $\log_5 \log_3 \log_2 (2x^3 + 5x^2 - 14x) > 0$, other condition's not need

$\log_3 \log_2 (2x^3 + 5x^2 - 14x) > 1$

$2x^3 + 5x^2 - 14x > 8$

$2x^3 + 5x^2 - 14x - 8 > 0$

$2x^2(x-2) + 9x(x-2) + 4(x-2) > 0$

$(x-2)(2x^2 + 9x + 4) > 0$

$(x-2)(2x^2 + 8x + x + 4) > 0$

$(x-2)(2x(x+4) + 1(x+4)) > 0$

$(x-2)(x+4)(2x+1) > 0$

$(x-2)(x+4)(2x+1) > 0$

$- \quad -4 \quad + \quad -\frac{1}{2} \quad - \quad 2 \quad +$

$\boxed{x \in \left(-4, -\frac{1}{2}\right) \cup (2, \infty)}$ Ans

Que $f(x) = \log_7 \log_5 \log_3 \log_2 (2x^3+5x^2-14x)$ find Domain. TAH-3

$\log_5 \log_3 \log_2 (2x^3+5x^2-14x) > 0$, $\log_3 \log_2 (2x^3+5x^2+14x) > 0$, $\log_2 (2x^3+5x^2+14x) > 0$, $2x^3+5x^2+14x > 0$

no need, no need, no need

$$\log_3 \log_2 (2x^3+5x^2-14x) > 1$$

$$\log_2 (2x^3+5x^2-14x) > 3$$

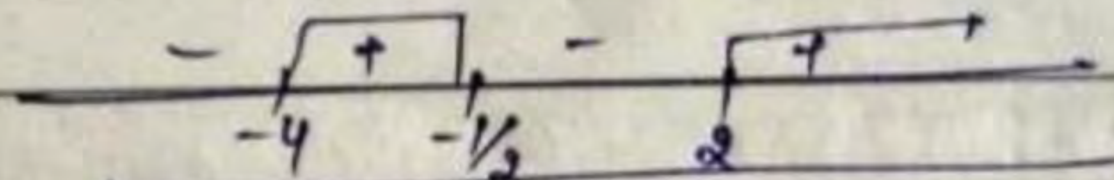
$$2x^3+5x^2-14x > 8$$

$$2x^3+5x^2-14x-8 > 0$$

$$2x^2(x-2)+9x(x-2)+4(x-2) > 0$$

$$(x-2)(2x^2+9x+4) > 0$$

$$(x-2)(x+4)\left(x+\frac{1}{2}\right) > 0$$



$$\boxed{x \in \left(-4, -\frac{1}{2}\right) \cup (2, \infty)}$$ Ans

(Solution to KTK)

QUESTION [JEE Mains 2024 (9 April)]

(KTK 1)

Let the range of the function $f(x) = \frac{1}{2+\sin 3x+\cos 3x}, x \in \mathbb{R}$ be $[a, b]$. If α and β are respectively the A.M. and the G.M. of a and b, then $\frac{\alpha}{\beta}$ is equal to

(A) π

(B) $\sqrt{\pi}$

(C) $\sqrt{2}$

(D) 2

Ans. C

Q) Let the range of the function $f(x) = \frac{1}{2+\sin 3x+\cos 3x}$, $x \in \mathbb{R}$ be $[a,b]$. If α & β are respectively the AM and the G.M of a and b, then $\frac{\alpha}{\beta}$ is equals to:

A) π B) $\sqrt{\pi}$ C) $\sqrt{2}$ D) 2

KTK 1

$\Rightarrow f(x) = \frac{1}{2+\sin 3x+\cos 3x}$

$\sin 3x+\cos 3x \rightarrow [-\sqrt{2}, \sqrt{2}]$

$2+\sin 3x+\cos 3x \rightarrow [2-\sqrt{2},\ 2+\sqrt{2}]$

$\rightarrow$ range of $f(x)$ is $\left[\frac{1}{2-\sqrt{2}}, \frac{1}{2+\sqrt{2}}\right]$

$\therefore a = \frac{1}{2-\sqrt{2}}$, $b = \frac{1}{2+\sqrt{2}}$

$\alpha = \frac{a+b}{2}$, $\beta = \sqrt{ab}$.

$\therefore \frac{\alpha}{\beta} = \frac{a+b}{2\sqrt{ab}} \Rightarrow \frac{\frac{2+\cancel{\sqrt{2}}+2-\cancel{\sqrt{2}}}{(2^2-2)}}{2\sqrt{(2^2-2)^{-1}}} = \frac{\cancel{4}}{\cancel{2}\times\cancel{2}\times(\sqrt{2})^{-1}}$

$= \sqrt{2}$ (Ans) (C)

Gourab Dutta
Howrah (WB)

KTK-1

Let the range of the fn $f(x) = \dfrac{1}{2+\sin 3x+\cos 3x}$, $x \in R$ be $[a, b]$. If α & β are respectively the A.M & G.M of a & b, then $\dfrac{\alpha}{\beta}$ is equal to

Soln $f(x) = \dfrac{1}{2+\sin 3x+\cos 3x} = \dfrac{1}{2+\sqrt{2}\sin\left(\frac{\pi}{4}+3x\right)}$

$[-1, 1]$

$[-\sqrt{2}, \sqrt{2}]$

$[2-\sqrt{2}, 2+\sqrt{2}]$

$\left[\dfrac{1}{2+\sqrt{2}}, \dfrac{1}{2-\sqrt{2}}\right]$

$a = \dfrac{1}{2+\sqrt{2}}$

$b = \dfrac{1}{2-\sqrt{2}}$

$A.M = \dfrac{a+b}{2} = \left(\dfrac{1}{2+\sqrt{2}} + \dfrac{1}{2-\sqrt{2}}\right) \times \dfrac{1}{2} \Rightarrow \dfrac{2}{4-2} = 1$

$G.M = \sqrt{ab} = \left(\dfrac{1}{2}\right)^{1/2} = \dfrac{1}{\sqrt{2}}$

$\dfrac{\alpha}{\beta} = \sqrt{2}$

(Ans.)

KTK-1. $f(x) = \dfrac{1}{2+\underbrace{\sin 3x+\cos 3x}}$

$\left\{ a\sin\theta + b\cos\theta = \pm\sqrt{a^2+b^2} \right\}$

$[-\sqrt{2}, \sqrt{2}]$

$[2-\sqrt{2}, 2+\sqrt{2}]$

$\left[\dfrac{1}{2-\sqrt{2}}, \dfrac{1}{2+\sqrt{2}}\right] \rightarrow b$

a

α & $\beta \Rightarrow$ AM & GM of a & b.

$\alpha = \dfrac{\dfrac{1}{2-\sqrt{2}} + \dfrac{1}{2+\sqrt{2}}}{2}$

$= \dfrac{\dfrac{2+\cancel{\sqrt{2}}+2-\cancel{\sqrt{2}}}{4+\cancel{2\sqrt{2}}-\cancel{2\sqrt{2}}-2}}{2}$

$= 4/4 = 1.$

$\alpha = 1$

$\beta = \sqrt{ab}$

$= \sqrt{\dfrac{1}{2-\sqrt{2}} \cdot \dfrac{1}{2+\sqrt{2}}}$

$= \sqrt{\dfrac{1}{4+\cancel{2\sqrt{2}}-\cancel{2\sqrt{2}}-2}} = \dfrac{1}{\sqrt{2}}.$

$\beta = 1/\sqrt{2}$

$\dfrac{\alpha}{\beta} = \sqrt{2}$ Ans.

QUESTION [JEE Mains 2024 (1 Feb)]

(KTK 2)

If the domain of the function

$f(x) = \frac{\sqrt{x^2-25}}{(4-x^2)} + \log_{10}(x^2 + 2x - 15)$ is $(-\infty, \alpha) \cup [\beta, \infty)$, then $\alpha^2 + \beta^3$ is equal to

(A) 140

(B) 175

(C) 125

(D) 150

Ans. D

Boby hr Ktk 2. 1

Date / /

Page No.

KTK ② $f(x) = \dfrac{\sqrt{x^2-25}}{4-x^2} + \log_{10}(x^2+2x-15)$ domain $(-\infty,\alpha)\cup[\beta,\infty)$

$\alpha^2+\beta^3 = ?$

Ans

Cond 1.

$4-x^2 \neq 0$

$x \neq +2, -2.$

Cond 2.

$x^2-25 \geq 0$

$x \in (-\infty,-5] \cup [5,\infty).$

Cond ③

$x^2-2x-15 > 0$

$(x+5)(x-3) > 0$

$+ \quad -5 \quad - \quad 3 \quad +$

$x \in (-\infty,-5) \cup (3,\infty)$

$\cap$

$x \in (-\infty,-5) \cup [5,\infty).$

$\alpha = -5 \qquad \beta = 5$

$\alpha^2+\beta^3 = 150.$

KTK-2

$:\rightarrow \quad x^2 - 25 \geq 0$

$(x-5)(x+5) \geq 0$

$x \in (-\infty, -5] \cup [5, \infty)$ — (i)

$\log_{10}(x^2 + 2x - 15)$

i.e

$x^2 + 2x - 15 > 0$

$(x+5)(x-3) > 0$

$4 - x^2 \neq 0$

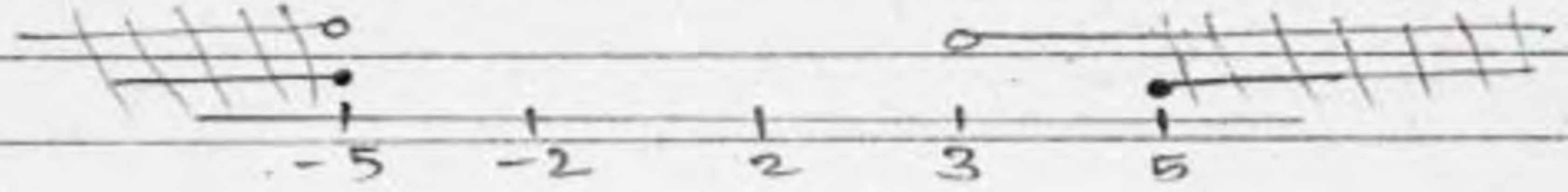
$(2+x)(2-x) \neq 0$

$x \neq \pm 2$ — (iii)

$x \in (-\infty, -5] \cup (3, \infty)$ — (ii)

from Eqn (i), (ii) and (iii) :

Divyanshu Sagar
Bihar



$x \in (-\infty, -5) \cup [5, \infty)$

then

$\alpha^2 + \beta^3 \equiv 25 + 125 \Rightarrow 150$ Ans

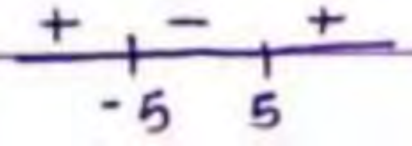
KTK-(2) If D_f is, of $f(x) = \dfrac{\sqrt{x^2-25}}{(4-x^2)} + \log_{10}(x^2+2x-15)$ is $(-\infty,\alpha)\cup[\beta,\infty)$, $\alpha^2+\beta^3 = ?$

$$f(x) = \frac{\sqrt{x^2-25}}{(4-x^2)} + \log_{10}(x^2+2x-15)$$

$x^2-25 \geq 0$

$(x-5)(x+5) \geq 0$

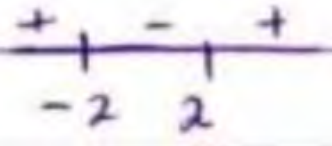
$x \in (-\infty,-5]\cup[5,\infty)$ — (i)



$4-x^2 > 0$

$x^2-4 < 0$

$(x-2)(x+2) < 0$



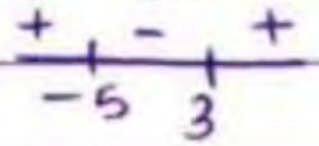
$x \in (-2,2)$ — (ii)

$x^2+2x-15 > 0$

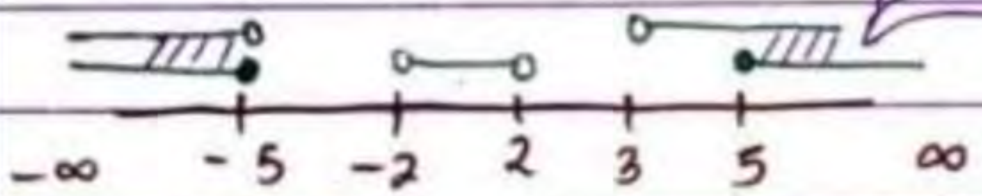
$x^2+5x-3x-15 > 0$

$x(x+5)-3(x+5) > 0$

$(x-3)(x+5) > 0$



$x \in (-\infty,-5)\cup(3,\infty)$ — (iii)



$x \in (-\infty,-5)\cup[5,\infty) = (-\infty,\alpha)\cup[\beta,\infty)$

so, $\alpha = -5$, $\beta = 5$.

$\alpha^2+\beta^3 = (-5)^2+5^3$

$= 25+125$

$\alpha^2+\beta^3 = 150$ Ans.

Khushi Sahani
from UP.

QUESTION [JEE Mains 2024 (30 Jan)]

(KTK 3)

If the domain of the function $f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right) + \{\log_e(3-x)\}^{-1}$ is $[-\alpha, \beta) - \{\gamma\}$, then $\alpha + \beta + \gamma$ is equal to :

(A) 11

(B) 12

(C) 9

(D) 8

Ans. A

KTK-3

Solⁿ :→ $\cos^{-1}\left(\frac{2-|x|}{4}\right)$

$\Downarrow$

$$-1 \le \frac{2-|x|}{4} \le 1$$

$\times 4$

$$-4 \le 2-|x| \le 4$$

(-2)

$$-6 \le -|x| \le 2$$

$-6 \le -|x|$

$|x| \le 6$

$x \le \pm 6$

$x \in [-6, 6]$ — (I)

$-|x| \le 2$

$|x| \ge -2$

~~x ∈ R~~

(Always true)

$\Rightarrow [\log_e(3-x)]^{-1}$

$\Rightarrow \frac{1}{\log_e(3-x)}$

$\Rightarrow \log_e(3-x) \ne 0$

$(3-x) \ne e^0$

$3-x \ne 1$

$\boxed{x \ne 2}$

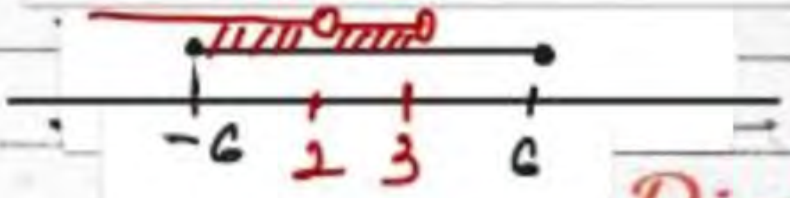
also

$3-x > 0$

$\boxed{x < 3}$

$x<3, x \ne 2$ — (II)

from (I) & (II)



$$\overset{\alpha}{\uparrow}\ \overset{\beta}{\uparrow}\ \overset{\gamma}{\uparrow}$$

$x \in [-6, 3) - \{2\}$

then

$\alpha+\beta+\gamma \Rightarrow -6+3+2 \Rightarrow \boxed{-1}$ Ans

Divyanshu Sagar

Bihar

QUESTION [JEE Mains 2021 (1 Sep)]

(KTK 4)

The range of the function,

$f(x) = \log_{\sqrt{5}}\left(3 + \cos\left(\frac{3x}{4} + x\right) + \cos\left(\frac{\pi}{4} + x\right) + \cos\left(\frac{\pi}{4} - x\right) - \cos\left(\frac{3\pi}{4} - x\right)\right)$ is

(A) $(0, \sqrt{5})$

(B) $[-2, 2]$

(C) $\left[\frac{1}{\sqrt{5}}, \sqrt{5}\right]$

(D) $[0, 2]$

Ans. D

KTK₄ $\log_{\sqrt{5}}\left(3+\cos\left(\frac{3\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{3\pi}{4}-x\right)-\cos\left(\frac{\pi}{4}-x\right)\right)$

find Range

Ans $\log_{\sqrt{5}}\left(3+\cos\left(\frac{3\pi}{4}+x\right)-\cos\left(\frac{3\pi}{4}-x\right)+\cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right)\right)$

$\log_{\sqrt{5}}\left(3+2\sin\left(\frac{3\pi}{4}\right)\sin(-x)+2\cos\left(\frac{\pi}{4}\right)\cos(x)\right)$

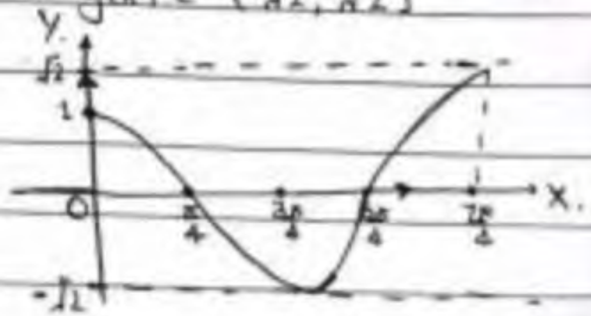
$\log_{\sqrt{5}}\left(3+\left(-\sqrt{2}\sin x+\sqrt{2}\cos x\right)\right)$

$\log_{\sqrt{5}}\left(3+\sqrt{2}\left(\cos x-\sin x\right)\right)$

$\log_{\sqrt{5}}\left(3+\sqrt{2}\,[-\sqrt{2},\sqrt{2}]\right)$

$g(x)=\cos x-\sin x$. ** Imp fxn (calculus)

$g(x)\in[-\sqrt{2},\sqrt{2}]$



$\log_{\sqrt{5}}\left(3+[-2,2]\right)$

$2\log_5\left([1,5]\right)$

$\log_5\left([1,5]^2\right)$

$\log_5\left([1,25]\right)$

$[0,2]$ Ans

Boby hr   Ktk 4

KTK– ④ The R_f.

$f(x)=\log_{\sqrt{5}}\left(3+\cos\left(\frac{3x}{4}+x\right)+\cos\left(\frac{\pi}{4}+x\right)+\cos\left(\frac{\pi}{4}-x\right)-\cos\left(\frac{3\pi}{4}-x\right)\right)$

$\cos\left(\frac{3\pi}{4}+x\right)=\cos\left(\pi-\frac{\pi}{4}+x\right)=\cos\left(\pi-\left(\frac{\pi}{4}-x\right)\right)=-\cos\left(\frac{\pi}{4}-x\right)$

$\cos\left(\frac{3\pi}{4}-x\right)=\cos\left(\pi-\frac{\pi}{4}-x\right)=\cos\left(\pi-\left(\frac{\pi}{4}+x\right)\right)=-\cos\left(\frac{\pi}{4}+x\right)$

$f(x)=\log_{\sqrt{5}}\left(3+\left(\cancel{-\cos\left(\frac{\pi}{4}-x\right)}\right)+\cos\left(\frac{\pi}{4}+x\right)+\cancel{\cos\left(\frac{\pi}{4}-x\right)}+\cos\left(\frac{\pi}{4}+x\right)\right)$

$f(x)=\log_{\sqrt{5}}\left(\underbrace{3+\underbrace{2\underbrace{\left(\cos\left(\frac{\pi}{4}+x\right)\right)}_{[-1,1]}}_{[-2,2]}}_{[1,5]}\right)$

$\log_{\sqrt{5}}1\le x\le\log_{\sqrt{5}}5 \rightarrow \left(2\log_5 5=2\right)$

$0\le x\le 2.$

Khushi Sahani from UP.

$x\in[0,2]$ Ans.

(K-TK-4) KTK-4 (Kamalakanta)

$f(x) = \log_{\sqrt5}\left(3 + \cos\left(\frac{3\pi}{4}+x\right) + \cos\left(\frac{\pi}{4}+x\right) + \cos\left(\frac{\pi}{4}-x\right) - \cos\left(\frac{3\pi}{4}-x\right)\right)$

$= \log_{\sqrt5}\left(3 + \cos\left(\frac{3\pi}{4}+x\right) - \cos\left(\frac{3\pi}{4}-x\right) + \cos\left(\frac{\pi}{4}+x\right) + \cos\left(\frac{\pi}{4}-x\right)\right)$

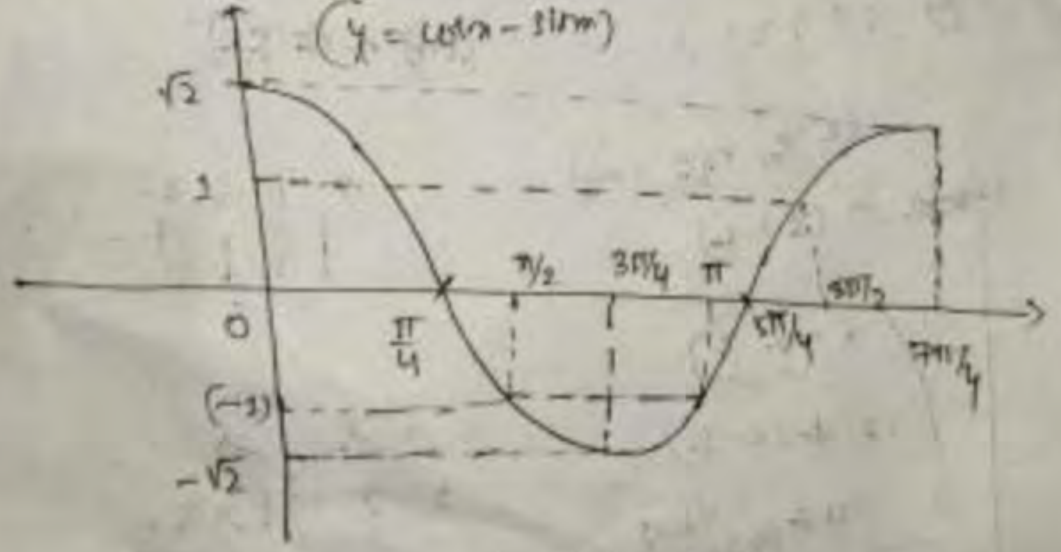
$= \log_{\sqrt5}\left(3 - 2\sin\left(\frac{3\pi}{4}\right)\cdot\sin x + 2\cos\frac{\pi}{4}\cdot\cos x\right)$

$= \log_{\sqrt5}\left(3 - 2\times\frac{1}{\sqrt2}\sin x + 2\times\frac{1}{\sqrt2}\cos x\right)$

$= \log_{\sqrt5}\left(3 + \sqrt2(\cos x - \sin x)\right)$

$3 + \sqrt2(\cos x - \sin x) > 0$

$(y = \cos x - \sin x)$



$-\sqrt2 \le \cos x - \sin x \le \sqrt2$

$f(x) = \log_{\sqrt5}\left(3 + \sqrt2[-\sqrt2, \sqrt2]\right)$

$= \log_{\sqrt5}\left(3 + [-2, 2]\right)$

$\Rightarrow y = \log_{\sqrt5}([1, 5]) \Rightarrow (\sqrt5)^y \in [1, 5]$

$\Rightarrow (\sqrt5)^y \in [(\sqrt5)^0, (\sqrt5)^2]$

$\boxed{y \in [0, 2]}$ (Ans)

KTK-04

$f(x) = \log_{\sqrt5}\left(3 + \cos\left(\frac{3\pi}{4}+x\right) + \cos\left(\frac{3\pi}{4}-x\right) + \cos\left(\frac{\pi}{4}-x\right) + \cos\left(\frac{\pi}{4}+x\right)\right)$

$f(x) = \log_{\sqrt5}\left(3 + \left(-2\sin\left(3\frac{\pi}{2}\right)\cdot\sin x\right) + \cancel{\cos\left(\frac{\pi}{2}\right)}^{0}\cos(2x)\right)$

$f(x) = \log_{\sqrt5}(3 + 2\sin x)$

$[-1, 1]$

$[-2, 2]$

$[1, 5]$

$\left[0, \log_{\sqrt5} 5\right] = [0, 2]$ Ans

Kripa Shankar maurya varanasi

(Solution to RPP)

QUESTION [JEE Mains 2024 (8 April)]

(RPP 1)

The sum of all the solutions of the equation $(8)^{2x} - 16 \cdot (8)^{x} + 48 = 0$ is :

(A) $1 + \log_8 (6)$

(B) $1 + \log_6 (8)$

(C) $\log_8 (6)$

(D) $\log_8 (4)$

Ans. A

RPP-1. The sum of all the solutions of the eqn $(8)^{2x}-16\cdot(8)^x+48=0$ is:

RPP 1

$$(8)^{2x}-16\cdot(8)^x+48=0$$

$$\Rightarrow (8)^{2x}-12\cdot(8)^x-4\cdot(8)^x+48=0$$

$$\Rightarrow (8^x-4)(8^x-12)=0$$

$\therefore 8^x=4$ | $8^x=12$

$\Rightarrow x=\log_8 4$ | $\therefore x=\log_8 12.$

$\therefore$ Sum of the solution $=\log_8 4+\log_8 12$

$$=\log_8 48=\log_8 8\cdot 6$$

$$=(1+\log_8 6)\text{ Ans.}$$

Sourik Maiti
West Bengal

QUESTION [JEE Mains 2024 (8 April)]

(RPP 2)

Let α, β be the roots of the equation $x^2 + 2\sqrt{2}x - 1 = 0$. The quadratic equation, whose roots are $\alpha^4 + \beta^4$ and $\frac{1}{10}(\alpha^6 + \beta^6)$, is:

(A) $x^2 - 180x + 9506 = 0$

(B) $x^2 - 195x + 9506 = 0$

(C) $x^2 - 190x + 9466 = 0$

(D) $x^2 - 195x + 9466 = 0$

Ans. B

RPP-02

$x^2+2\sqrt{2}x-1=0 \quad S_n=\alpha^n+\beta^n$ ∵ α,β roots

By Newton formulas

$S_1=\alpha+\beta=-2\sqrt{2} \quad S_2=\alpha^2+\beta^2=10$

Now

$\underline{n=1}$ $\quad S_{n+2}+2\sqrt{2}S_{n+1}-S_n=0$

$S_3+2\sqrt{2}S_2-S_1=0$

$S_3=-2\sqrt{2}S_2+S_1=-22\sqrt{2}$

again $n=2$

$S_4+2\sqrt{2}S_3-S_2=0$

$S_4=-2\sqrt{2}S_3+S_2=\boxed{98=\alpha^4+\beta^4}$

Again.

$\underline{n=3}$

$S_5+2\sqrt{2}S_4-S_3=0$

$S_5=-2\sqrt{2}S_4-S_3=-218\sqrt{2}$

Now $n=4$

$S_6=S_4-2\sqrt{2}S_5$

$=98+2\times218\times2=970$

$S_6=\alpha^6+\beta^6=970$

But $\boxed{\frac{S_6}{10}=\frac{970}{10}}\boxed{=97}$

we get root of requir eqn

98 and 97

Eqn $=x^2-(98+97)x+97\times98=0$

$x^2-195x+9506$ Ans

Kripa Shankar maurya Varanasi

RPP 2

RPP-2 Let α,β be the roots of the eqn $x^2+2\sqrt{2}x-1=0$. The Quadratic eqn, whose roots are $\alpha^4+\beta^4$ and $\frac{1}{10}(\alpha^6+\beta^6)$, is—

$\rightarrow \quad \alpha+\beta=-2\sqrt{2}$

$\alpha\beta=-1$

Now, $\alpha^4+\beta^4=(\alpha^2+\beta^2)^2-2(\alpha\beta)^2$

$=\left[(\alpha+\beta)^2-2\alpha\beta\right]^2-2\times1$

$=\left[8+2\right]^2-2=100-2$

$=98$

$(\alpha^6+\beta^6)=(\alpha^2)^3+(\beta^2)^3=(\alpha^2+\beta^2)(\alpha^4+\beta^4-(\alpha\beta)^2)$

$=10[98-1]$

$=10\times97$

Now, $\frac{1}{10}(\alpha^6+\beta^6)$

$=\frac{1}{10}\times10\times97=97.$

∴ The quadratic eqn $=x^2-\left[(\alpha^4+\beta^4)+\frac{1}{10}(\alpha^6+\beta^6)\right]x+(\alpha^4+\beta^4)\cdot\frac{1}{10}(\alpha^6+\beta^6)$

$=x^2-(97+98)x+97\times98$

$=(x^2-195x+9506)$ — Ans.

Sourik Maiti West Bengal

QUESTION [JEE Mains 2024 (1 Feb)]

(RPP 3)

If $\tan A = \frac{1}{\sqrt{x(x^2+x+1)}}$, $\tan B = \frac{\sqrt{x}}{\sqrt{x^2+x+1}}$ and $\tan C = \left(x^{-3}+x^{-2}+x^{-1}\right)^{1/2}$, $0 < A, B, C < \frac{\pi}{2}$, then $A + B$ is equal to :

(A) C

(B) $\pi - C$

(C) $2\pi - C$

(D) $\frac{\pi}{2} - C$

Ans. C

RPP-3. If $\tan A = \dfrac{1}{\sqrt{x(x^2+x+1)}}$, $\tan B = \dfrac{\sqrt{x}}{\sqrt{x^2+x+1}}$ and

RPP 3

$\tan C = \left(x^{-3}+x^{-2}+x^{-1}\right)^{1/2}$ $0<A,B,C<\pi/2$, then $A+B$ is equal to –

$\rightarrow \tan A = \dfrac{1}{\sqrt{x}\cdot\sqrt{x^2+x+1}}$, $\tan B = \dfrac{\sqrt{x}}{\sqrt{x^2+x+1}}$, $\tan C = \left(\dfrac{1}{x^3}+\dfrac{1}{x^2}+\dfrac{1}{x}\right)^{1/2}$

$= \left(\dfrac{1+x+x^2}{x^3}\right)^{1/2}$

$\boxed{\tan C = \dfrac{\sqrt{1+x+x^2}}{x\sqrt{x}}}$

$\tan(A+B) = \dfrac{\tan A+\tan B}{1-\tan A\cdot\tan B}$

$= \dfrac{\dfrac{1}{\sqrt{x}\cdot\sqrt{x^2+x+1}}+\dfrac{\sqrt{x}}{\sqrt{x^2+x+1}}}{1-\dfrac{1}{\sqrt{x}\cdot\sqrt{x^2+x+1}}\cdot\dfrac{\sqrt{x}}{\sqrt{x^2+x+1}}}$

[as $x\neq 0$. $x\neq R^-$ $0<B<\pi/2$ ↳ as $\tan B\in R^+$ ↳ defined.]

$= \dfrac{\dfrac{1+x}{\sqrt{x}\cdot\sqrt{x^2+x+1}}}{\dfrac{x^2+x+1-1}{x^2+x+1}} = \dfrac{(1+x)\left(\sqrt{x^2+x+1}\right)}{\sqrt{x}\cdot x(1+x)}$

$= \dfrac{\sqrt{x^2+x+1}}{x\sqrt{x}}$

$= \tan C$

$\Rightarrow \tan(A+B) = \tan C$

$\boxed{\therefore A+B = C}$ Ans

sourik Maiti
West Bengal

RPP-03

$\tan A = \dfrac{1}{\sqrt{x(x^2+x+1)}}$, $\tan B = \dfrac{\sqrt{x}}{\sqrt{x^2+x+1}}$ $\tan C = (x^{-3}+x^{-2}+x^{-1})^{1/2}$

then $A+B = ?$

$\tan(A+B) = \dfrac{\tan A+\tan B}{1-\tan A\cdot\tan B}$

$= \dfrac{\dfrac{1}{\sqrt{x(x^2+x+1)}}+\dfrac{\sqrt{x}}{\sqrt{x^2+x+1}}}{1-\dfrac{\cancel{\sqrt{x}}}{\cancel{\sqrt{x}}(x^2+x+1)}}$

$= \dfrac{\sqrt{x^2+x+1}+x\sqrt{x^2+x+1}}{\sqrt{x}\,\cancel{(x^2+x+1)}}\times\dfrac{\cancel{(x^2+x+1)}}{(x^2+x)}$

$= \dfrac{\sqrt{x^2+x+1}\,\cancel{(1+x)}}{x\sqrt{x}\,\cancel{(x+1)}} = \dfrac{\sqrt{x^2+x+1}}{x^{3/2}}$

$\tan(A+B) = \left(x^{-3}+x^{-2}+x^{-1}\right)^{1/2}$

$\tan(A+B) = \tan C$

$\underline{A+B = C}$ Ans

Kripa Shankar maurya
Varanasi

Bhoolna मत

Yeh bataaya thaa

Elementary row operation (ERT)

1. Interchanging of rows
2. Multiplying a row by same non zero number
3. Adding multiple of one row to another row

Elementary matrix : Matrix obtained by applying a single ERT on a entity matrix is called elementary matrix. (it is represented by E)

EA gives a matrix obtained by applying the same ERT on A as we applied on T to get E.


THANK
YOU

PRAYAS

JEE 2025

Lecture- 08 | Mathematics

Relation & Functions

By- Ashish Agarwal Sir (IIT Kanpur)

Topics *to be covered*

1. Domain & Range Problems
2. Classification of Functions

Recap *of previous lecture*

1. If $|x| \le a,\ a \in R^+$ then $x \in$ $[-a, a]$
2. If $|x| \ge a,\ a \in R^+$ then $x \in$ $(-\infty, -a] \cup [a, \infty)$
3. If $f(x) = \frac{ax+b}{cx+d} \cdot \frac{a}{c} \ne \frac{b}{d}$ then range of $f(x)$ is $R - \left\{\frac{a}{c}\right\}$
4. $\log_{\frac{1}{2}} x < -1$ then $x \in$ $(2, \infty)$

$\log_{\frac{1}{2}} x < -1 \rightarrow x > \left(\frac{1}{2}\right)^{-1}$ & $x > 0$

$x > 2$ $\Downarrow$ (No need)

5. $x < |x|$ if $x \in$ $(-\infty, 0)$
6. $x > |x|$ if $x \in$ ϕ

Recap of previous lecture

Domain:
$|x| - x > 0$
$|x| > x \Rightarrow x \in R^-$

7. $f(x) = \frac{1}{\sqrt{|x|-x}}$ then domain of f is $(-\infty, 0)$

Domain $= R - \{7, 4/3\}$

8. $f(x) = \frac{(2x-3)\cancel{(x-7)}}{\cancel{(x-7)}(3x-4)}$ then range of f $R - \{2/3, \frac{11}{17}\}$

$f(x) = \frac{2x-3}{3x-4}, x \neq 7$

Range: $R - \{\frac{2}{3}, f(7)\}$

9. $f(x) = 2 \tan x \cdot \cos x$ then range of $(-2, 2)$

$f(x) = 2\frac{\sin x}{\cancel{\cos x}} \cdot \cancel{\cos x}$

10. Range of $f \subseteq$ codomain of f (T/F)

$f(x) = 2\sin x, x \neq (2n+1)\frac{\pi}{2}$

$(-1, 1)$

$(-2, 2)$

11. $f(x) = \sqrt{(x-1)(x-5)(x-3)^2}$ then domain $x \in (-\infty, 1] \cup [5, \infty) \cup \{3\}$

12. Range of odd degree polynomial defined over R is $(-\infty, \infty)$

(11) $(x-1)(x-5)(x-3)^2 \geq 0$

$(x-5)(x-1) \geq 0$ $\quad x=3$ possible

$+ \quad 1 \quad - \quad 5 \quad +$

$x \in (-\infty, 1] \cup [5, \infty) \cup \{3\}$

(16) $\log(2x-3) \neq 0$

$2x-3 \neq 1$ & $2x-3 > 0$.

$x \neq 2$ & $x > 3/2$

$x \in \left(\frac{3}{2}, \infty\right) - \{2\}$.

★ $\dfrac{1}{f(x)}$ Domain $\downarrow$ $f(x) \neq 0$

f defined bhi toh honaa chahiyay

Gadho | Gadhiyo aisaa naa karo.

$\log(2x-3) \neq 0$

$2x-3 \neq 1$

$x \neq 2$ Ans.

Recap of previous lecture

13. Range of even degree polynomial is always a proper subset of R

14. If a polynomial function f satisfies $f(x)+f(1/x) = f(x) \cdot f(1/x)\ \forall\ x \in R_0$ then f may be $1 \pm x^n, n \in I^+$ or $f(x)=0$ or $f(x)=2$.

15. If f(x) has domain [–1, 1] then domain of f(2x + 3) is $[-2,-1]$

and $R_f = [-3, 4]$

$-1 \le 2x+3 \le 1$

$-4 \le 2x \le -2$

$-2 \le x \le -1$

Range of (a) $f(2x+3)$ = Range $[-3,4]$

(b) $g(x) = 5 + 2f(2x+3)$

$[-3,4]$

$[-6,8]$

Range: $[-1,13]$

16. If log (2x-3) ≠ 0 then $x \in (3/2, \infty) - \{2\}$.

17. Domain of $f(x) = \dfrac{1}{e^{2\log_e x} - 2x - 3}$ is $x \in (0,\infty) - \{3\}$

$$f(x) = \frac{1}{e^{2\log_e x} - 2x - 3}$$

$$e^{2\log_e x} - 2x - 3 \neq 0 \quad \& \quad x > 0$$

$$e^{\log_e x^2} - 2x - 3 \neq 0 \quad \& \quad x > 0$$

$$x^2 - 2x - 3 \neq 0$$

$$(x-3)(x+1) \neq 0 \quad \& \quad x > 0$$

$$x \neq 3, -1 \quad \& \quad x > 0$$

$x \neq 3$ $\& \, x > 0$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

e^x polynomial??

$\Downarrow$

Not a polynomial

Quad lect -10

Range of Rational function

$$y = \frac{ax+b}{cx+d}$$

$$cxy + dy = ax + b$$

$$dy - b = (a - cy)x$$

$$x = \frac{dy - b}{a - cy}$$

$$y \in R - \{a/c\}$$

Galti Batao

2) $f(x) = \log_2\left(-\log_{1/2}\left(1+\frac{1}{\sqrt[4]{x}}\right)-1\right)$

$-\log_{1/2}\left(1+\frac{1}{\sqrt[4]{x}}\right)-1>0$ $\quad 1+\frac{1}{\sqrt[4]{x}}>0$ $\quad x>0$

always true

$\log_{1/2}\left(1+\frac{1}{\sqrt[4]{x}}\right)<-1$

sign reverse

$1+\frac{1}{\sqrt[4]{x}}<2$

$\frac{1}{\sqrt[4]{x}}<1$

$\frac{1}{x}<1$

$\boxed{x>1}$

$x \in (1, \infty)$ Ans

Galti Batao

Q. The domain of $f(x) = \dfrac{\log_{(x+1)}(x-2)}{e^{2\log_e x} - (2x+3)}$, $x \in R$. [JM-23]

$\Rightarrow f(x) = \dfrac{\log_{(x+1)}(x-2)}{x^2 - (2x+3)}$

$\Rightarrow x+1 \neq 1$

$x \neq 0$

$\Rightarrow x-2>0$

$\Rightarrow x>2$

$x \in (2, \infty)$

— Eq (1)

$x^2-2x-3 \neq 0$, $x>0$

$x^2-3x+x-3 \neq 0$, $x>0$

$(x+1)(x-3) \neq 0$, $x>0$

$x \neq -1$ $x \neq 3$

Then

$x \in (2, \infty) - \{3\}$.

Doubts

Dipankar Paul 17 hr ago

sir slide no. 9 me agar ham x axis ko 1 unit niche le aye to fir to origin 0,-1 pe shift ho jayega na?
to fir to ye point 0,0 kaise ho sakta hai.?

graph ko shift karne ka matlab samajh gaya lekin x axis shift karne ka matlab clear nhi hua🙏🙏🙏🙏🙏🙏🙏🙏

❤ 1 Likes ⚠ Report

Pratham Vishwakarma 16 hr ago

sir apne Jo fx gx nikalne ke liye cases liye the jaise x<2 , x€[1,2) etc ye cases kaise liye sir mai soch nhi pa rha hu ye ...

❤ Like ⚠ Report

Shrey Prayas One Point O 13 hr ago

graphical transformation krne pr range bhi change ho jati h na...like range of sinx is [-1,1] but that of 1 sinx is [0,2]...well if function has changed so range should also have changed ...

❤ Like ⚠ Report

abhijeet 3 hr ago

sir in the time interval 23:49
we should take the intersection na
because in for fun √(fx/gx)
domain = Df intersection Dg
??????

❤ Like ⚠ Report

Anshul 15 hr ago

sir graph smj nhi aa rhe 🥺🥺 sab lecture dekhne ke baad bhi

❤ Like ⚠ Report

Discussion of Homework of Previous Class/Doubts

QUESTION

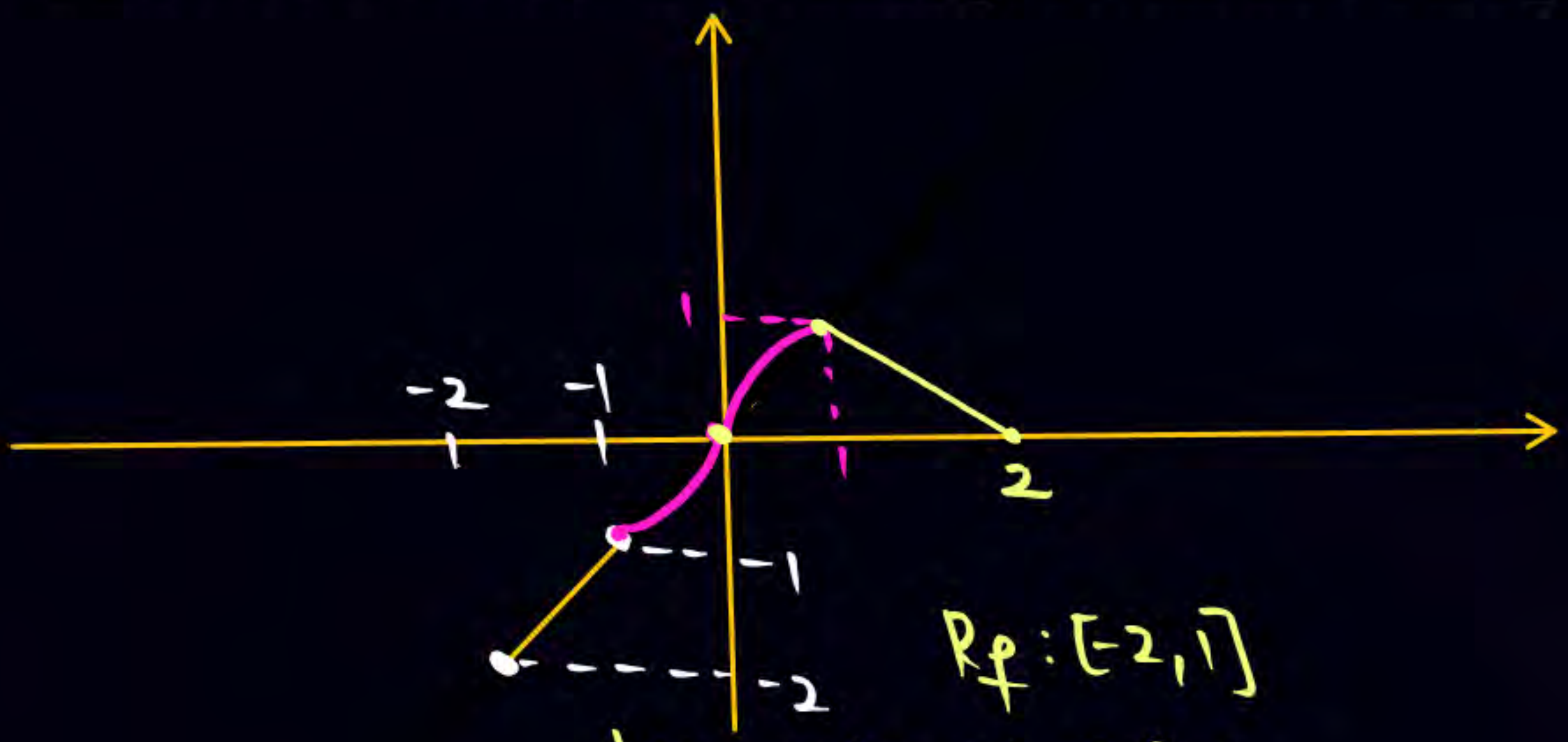
(a) Let $f(x) = \begin{cases} x, & -2 \le x \le -1 \\ x^2 + 2x, & -1 < x \le 0 \\ 2x - x^2, & 0 < x \le 1 \\ 2 - x, & 1 < x \le 2 \end{cases}$

Find the number of integers in the range of f(x).

$y = x^2 + 2x$
$= x(x+2)$

$y = 2x - x^2$
$= x(2-x)$ Downward open.
roots $= 0, 2$

$y = 2 - x.$



$R_f : [-2, 1]$

Integers in Range = $-2, -1, 0, 1$

QUESTION

Tan 1

P Let $g(x) = \begin{cases} x^2 - 2, & -\infty < x < 0 \\ x, & 0 \le x < 2 \\ (x-2)^2, & 2 \le x < 4 \\ x - 4, & 4 \le x < \infty \end{cases}$.

If the equation $g(x) = k$ has four real and distinct roots, then find the sum of all possible integral values of k.

Q. find the Range of K for which

$$||x^2-6x+8|-12| = K$$ has

(1) No soln

(2) Exactly 2 real soln.

(3) Exactly 4 real solns.

(4) Exactly 6 real solutions.

Tanz

QUESTION

Gadho/Gadhiyoo aisaa na karo!!

$f(x) = \frac{1}{1+\frac{1}{x}} = \frac{1}{\frac{x+1}{x}}$

$f(x) = \frac{x}{1+x} = g(x)$

$D_f = R - \{-1\} = D_g$

$\Downarrow$

Identical.

Identify the equal function

(i) $f(x) = \log_x e; g(x) = \frac{1}{\log_e x}$

(ii) $f(x) = \log_e x; g(x) = \frac{1}{\log_x e}$

(iii) $f(x) = \sqrt{x^2 - 1}; g(x) = \sqrt{x-1}\sqrt{x+1}$

(iv) $f(x) = \log(x+2) + \log(x-3); g(x) = (x^2 - x - 6)$

(v) $f(x) = x|x|; g(x) = x^2 \operatorname{sgn} x$

$D_f : R - \{0, -1\}$

(vi) $f(x) = \frac{1}{1+\frac{1}{x}}; g(x) = \frac{x}{1+x}$ — $0 \notin D_f$ but $0 \in D_g \Rightarrow D_f \neq D_g \Rightarrow (N.I)$

(vii) $f(x) = [\{x\}]; g(x) = \{[x]\}$

sometimes simplification

leads to change in fn

QUESTION

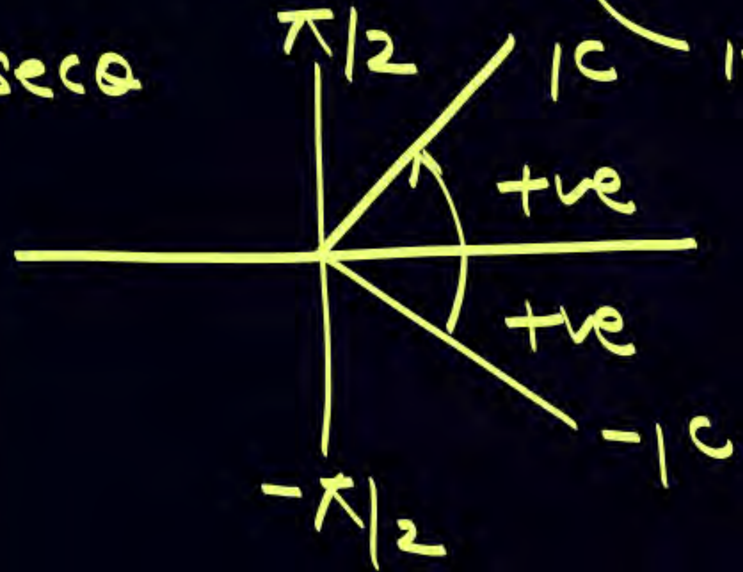
$$f(x) = \frac{1}{[x]} + \log_{1-\{x\}}(x^2 - 3x + 10) + \frac{1}{\sqrt{2-|x|}} + \frac{1}{\sqrt{\sec(\sin x)}}$$

$\sec\theta \in (-\infty, -1] \cup [1, \infty)$
$\theta \in R - (2n+1)\pi/2$

$\sec(\underbrace{\sin x}_{[-1,1]}) = \sec\underbrace{\theta}_{[-1,1]} = +ve$

if $x \in (-\infty, \infty)$

$y = \sec\theta$



QUESTION [JEE Mains 2024 (30 Jan)]

(KTK 3)

If the domain of the function $f(x) = \cos^{-1}\left(\frac{2-|x|}{4}\right) + \{\log_e(3-x)\}^{-1}$ is $[-\alpha, \beta) - \{\gamma\}$, then $\alpha + \beta + \gamma$ is equal to :

(A) 11

(B) 12

(C) 9

(D) 8

Ans. A

KTK-3

Sol^n :→ $\cos^{-1}\left(\frac{2-|x|}{4}\right)$

⇓

$$-1 \le \frac{2-|x|}{4} \le 1$$

×4

$$-4 \le 2-|x| \le 4$$

(−2)

$$-6 \le -|x| \le 2$$

$-6 \le -\|x\|$	$-\|x\| \le 2$
$\|x\| \le 6$	$\|x\| \ge -2$
$x \le \pm 6$	
$x \in [-6, 6]$ — (I)	(Always true)

$\Rightarrow [\log_e(3-x)]^{-1}$

$\Rightarrow \frac{1}{\log_e(3-x)}$

$\Rightarrow \log_e(3-x) \ne 0$

$(3-x) \ne e^0$

$3-x \ne 1$

$\boxed{x \ne 2}$

also

$3-x > 0$

$\boxed{x < 3}$

$x < 3, x \ne 2$ — (II)

from (I) & (II)

$-6 \quad 2 \quad 3 \quad 6$

$$x \in [-6, 3) - \{2\}$$

$-\alpha = -6$, $\beta = 3$, $\gamma = 2$

then

$$\alpha + \beta + \gamma \Rightarrow +6 + 3 + 2 \Rightarrow \boxed{11} \text{ Ans}$$

Divyanshu Sagar
Bihar

Range Finding Method

M1: Put $y = f(x)$ and then & solve x in terms of y and then use the condition $x \in R$.

M2: For continuous function interval from minimum to maximum value gives range.

M3: Find Domain & try to find outputs as per domain.

M4: Draw graph..

M5: Use Gola Method

Problems on Range of Functions

QUESTION

$$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$$
$$\alpha^4+\beta^4=(\alpha^2+\beta^2)^2-2\alpha^2\beta^2$$

$$|x|^2=x^2$$

Find Range of

$$f(x)=\frac{1}{\sin^4 x+\cos^4 x}$$

$$f(x)=\frac{1}{(\sin^2 x+\cos^2 x)^2-2\sin^2 x\cos^2 x}$$

$$=\frac{1}{1-2\sin^2 x\cos^2 x}=\frac{2}{2-4\sin^2 x\cos^2 x}$$

$$=\frac{2}{2-(2\sin x\cos x)^2}=\frac{2}{2-(\sin 2x)^2}$$

$[-1,1]=[-1,0]\cup[0,1]$

$[0,1]$

$[1,2]$

$2\left[\frac{1}{2},1\right]=[1,2]=R_f.$

$$f(x)=|\sin x|+|\cos x|$$

$y=|\sin x|+|\cos x|$ – clearly $y\geq 0$

$y^2=|\sin x|^2+|\cos x|^2+2|\sin x|\cdot|\cos x|$

$=\sin^2 x+\cos^2 x+|\sin 2x|$

$=1+|\sin 2x|$

$[-1,1]$

$[0,1]$

$[1,2]$

$y^2\in[1,2]$

$y\in[\sqrt{1},\sqrt{2}]$

$y\in[1,\sqrt{2}]$

(a) $y = \sin(2x)$

$(-\infty, \infty)$

Range: $[-1, 1]$

(b) $y = 2\sin x$

$[-1, 1]$

Range: $[-2, 2]$

(c) $y = \sin(3x+2)$

$(-\infty, \infty)$

Range: $[-1, 1]$

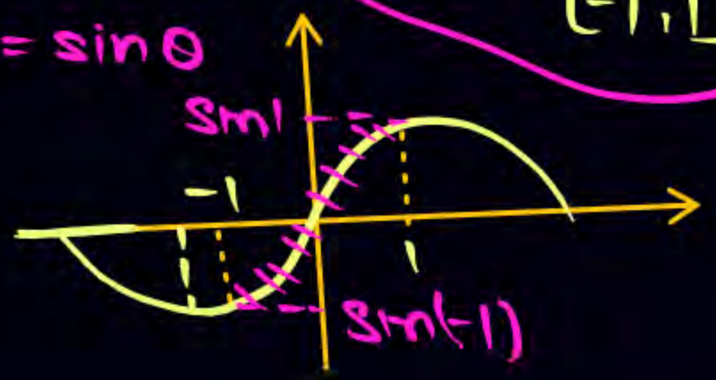
(P) Find range of (i) $y = \sin(\cos x)$

$[-1, 1]$

$[\sin(-1), \sin 1]$

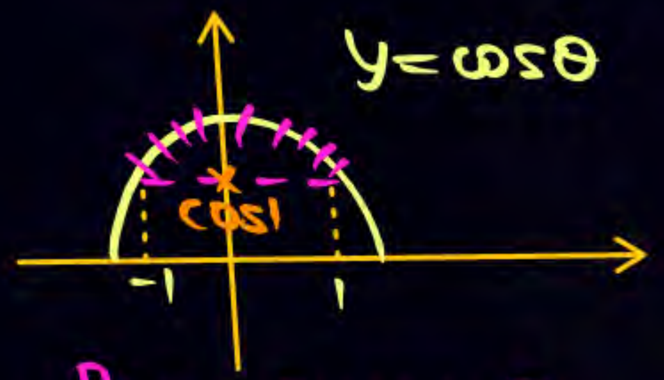
$\parallel$

$[-\sin 1, \sin 1]$



(ii) $y = \cos(\sin x)$

$[-1, 1]$

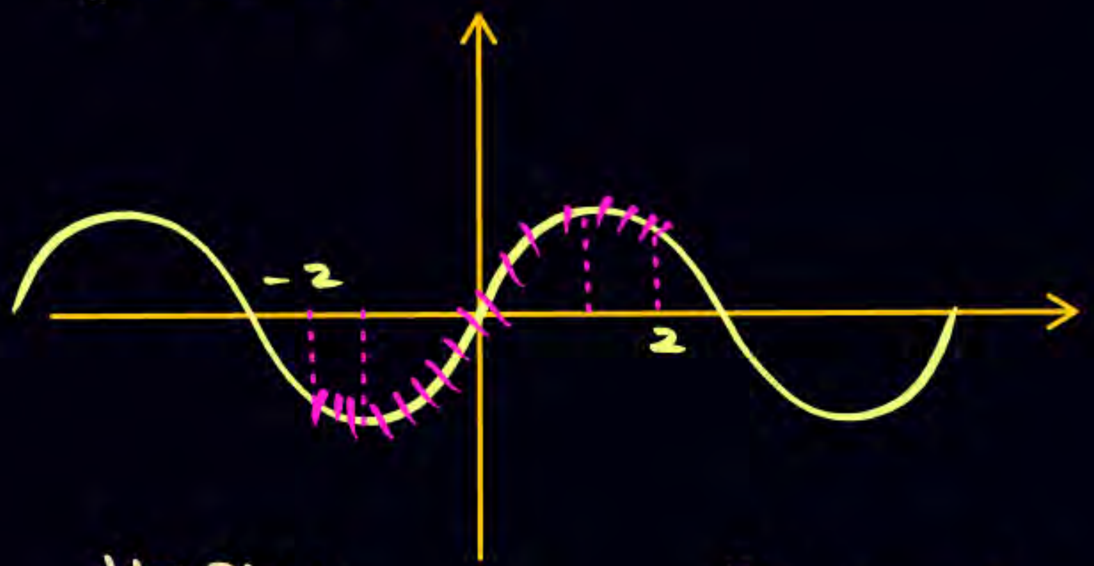


Range $= [\cos 1, 1]$

(iii) $y = \sin(2\cos x)$

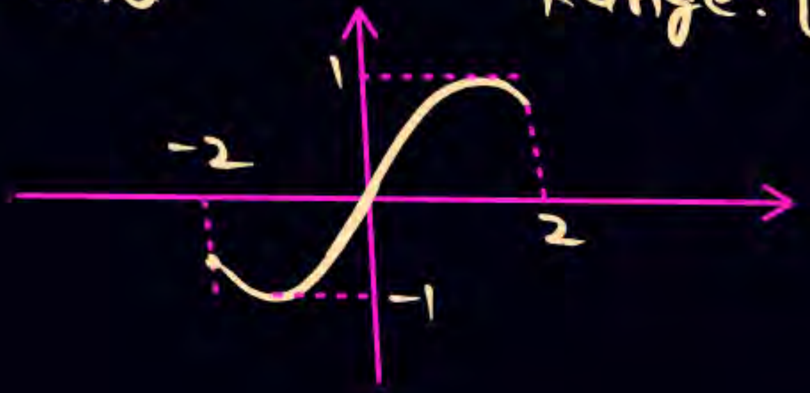
$2\cos x \in [-2, 2]$

$y = \sin\theta$



$y = \sin\theta$

Range: $[-1, 1]$



$\frac{\pi}{2} \approx 1.57$

$\frac{\pi}{4} \approx 0.77$

$\frac{3\pi}{2} \approx 4.7$

$2\pi \approx 6.28$

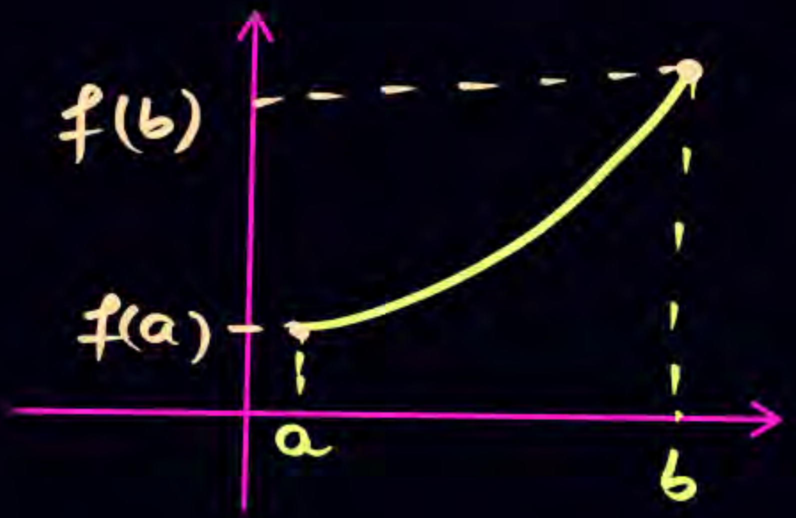
(iv) $f(x) = 2 - x \quad x \in [2, 5]$

MAX $= 2 - 5 = -3$

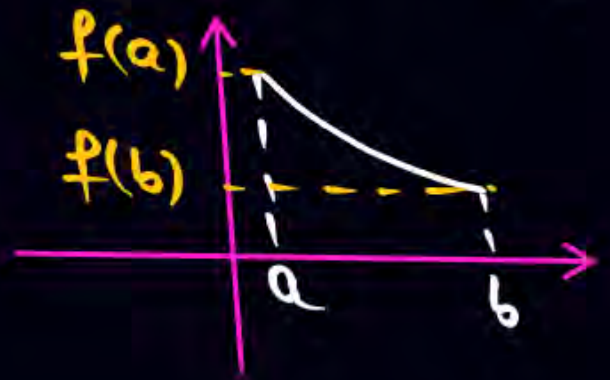
MIN $= 2 - 2 = 0$

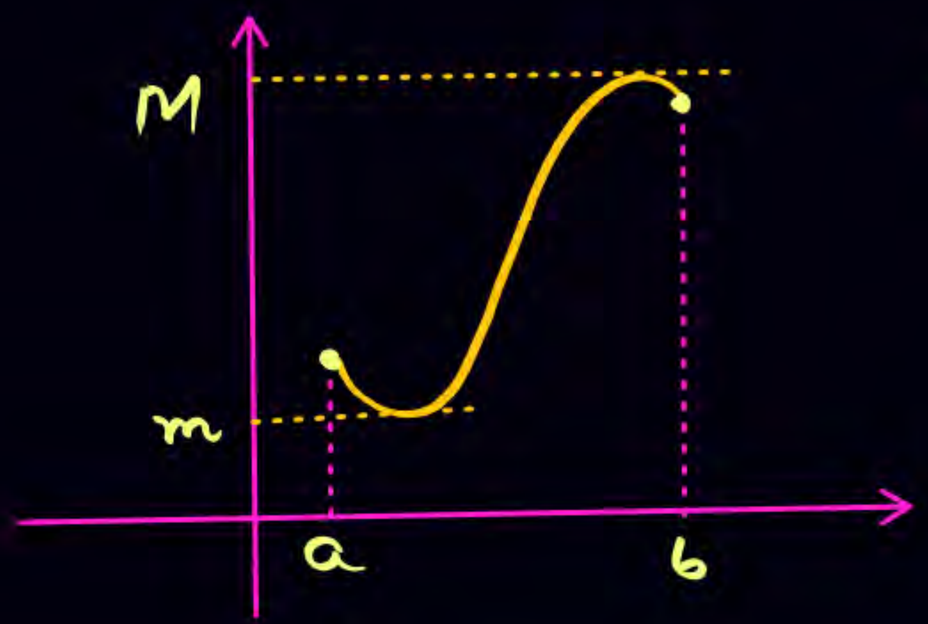
Gadho | Gadhiyo aisaa naa karo

★ If f is inc & continuous in $[a, b]$ then value of f over this interval is $[f(a), f(b)]$



★ If f is dec & continuous in $[a, b]$ then value of f over this interval is $[f(b), f(a)]$





★ if f is neither inc nor dec on $[a,b]$ but continuous then the value of f over this interval varies in $[m, M]$

★ $\sin x, \cos x \in [-1,1]$

★ $\sin^{2n} x, \cos^{2n} x \in [0,1]$

★ $(\sin x)^{2n+1}, (\cos x)^{2n+1} \in [-1,1]$

$n \in I^+$

QUESTION find Range

$$f(x) = \ln\left(5x^2 - 8x + 4\right)$$

$= \ln\left(5\left(x^2 - \frac{8}{5}x\right) + 4\right)$

$= \ln\left(5\left(x^2 - \frac{8}{5}x + \frac{16}{25} - \frac{16}{25}\right) + 4\right)$

$= \ln\left(5\left(x - \frac{4}{5}\right)^2 - \frac{16}{5} + 4\right)$

$= \ln\left(5\left(x - \frac{4}{5}\right)^2 + \frac{4}{5}\right)$

$[0, \infty) \to [4/5, \infty) \to [\ln\frac{4}{5}, \infty)$

$y = \ln x$

(ASRQ)

$$f(x) = \log_2\left(2 - \log_{\sqrt{2}}\left(16\sin^2 x + 1\right)\right)$$

$y = \log_2\left(2 - \log_{\sqrt{2}}\left(16\sin^2 x + 1\right)\right)$

$[0, 1]$

$[1, 17]$

$[\log_{\sqrt{2}} 1, \log_{\sqrt{2}} 17]$

$= [0, 8.\ldots]$

$= [0, 2)$

$(0, 2]$

$(-\infty, 1]$

$=$ Range

Reducing Range

QUESTION

Find Range of following functions:

(a) $f(x) = e^{(x-1)^2}$ (b) $f(x) = 2^{x^2} + 1$ (c) $f(x) = \dfrac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}$

Tan 3

$\Downarrow$

$y = \dfrac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}$

$y = \dfrac{e^{2x}+e^{x}+1-2e^{x}}{e^{2x}+e^{x}+1}$

$y = 1 - \dfrac{2e^{x}}{e^{2x}+e^{x}+1}$

$y = 1 - \dfrac{2}{\dfrac{e^{2x}+e^{x}+1}{e^{x}}} = 1 - \dfrac{2}{e^{x}+1+\dfrac{1}{e^{x}}}$

$y = 1 - \dfrac{2}{e^{x}+\frac{1}{e^{x}}+1}$

≥ 2

$[3, \infty)$

$(0, \tfrac{1}{3}]$

$2(0, \tfrac{1}{3}] = (0, \tfrac{2}{3}]$

Range $= [\tfrac{1}{3}, 1)$

$$a\sin x + b\cos x \in \left[-\sqrt{a^2+b^2}, \sqrt{a^2+b^2}\right]$$

QUESTION [JEE Mains 2023 (30 Jan)]

$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}, \; a, b \geq 0$

The range of the function $f(x) = \sqrt{3-x} + \sqrt{2+x}$ is :

$3 - x \geq 0$, $x + 2 \geq 0$ $\Rightarrow x \in [-2, 3]$

(A) $[2\sqrt{2}, \sqrt{11}]$

(B) $[\sqrt{5}, \sqrt{13}]$

(C) $[\sqrt{2}, \sqrt{7}]$

(D) $[\sqrt{5}, \sqrt{10}]$

$y = \sqrt{3-x} + \sqrt{2+x}$ — Domain $= [-2, 3]$

Clearly y is non-ve

S.B.S

$$y^2 = 3 - x + 2 + x + 2\sqrt{3-x} \cdot \sqrt{2+x}$$

$$= 5 + 2\sqrt{6 + x - x^2}$$

$$= 5 + 2\sqrt{6 - (x^2 - x + 1/4 - 1/4)}$$

$$= 5 + 2\sqrt{6 - [(x - 1/2)^2 - 1/4]}$$

$$= 5 + 2\sqrt{6 + 1/4 - (x - 1/2)^2}$$

$$= 5 + 2\sqrt{25/4 - (x - 1/2)^2}$$

$$y^2 = 5 + 2\sqrt{\frac{25}{4} - \left(x - \frac{1}{2}\right)^2}$$

$[-2, 3]$

$[0, 5/2]$

$[0, 5]$

$[5, 10]$

$y^2 \in [5, 10]$

$y \in [\sqrt{5}, \sqrt{10}]$

$x - \frac{1}{2} \in \left[-\frac{5}{2}, \frac{5}{2}\right] = \left[-\frac{5}{2}, 0\right] \cup \left[0, \frac{5}{2}\right]$

$\left(x - \frac{1}{2}\right)^2 \in [0, 25/4]$

$\frac{25}{4} - (x - 1/2)^2 \in \left[0, \frac{25}{4}\right]$

$\sqrt{25/4 - (x - 1/2)^2} \in [0, 5/2]$

QUESTION [JEE Mains 2023 (30 Jan)]

$\sqrt{ab} = \sqrt{a}\cdot\sqrt{b},\ a, b \geq 0$

$\left.\begin{matrix} 3-x \geq 0 \\ x+2 \geq 0 \end{matrix}\right\rangle x \in [-2, 3]$

The range of the function $f(x) = \sqrt{3-x} + \sqrt{2+x}$ is :

(A) $[2\sqrt{2}, \sqrt{11}]$

(B) $[\sqrt{5}, \sqrt{13}]$

(C) $[\sqrt{2}, \sqrt{7}]$

(D) $[\sqrt{5}, \sqrt{10}]$

M 2 $\quad y = \sqrt{3-x} + \sqrt{2+x}$ — Domain $= [-2, 3]$

Clearly y is non -ve

S.B.S

$$y^2 = 3 - x + 2 + x + 2\sqrt{3-x}\cdot\sqrt{2+x}$$

$$= 5 + 2\sqrt{3-x}\cdot\sqrt{2+x}$$

A.M G.M Ineq.

$$\frac{\underbrace{3-x}_{\checkmark} + \underbrace{2+x}_{\checkmark}}{2} \geq \sqrt{(3-x)(2+x)} \Rightarrow \sqrt{3-x}\cdot\sqrt{2+x} \leq 5/2.$$

Non-ve.

$$\sqrt{3-x}\cdot\sqrt{2+x}\Big|_{MAX} = 5/2$$

$$\sqrt{3-x}\sqrt{2+x}\Big|_{MIN} = 0$$

$$y^2 = 5 + 2\underbrace{\sqrt{3-x}\,\sqrt{2+x}}$$

$max = \frac{5}{2}$ $min = 0$

$[0, 5]$

$[5, 10]$

$y^2 \in [5, 10]$

$y \in [\sqrt{5}, \sqrt{10}]$ = Range

QUESTION [JEE Mains 2023 (30 Jan)]

$\sqrt{ab} = \sqrt{a}\cdot\sqrt{b},\ a, b \geq 0$

$3 - x \geq 0$, $x + 2 \geq 0$ $\Rightarrow x \in [-2, 3]$

The range of the function $f(x) = \sqrt{3-x} + \sqrt{2+x}$ is :

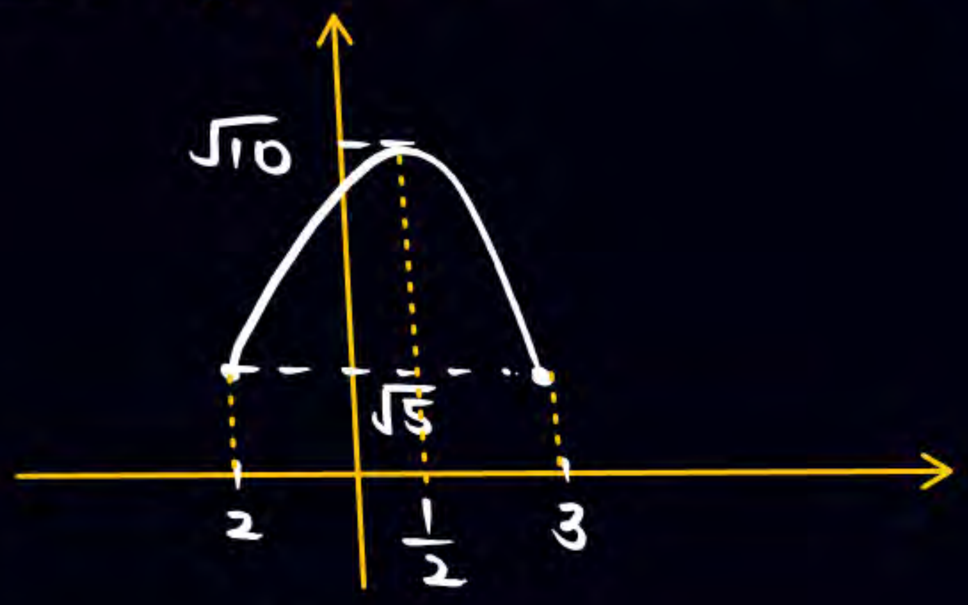
(A) $[2\sqrt{2}, \sqrt{11}]$

(B) $[\sqrt{5}, \sqrt{13}]$

(C) $[\sqrt{2}, \sqrt{7}]$

(D) $[\sqrt{5}, \sqrt{10}]$

M 3 $y = \sqrt{3-x} + \sqrt{2+x}$ — Domain $= [-2, 3]$



$[\sqrt{5}, \sqrt{10}]$

$$\frac{dy}{dx} = \frac{-1}{2\sqrt{3-x}} + \frac{1}{2\sqrt{2+x}}$$

$$= \frac{\sqrt{3-x} - \sqrt{2+x}}{2\sqrt{2+x}\sqrt{3-x}} = 0$$

$$\sqrt{3-x} = \sqrt{2+x}$$

$$3 - x = 2 + x$$

$$x = 1/2$$

QUESTION [JEE Mains 2023 (30 Jan)]

$\sqrt{ab} = \sqrt{a}\cdot\sqrt{b},\ a, b \geq 0$

$3 - x \geq 0$, $x + 2 \geq 0$ $\Rightarrow x \in [-2, 3]$

The range of the function $f(x) = \sqrt{3 - x} + \sqrt{2 + x}$ is :

(A) $[2\sqrt{2}, \sqrt{11}]$

(B) $[\sqrt{5}, \sqrt{13}]$

(C) $[\sqrt{2}, \sqrt{7}]$

(D) $[\sqrt{5}, \sqrt{10}]$

M(4) $y = \sqrt{3-x} + \sqrt{2+x}$

Domain $[-2, 3]$

let $x = 3\cos^2\theta - 2\sin^2\theta$

$$y = \sqrt{3 - (3\cos^2\theta - 2\sin^2\theta)} + \sqrt{2 + 3\cos^2\theta - 2\sin^2\theta}$$

$$= \sqrt{3(1-\cos^2\theta) + 2\sin^2\theta} + \sqrt{2(1-\sin^2\theta) + 3\cos^2\theta}$$

$$= \sqrt{5\sin^2\theta} + \sqrt{5\cos^2\theta}$$

$$= \sqrt{5}\,\underbrace{(|\sin\theta| + |\cos\theta|)}_{[1, \sqrt{2}]} \rightarrow [\sqrt{5}, \sqrt{10}]$$

QUESTION

Tan5

Find the domain & range of the following functions :

$y = \sqrt{2 - x} + \sqrt{1 + x}$

QUESTION [JEE Mains 2023 (25 Jan)]

Tan 6

Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \log_{\sqrt{m}}\{\sqrt{2}(\sin x - \cos x) + m - 2\}$, for some m, such that the range of f is [0, 2]. Then the value of m is

(A) 4

(B) 3

(C) 5

(D) 2

QUESTION [JEE Mains 2020 (8 Jan)]

(KTK 5)

codomain

Let $f:(1,3) \to R$ be a function defined by $f(x) = \frac{x[x]}{1+x^2}$, where $[x]$ denotes the greatest integer $\leq x$. Then the range of f is

Domain: $(1,3)$

(A) $\left(\frac{2}{5},\frac{1}{2}\right) \cup \left(\frac{3}{5},\frac{4}{5}\right]$

(B) $\left(\frac{3}{5},\frac{4}{5}\right)$

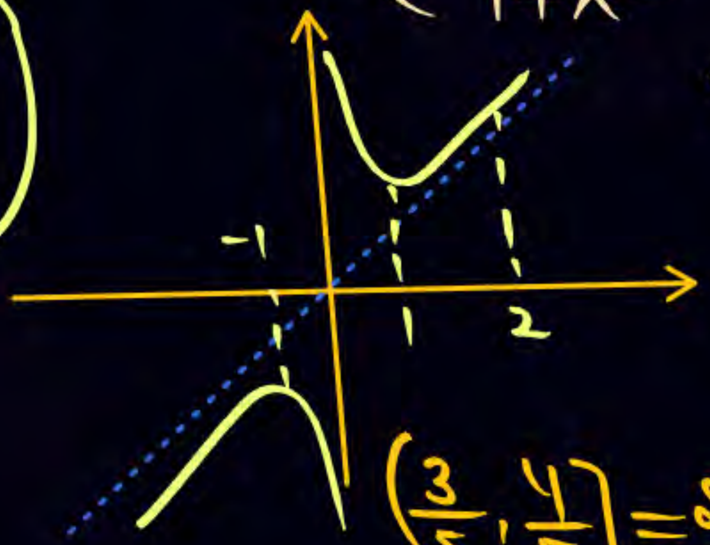
(C) $\left(\frac{2}{5},\frac{4}{5}\right]$

(D) $\left(\frac{2}{5},\frac{3}{5}\right] \cup \left(\frac{3}{4},\frac{4}{5}\right)$

$$f(x) = \frac{x[x]}{1+x^2} = \begin{cases} \frac{x}{1+x^2} & \text{if } 1<x<2 \\ \frac{2x}{1+x^2} & \text{if } 2\leq x<3 \end{cases}$$

$\underbrace{\text{If } x\in(1,2)}$ $y = \frac{x}{1+x^2} = \frac{1}{x+\frac{1}{x}}$ $(2, 5/2)$

$\left(\frac{2}{5},\frac{1}{2}\right)$



$y = \frac{2x}{1+x^2}$ $x\in[2,3)$

$y = \frac{2}{x+1/x}$ $[5/2, \frac{10}{3})$

$\left(\frac{3}{5},\frac{4}{5}\right] = 2\left(\frac{3}{10},\frac{2}{5}\right]$

Ans. A

Range of f : $\left(\frac{2}{5}, \frac{1}{2}\right) \cup \left(\frac{3}{5}, \frac{4}{5}\right]$

QUESTION [JEE Mains 2023 (31 Jan)]

Tan 7

ASRQ

If the domain of the function $f(x) = \frac{[x]}{1+x^2}$, where $[x]$ is greatest integer $\leq x$, is $[2, 6)$. then its range is

(A) $\left(\frac{5}{37}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$

(B) $\left(\frac{5}{37}, \frac{2}{5}\right]$

(C) $\left(\frac{5}{26}, \frac{2}{5}\right]$

(D) $\left(\frac{5}{26}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$

QUESTION

ASRQ

Find the domain & range of the following functions :

(1) $f(x) = \dfrac{x}{1+|x|}$ — Tan 7 (b)

(2) $f(x) = \dfrac{\sqrt{x+4}-3}{x-5}$ — Domain: $x+4 \geq 0$ & $x-5 \neq 0$ $\Downarrow$

Domain $= [-4, \infty) - \{5\}$

$$f(x) = \frac{\sqrt{x+4}-3}{x-5} \cdot \frac{\sqrt{x+4}+3}{\sqrt{x+4}+3}$$

$$f(x) = \frac{\cancel{x-5}}{(\cancel{x-5})(\sqrt{x+4}+3)}$$

$$f(x) = \frac{1}{\sqrt{x+4}+3}.$$

$$f(x) = \frac{1}{\sqrt{x+4}+3}$$

$[0,\infty)-\{9\}$

$[0,\infty)-\{3\}$

$[3,\infty)-\{6\}$

Range $= (0, 1/3] - \{1/6\}$

QUESTION [JEE Mains 2024 (6 April)]

Tan 7 C

Let $f(x) = \frac{1}{7-\sin 5x}$ be a function defined on R. Then the range of the function $f(x)$ is equal to :

(A) $\left[\frac{1}{8}, \frac{1}{5}\right]$

(B) $\left[\frac{1}{7}, \frac{1}{6}\right]$

(C) $\left[\frac{1}{7}, \frac{1}{5}\right]$

(D) $\left[\frac{1}{8}, \frac{1}{6}\right]$

Ans. D

QUESTION

Jan 8

The range of the function $y = \frac{8}{9-x^2}$ is

(A) $(-\infty, \infty) - \{\pm 3\}$

(B) $\left[\frac{8}{9}, \infty\right)$

(C) $\left(0, \frac{8}{9}\right)$

(D) $(-\infty, 0) \cup \left[\frac{8}{9}, \infty\right)$

Ans. D

QUESTION

Tan 9

Find range of :

(1) $f(x) = \frac{2x-3}{x-1}$

(2) $f(x) = \frac{x+3}{2-5x}$

(3) $f(x) = \frac{6}{4x+7}$

(4) $f(x) = \frac{7x+5}{3}$

QUESTION [JEE Mains 2023 (31 Jan)]

Tan 10

Let $f : \mathbb{R} - \{2, 6\} \to \mathbb{R}$ be real valued function defined as $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$. Then range of f is

WATCH lect-10,11 of Quadratic

(A) $\left(-\infty, -\frac{21}{4}\right] \cup [1, \infty)$

(B) $\left(-\infty, -\frac{21}{4}\right) \cup (0, \infty)$

(C) $\left(-\infty, -\frac{21}{4}\right] \cup [0, \infty)$

(D) $\left(-\infty, -\frac{21}{4}\right] \cup \left[\frac{21}{4}, \infty\right)$

QUESTION [JEE Mains 2019]

Let $[x]$ denote the greatest integer less than or equal to x. Then the values of $x \in R$ satisfying the equation $[e^x]^2 + [e^x + 1] - 3 = 0$ lies in the interval:

(A) $\left[0, \frac{1}{e}\right)$

(B) $[\log_e 2, \log_e 3)$

(C) $[1, e)$

(D) $[0, \log_e 2)$

$$[x+m] = [x] + m$$
$$m \in I$$

$$[e^x]^2 + [e^x] + 1 - 3 = 0$$

let $[e^x] = t$

$$t^2 + t - 2 = 0$$

$$(t+2)(t-1) = 0$$

$$t = 1, -2$$

$$[e^x] = 1, -2$$

$1 \le e^x < 2$ $\quad$ $-2 \le e^x < -1$ Not possible $\because e^x > 0$

$\ln 1 \le x < \ln 2$

$x \in [0, \ln 2)$

ASNC

$$[x+m]=[x]+m\ ,\ m\in I$$

$$\underbrace{[x+m]-[x]}_{\text{difference}=m}=m$$

$$\begin{array}{r} -1.56 \\ -\ 2.44 \\ \hline 4.00 \\ \hline \end{array}$$

Ex $[1.26]-[2.26]=-1$

$[-1.56]-[2.44]=-4$

QUESTION

ASRQ

Let [x] = the greatest integer less than or equal to x. If all the values of x such that the product $\left[x-\frac{1}{2}\right]\left[x+\frac{1}{2}\right]$ is prime, belongs to the set $[x_1, x_2) \cup [x_3, x_4)$, find the values of $x_1^2 + x_2^2 + x_3^2 + x_4^2$.

Greatest Integer x is always integer

$y = \left[x+\frac{1}{2}\right]\left[x-\frac{1}{2}\right] \in$ prime

diff 1

⇓

y is product of 2 consecutive Integers.

$1 \cdot 2$, $-2 \cdot -1 \in$ prime

$[x+1/2] = 2$ or $[x+1/2] = -1$

$2 \le x+1/2 < 3$ or $-1 \le x+1/2 < 0$

$2-\frac{1}{2} \leq x < 3-\frac{1}{2}$ or $-1-\frac{1}{2} \leq x < 0-\frac{1}{2}$

$x \in \left[\frac{3}{2}, \frac{5}{2}\right)$ or $x \in \left[-\frac{3}{2}, -\frac{1}{2}\right)$

$$x \in \left[-\frac{3}{2}, -\frac{1}{2}\right) \cup \left[\frac{3}{2}, \frac{5}{2}\right)$$

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 = \frac{9}{4} + \frac{1}{4} + \frac{9}{4} + \frac{25}{4}$$

$$= 5/2 + \frac{17}{2}$$

$$= 11 \text{ Ans}$$

QUESTION

Jan 11

Read the symbols [ ] and { } as greatest integer function less than or equal to x and fractional part function..

(i) Find the number of real values of x, satisfying the equation $(x-2)[x] = \{x\} - 1$.

(ii) Find the number of solutions of the equation, $x^2 - 3x + [x] = 0$ in the interval $[0, 3]$.

(iii) If $[x]^2 + 3[x] - 10 \geq 0$, then find the range of x.

(iv) If $y = \sqrt{\operatorname{sgn}(x^2 - 2(k+1)x + 4)}$ is defined for all $x \in R$ then find number of integral values of k.
[Note: $\operatorname{sgn}(k)$ denotes signum function of k]

(Revision Practice Problems)

QUESTION [JEE Mains 2024 (6 April)]

If A is a square matrix of order 3 such that $\det(A) = 3$ and $\det\left(\operatorname{adj}\left(-4\operatorname{adj}\left(-3\operatorname{adj}\left(3\operatorname{adj}\left((2A)^{-1}\right)\right)\right)\right)\right) = 2^m 3^n$, then $m + 2n$ is equal to :

(A) 2

(B) 4

(C) 3

(D) 6

Ans. B

QUESTION [JEE Mains 2024 (6 April)]

For $\alpha, \beta \in \mathbb{R}$ and a natural number n, let $A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$. Then

$2A_{10} - A_8$ is

(A) $4\alpha + 2\beta$

(B) 0

(C) $2n$

(D) $2\alpha + 4\beta$

Ans. A

QUESTION [JEE Mains 2024 (5 April)]

Let $\alpha\beta \neq 0$ and $A = \begin{bmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\alpha \end{bmatrix}$. If $B = \begin{bmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{bmatrix}$ is the matrix of cofactors of the elements of A, then det(AB) is equal to:

(A) 64

(B) 343

(C) 125

(D) 216

Ans. D

Previous TAH

Solutions

QUESTION [JEE Mains 2024 (30 Jan)]

If the domain of the function $f(x) = \log_e\left(\frac{2x+3}{4x^2+x-3}\right) + \cos^{-1}\left(\frac{2x-1}{x+2}\right)$ is $(\alpha, \beta]$, then the value of $5\beta - 4\alpha$ is equal to

(A) 9

(B) 12

(C) 11

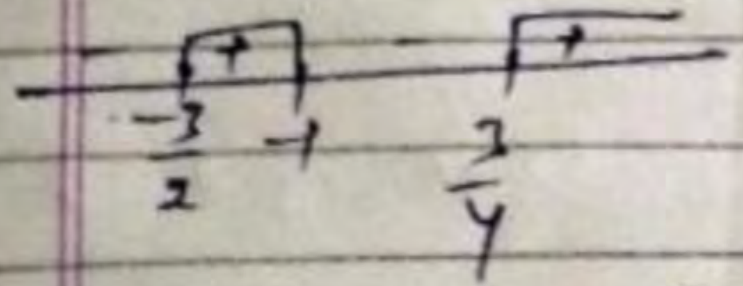
(D) 10

Ans. B

Que If the Domain of the func $f(x) = \log_e\left(\frac{2x+3}{4x^2+x-3}\right) + \cos^{-1}\left(\frac{2x-1}{x+2}\right)$ is $(\alpha, \beta]$ then the value of $5\beta - 4\alpha$ is Equal to : TAH-1

$$\frac{2x+3}{4x^2+x-3} > 0$$

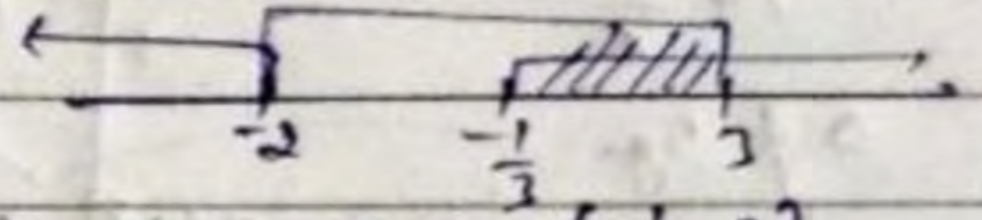
$$\frac{2x+3}{(x+1)(4x-3)} > 0$$



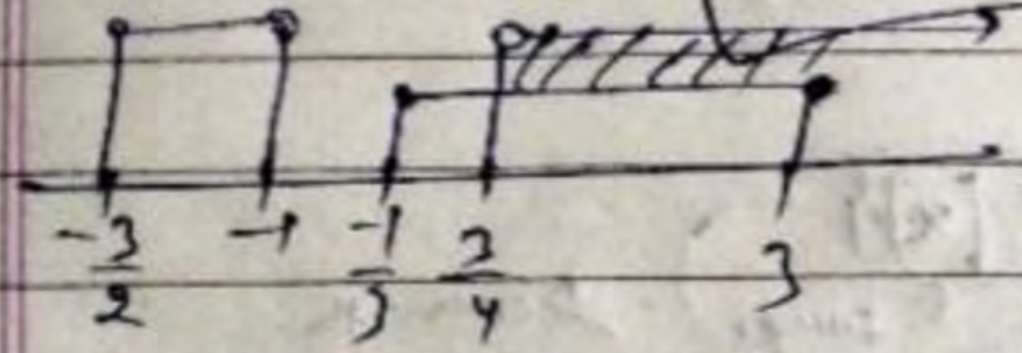
$$-1 \le \frac{2x-1}{x+2} \le 1$$

$$\frac{2x-1}{x+2} \ge -1 \qquad \frac{2x-1}{x+2} \le 1$$

$$\frac{3x+1}{x+2} \ge 0 \qquad \frac{x-3}{x+2} \le 0$$



$$x \in \left[-\frac{1}{3}, 3\right]$$



$$x \in \left(\underset{\alpha}{\frac{3}{4}}, \underset{\beta}{3}\right]$$

$$5\beta - 4\alpha = 15 - 4 \times \frac{3}{4} = \boxed{12} \text{ Ans}$$

AJEET JAIN
AGRA

TAH-1

QUESTION [JEE Mains 2023 (29 Jan)]

The domain of $f(x) = \frac{\log_{(x+1)}(x-2)}{e^{2\log_e x} - (2x+3)}, x \in \mathbb{R}$ is

(A) $(-1, \infty) - \{3\}$

(B) $\mathbb{R} - \{-1, 3\}$

(C) $(2, \infty) - \{3\}$

(D) $\mathbb{R} - \{3\}$

Ans. C

Boby. Hr

Tah 2

Th② $\dfrac{\log_{(x+1)}(x-2)}{e^{2\log x}-(2x+3)}$ domain are.

Ans

Cond ①

$x+1>0$ with $x\neq 0$

$x>-1$

Cond② $x-2>0$

$x>2$

Cond ③

$e^{2\log x}-(2x+3)\neq 0$

$e^{\log x^2}-(2x+3)\neq 0$

$x^2-2x-3\neq 0$

$x\neq 3,-1$

Cond 4 $x>0$

$\cap$ of Cond ① $\cap$ cond② $\cap$ cond③ $\cap$ Cond ④

$x\in(2,\infty)-\{3\}$.

QUESTION

$$f(x)=\begin{cases}x+1 & x<2\\ x+3 & x\geq 2\end{cases} \;\&\; g(x)=\begin{cases}x^2+2x+7 & x<1\\ x^2+5x+7 & x\geq 1\end{cases}$$

Find $f(x)\pm g(x)$ and $\dfrac{f(x)}{g(x)}$.

Tah(4) ⇒

$$f(x) = \begin{cases} x+1, & x<2 \\ x+3, & x\geq 2 \end{cases} \qquad g(x) = \begin{cases} x^2+2x+7, & x<1 \\ x^2+5x+7, & x\geq 1 \end{cases}$$

$$f(x)+g(x) = \begin{cases} x+1+x^2+2x+7, & x<1 \\ x+1+x^2+5x+7, & 1\leq x<2 \\ x+3+x^2+5x+7, & x\geq 2 \end{cases}$$

$$f(x)+g(x) = \begin{cases} x^2+3x+8, & x<1 \\ x^2+6x+8, & 1\leq x<2 \\ x^2+6x+10, & x\geq 2 \end{cases}$$

$$f(x)-g(x) = \begin{cases} x+1-x^2-2x-7, & x<1 \\ x+1-x^2-5x-7, & 1\leq x<2 \\ x+3-x^2-5x-7, & x\geq 2 \end{cases}$$

$$f(x)-g(x) = \begin{cases} -x^2-x-6, & x<1 \\ -x^2-4x-6, & 1\leq x<2 \\ -x^2-4x-4, & x\geq 2 \end{cases}$$

Amol

$$f(x).g(x) = \begin{cases} (x+1)(x^2+2x+7), & x<1 \\ (x+1)(x^2+5x+7), & 1\leq x<2 \\ (x+3)(x^2+5x+7), & x\geq 2 \end{cases}$$

$$\frac{f(x)}{g(x)} = \begin{cases} \dfrac{x+1}{x^2+2x+7}, & x<1 \\ \dfrac{x+1}{x^2+5x+7}, & 1\leq x<2 \\ \dfrac{x+3}{x^2+5x+7}, & x\geq 2 \end{cases}$$

TAH (4) Solⁿ:- $f(x) = \begin{cases} x+1 & x<2 \\ x+3 & x\geq 2 \end{cases}$ & $g(x) = \begin{cases} x^2+2x+7 & x<1 \\ x^2+5x+7 & x\geq 1 \end{cases}$

find $f(x) \pm g(x)$ and $\dfrac{f(x)}{g(x)}$.

$$(f+g)x = f(x)+g(x) = \begin{cases} x+1+x^2+2x+7 & x<1 \\ x+1+x^2+5x+7 & 1\leq x<2 \\ x+3+x^2+5x+7 & x\geq 2 \end{cases}$$

$$= \begin{cases} x^2+3x+8 & x<1 \\ x^2+6x+8 & 1\leq x<2 \\ x^2+6x+10 & x\geq 2 \end{cases}$$

$$(f-g)x = f(x)-g(x) = \begin{cases} x+1-x^2-2x-7 & x<1 \\ x+1-x^2-5x-7 & 1\leq x<2 \\ x+3-x^2-5x-7 & x\geq 2 \end{cases}$$

Shivani
From bihar

$$= \begin{cases} -(x^2+x+6) & x<1 \\ -(x^2+4x+6) & 1\leq x<2 \\ -(x^2+4x+4) & x\geq 2 \end{cases}$$

$$\frac{f(x)}{g(x)} = \begin{cases} \dfrac{x+1}{x^2+2x+7} & x<1 \\ \dfrac{x+1}{x^2+5x+7} & 1\leq x<2 \\ \dfrac{x+3}{x^2+5x+7} & x\geq 2 \end{cases}$$

QUESTION

TAH

Find the domain of the following function :

(i) $y = \log_{(x-4)}\left(x^2 - 11x + 24\right)$

(ii) $f(x) = \log_2\left(-\log_{\frac{1}{2}}\left(1 + \frac{1}{\sqrt[4]{x}}\right) - 1\right)$

$x \in (2,\infty) - \{3\}$ Ans

Tah-5 (i) $y = \log_{(x-4)}(x^2-11x+24)$. Domain = ?

$\Rightarrow$ $x-4>0$, $x-4\neq 1$ & $x^2-11x+24>0$

$x>4$, $x\neq 5$

$(x-8)(x-3)>0$

$x \in (-\infty,3)\cup(8,\infty)$

$\cap$ $x \in (8,\infty)$ Ans

(ii) $f(x) = \log_2\left(-\log_{1/2}\left(1+\frac{1}{\sqrt[4]{x}}\right)-1\right)$

$-\log_{1/2}\left(1+\frac{1}{\sqrt[4]{x}}\right)-1>0$

$\Rightarrow \log_{1/2}\left(1+\frac{1}{\sqrt[4]{x}}\right)<-1$

$\Rightarrow \left(1+\frac{1}{\sqrt[4]{x}}\right)>2$

$\Rightarrow \frac{1}{\sqrt[4]{x}}>1$

$\Rightarrow \frac{1-\sqrt[4]{x}}{\sqrt[4]{x}}>0$

$\Rightarrow \sqrt[4]{x}\in(0,1)$

$\boxed{\therefore\ x\in(0,1)}$ Ans.

sourik Maiti
West Bengal

Q. find the Domain of the following function !

(i) $y = \log_{(x-4)}(x^2-11x+24)$

$x-4>0$ | $x \neq 4$ | $x^2-11x+24>0$

$x>4$ | | $(x-8)(x-3)>0$

$x \in (4,\infty)$ | | $x \in (-\infty,3)\cup(8,\infty)$

3 4 8

$\boxed{x \in (8,\infty)}$ Ans

(iii) $f(x) = \log_2\left(-\log_{\frac{1}{2}}\left(1+\frac{1}{\sqrt[4]{x}}\right)-1\right)$

$\Rightarrow -\log_{\frac{1}{2}}\left(1+\frac{1}{\sqrt[4]{x}}\right)-1>0$

$\Rightarrow \log_{\frac{1}{2}}\left(1+\frac{1}{\sqrt[4]{x}}\right)<-1$

$1+\frac{1}{\sqrt[4]{x}}>2$

$\frac{1}{\sqrt[4]{x}}-1>0$

$\frac{1-\sqrt[4]{x}}{\sqrt[4]{x}}>0$

$\frac{\sqrt[4]{x}-1}{\sqrt[4]{x}}<0$

$\boxed{x \in (0,1)}$

Ans

AJEET JAIN
AGRA

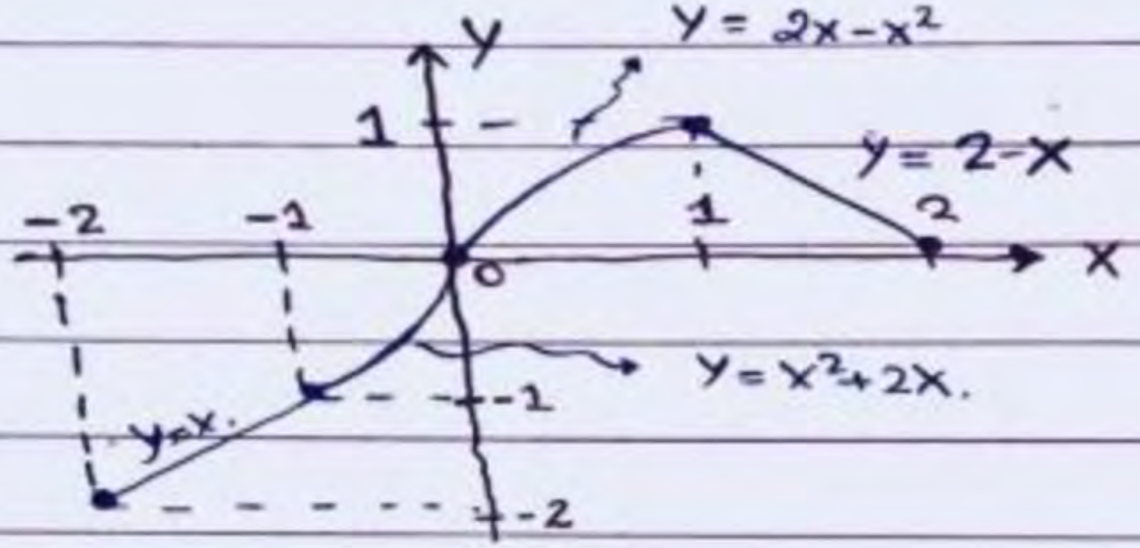
TAH-5

QUESTION

(a) Let $f(x) = \begin{cases} x, & -2 \leq x \leq -1 \\ x^2 + 2x, & -1 < x \leq 0 \\ 2x - x^2, & 0 < x \leq 1 \\ 2 - x, & 1 < x \leq 2 \end{cases}$

Find the number of integers in the range of $f(x)$.

Tah 7 $f(x) = \begin{cases} x & -2 \le x \le -1 \\ x^2 + 2x. & -1 < x \le 0 \\ 2x - x^2 & 0 < x \le 1 \\ 2 - x & 1 < x \le 2 \end{cases}$



Boby hr

Tah 7

Range $f(x) \in [-2, 1]$

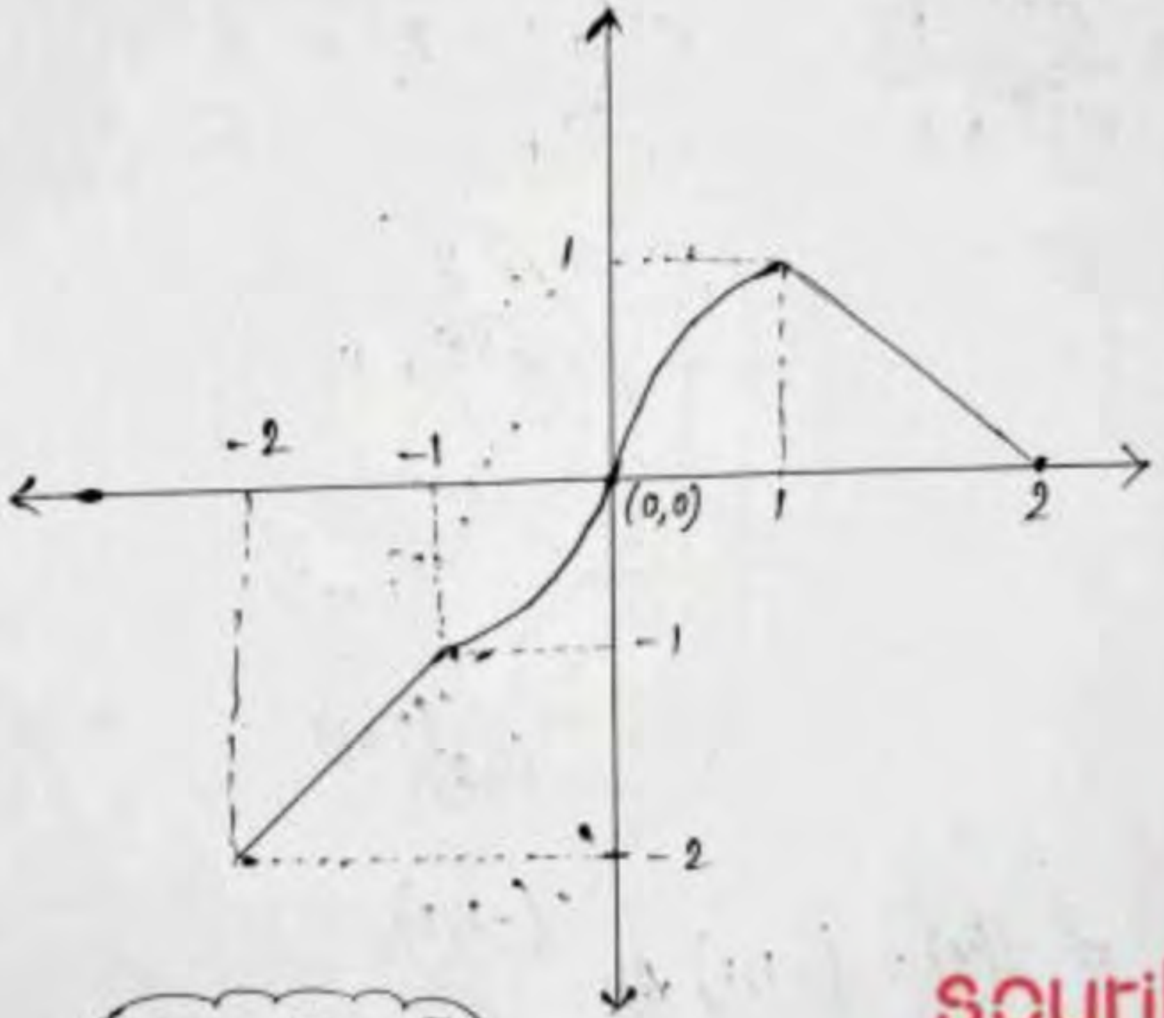
No of Integer in Range $-2, -1, 0, 1$

TAH-7

let $f(x) = \begin{cases} x & , -2 \le x \le -1 \\ x^2+2x & , -1 < x \le 0 \\ 2x - x^2 & , 0 < x \le 1 \\ 2-x & , 1 < x \le 2 \end{cases}$

TAH 7

Find the numbers of integers in the range of $f(x)$.



$f(x) \in [-2, 1]$

$\hookrightarrow$ Number of integers $= (4)$ Ans

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West Bengal

QUESTION

(b) Let $g(x) = \begin{cases} x^2 - 2, & -\infty < x < 0 \\ x, & 0 \le x < 2 \\ (x-2)^2, & 2 \le x < 4 \\ x - 4, & 4 \le x < \infty \end{cases}$.

If the equation $g(x) = k$ has four real and distinct roots, then find the sum of all possible integral values of k.

QUESTION

Find Range of

$$f(x) = \frac{2e^x}{3e^x + 5}$$

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

TAH-6

Find the range –

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

TAH 6

$$\Rightarrow f(x) = \frac{e^{2x} - 1}{e^{2x} + 1} = \frac{e^{2x} + 1 - 2}{e^{2x} + 1}$$

$$= 1 - \frac{2}{e^{2x} + 1}$$

$e^{2x} \to (0, \infty)$

$e^{2x}+1 \to (1, \infty)$

$\frac{2}{e^{2x}+1} \to (0, 2)$

$1 - \frac{2}{e^{2x}+1} \to (-1, 1)$

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$\therefore$ Range of $f(x) = (-1, 1)$ Ans

$\boxed{\text{TAH-6}}$

$$f(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}} = \frac{e^{2x}-1}{e^{2x}+1} = \frac{e^{2x}+1-2}{e^{2x}+1} = 1 - \frac{2}{e^{2x}+1}$$

$e^{2x} \to (0,\infty)$

$e^{2x}+1 \to (1,\infty)$

$\frac{2}{e^{2x}+1} \to (0,2)$

$1-\frac{2}{e^{2x}+1} \to (-1,1)$

Souvik Mondal
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Murshidabad.

Homework from Module

Chapter: SETS

Prarambh: COMPLETE

Prabal : COMPLETE


THANK
YOU

PRAYAS

JEE 2025

Lecture- 09 | Mathematics

Relation & Functions

By- Ashish Agarwal Sir (IIT Kanpur)

Topics *to be covered*

1. Classification of Functions
2. Even and Odd Functions

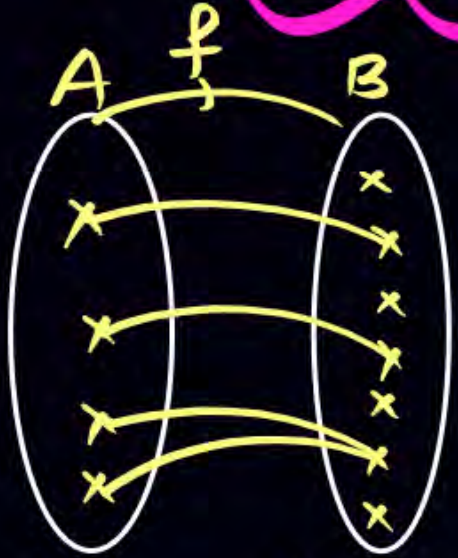
Classification of fns.

(1) Injective fn / Injection / one-one fn.

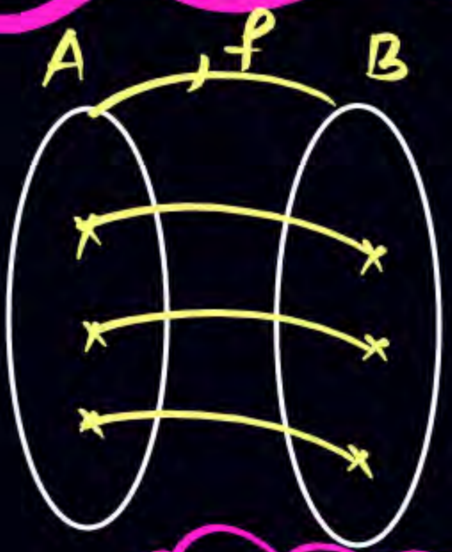
Let $f : A \to B$ s.t different elements of A have different images in B

Yaani A ke kinhi 2 elements ki image same nahi hoti | Yaani Alag Alag elements ki image Alag Alag hoti hai

Yaani 2 elements ki image same tabhi ho sakti jab voh dono elements bhi same ho



Yes it is function
But not one-one



One-one

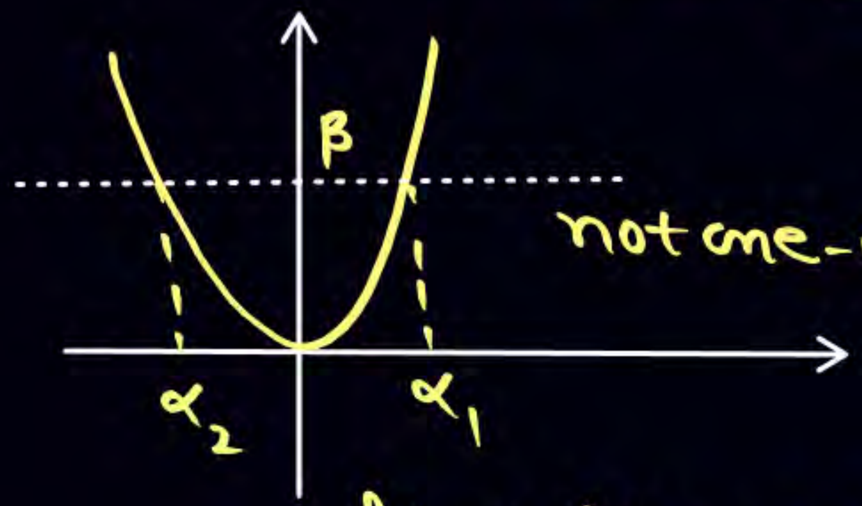
Not a fn.

(Yes one-one)

$f: A \longrightarrow B$ is injective if $x_1 \neq x_2 \Longleftrightarrow f(x_1) \neq f(x_2) \ \forall \ x_1, x_2 \in A$

OR

$f: A \rightarrow B$ is one-one if $f(x_1) = f(x_2) \Longrightarrow x_1 = x_2$

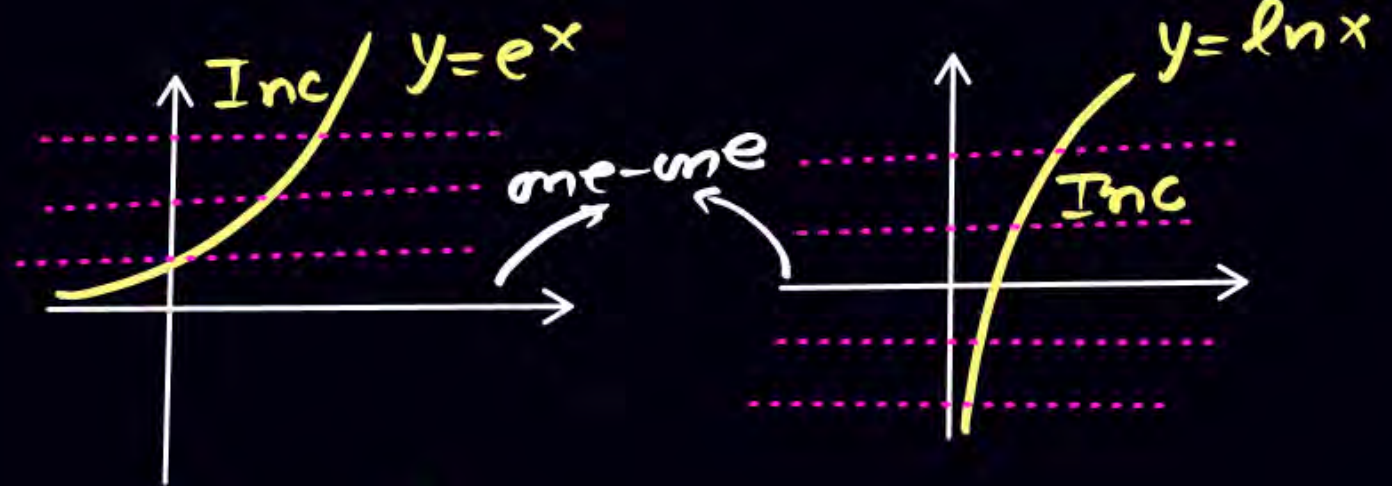


not one-one

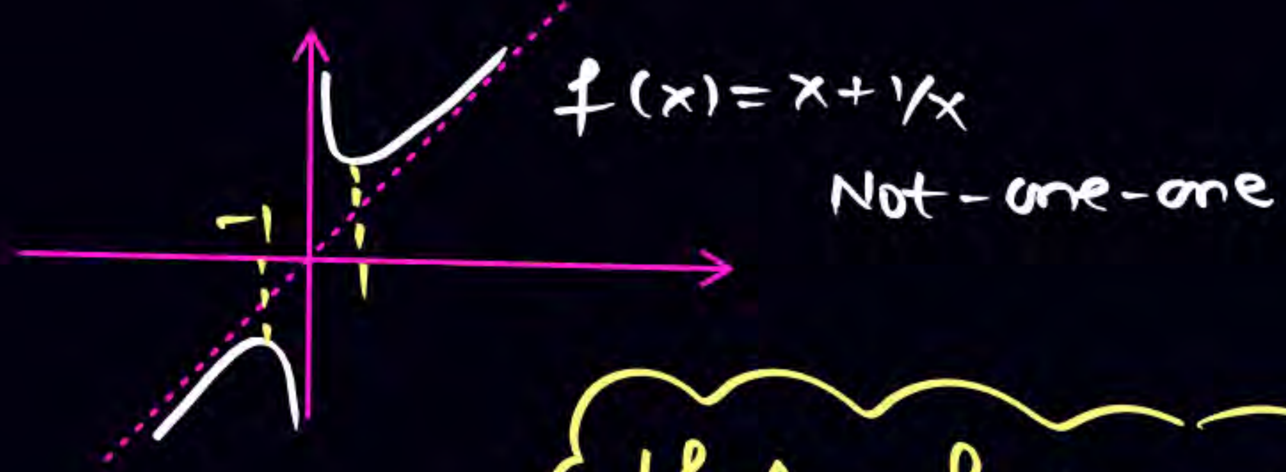
$f(\alpha_1) = f(\alpha_2)$ But $\alpha_1 \neq \alpha_2$

Graphical Pechaan

If no horizontal line intersects the graph of f in two or more points then it is one-one.



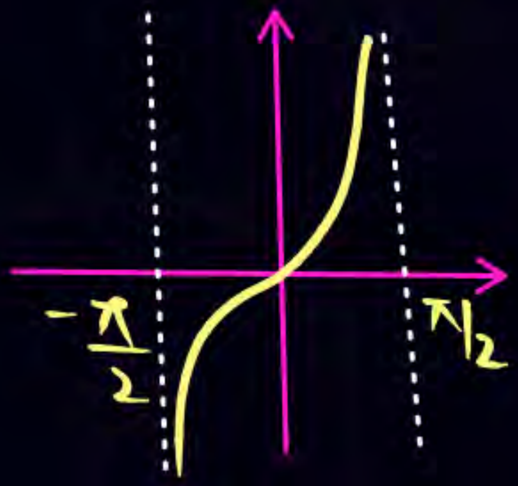
Every horizontal Intersects graph of $y=f(x)$ at atmost one point $\Longrightarrow f$ is one-one



If Any fn is continuous and Inc or dec on an Interval then it is one-one on the Interval

Ex: $f(x)=\tan x \quad x\in(-\pi/2, \pi/2)$

is inc on this interval hence it is one-one



$f(x) = \sin x \quad x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ is one-one

Let $y = f(x)$ be derivable on (a,b) (i.e graph is continuous & has no sharp ends)

& $\frac{dy}{dx} = f'(x) \geq 0 \,/\, \leq 0$ on (a,b) where equality holds at some discrete points of (a,b) the f is inc/dec on (a,b) & hence one-one

Ex: $y = f(x) = x^3$ on $(-\infty, \infty)$

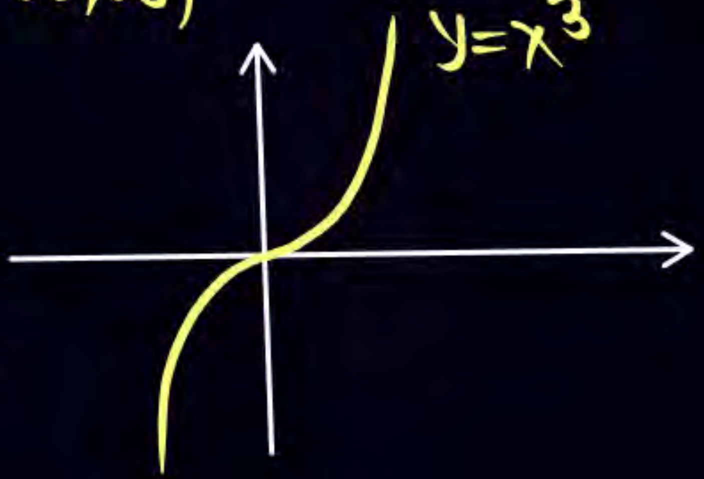
$\frac{dy}{dx} = 3x^2 \geq 0$

$\Downarrow$

$f(x) = x^3$ is inc

$\Downarrow$

one-one



Ex: $f(x) = e^x \quad x \in (-\infty, \infty)$

$\frac{dy}{dx} = e^x > 0$ on $(-\infty, \infty)$

$\downarrow$

Inc on R $\rightarrow$ one-one.

Many one fn. : A fn $f : A \to B$ which is not one-one is many one.

No: of one-one fns from A to B + No: of many one fns from A to B = Total no: of fns from A to B

★ Graphical Pechaan: If a horizontal intersect the graph of a fn at 2 or more points then it is many one.

If a continuous fns has a local-MAX | MIN then it is many-one

If $y = f(x)$ has same value at x_1, x_2 $(x_1 \neq x_2)$ then it is many-one

Ex: $f(x) = x^2 - 6x + 8$

$f(2) = 0 = f(4)$

$\Downarrow$

many one.

Ex: $f(x) = |x|$

$f(1) = f(-1)$ many one.

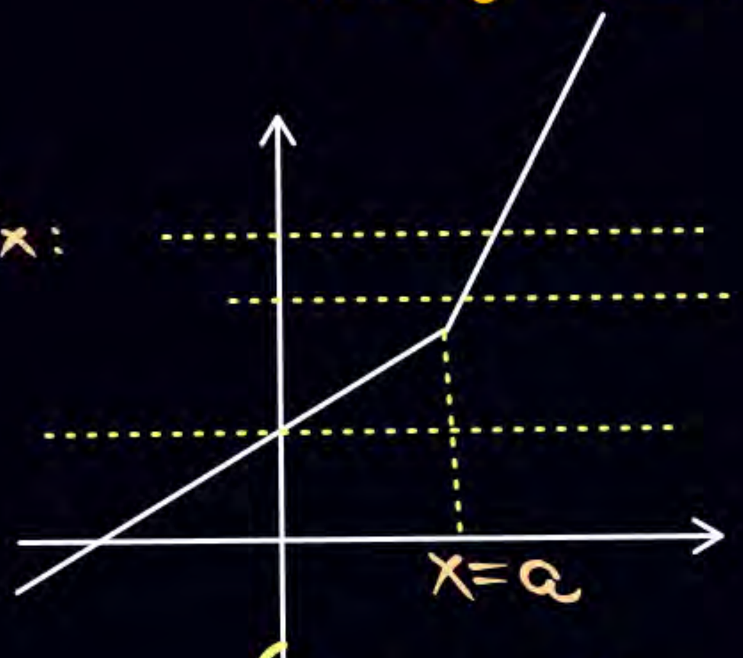
Ex: $f(x) = \dfrac{x^2 - 5x + 6}{x^2 + x + 1}$

$f(2) = f(3) = 0$

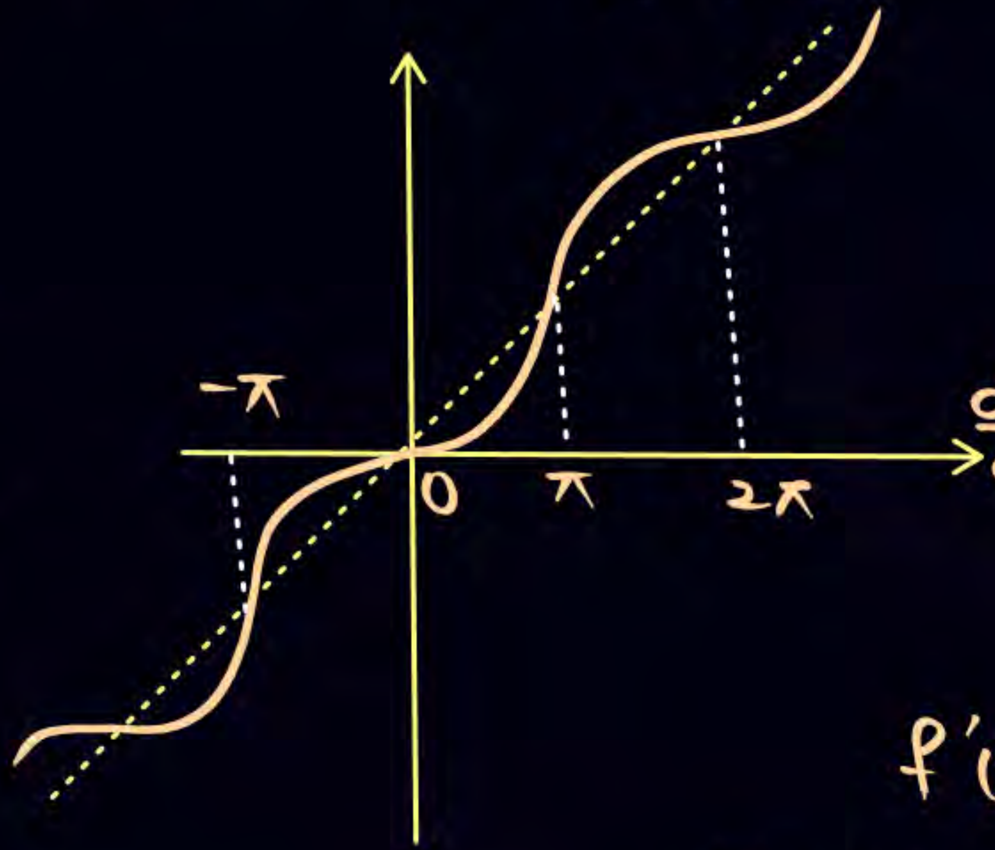
$\Downarrow$

many-one.

Ex:



(one-one)



$f(x) = x + \sin x$

$\dfrac{dy}{dx} = f'(x) = 1 + \cos x \geq 0$

$\Downarrow$ (inc)

one-one

$f'(x) = 0$ at $x = (2n+1)\pi, n \in I$.

$\dfrac{dy}{dx} = 1 + \cos x = 0$

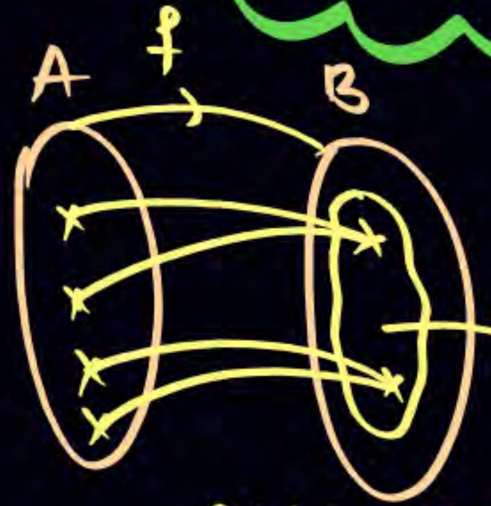
$\cos x = -1 \rightarrow x = \ldots -\pi, \pi, 3\pi, 5\pi \ldots$

ON TO Fn / Surjective fn / Surjection.

let f be a fn $f: A \to B$ such that each & every element of B has a preimage in A then f is onto.

$\Downarrow$

f is onto $\iff$ Range of f = codomain of f



Range = B = codomain

onto + MANY one.

Range $\neq$ B (codomain)

has no preimage

Not onto + MANYONE

★ Every odd degree polynomial $f: R \to R$ is onto fn.

★ Every even degree polynomial with codomain R is never onto.

Into fn : A fn $f: A \to B$ which is not onto is into

No: of onto fns $f: A \to B$ + No: of Into fns $f: A \to B$ = Total no: of fns $f: A \to B$

A fn which is one-one + onto is called a Bijective fn or Bijection or Invertible fn or Non Singular or Bi-uniform fn.

QUESTION

Classify the following functions $f : R \to R$

(a) $f(x) = e^x + e^{-x}$

(b) $f(x) = \sqrt{1 + x^2}$

(c) $f(x) = x^3$

(d) $f(x) = |x| \text{ Sgn } x$

(e) $f(x) = x^3 - 2x^2 + 5x + 13$

(f) $f(x) = 2x^3 - 6x^2 - 18x + 17$

(g) $f(x) = \frac{x^2+4x+30}{x^2-8x+18}$

ⓐ $f(x) = e^x + e^{-x}$

clearly $1 \notin R_f$

$R_f \neq R \Rightarrow$ Into

M① $f(1) = e + e^{-1} = f(-1) \Rightarrow$ many one.

M② $y = e^x + e^{-x}$

$\frac{dy}{dx} = e^x - e^{-x} = \frac{e^{2x} - 1}{e^x}$ +ve $e^{2x} > 1 \Rightarrow x > 0$; -ve $e^{2x} < 1 \Rightarrow x < 0$

$\Rightarrow$ Many one.

M③ Graph $y = e^x + e^{-x} = e^x + \frac{1}{e^x}$

$x \to \infty$, $y \to \infty$; $x \to -\infty$, $y \to \infty$

$e^{-\infty} + \frac{1}{e^{-\infty}}$

$= 0 + e^{\infty}$

∞

$f: R \to R$

(b) $f(x) = \sqrt{1+x^2}$

$f(-1) = f(1) = \sqrt{2} \Rightarrow$ many one

\+

$-1 \notin R_f \Rightarrow R_f \neq R \Rightarrow$ Into

(c) $f(x) = x^3$ — odd degree polynomial

$\frac{dy}{dx} = 3x^2 \geq 0$

$\Downarrow$

Inc

$\Downarrow$

one-one

Range = R = codomain

$\Downarrow$

onto.

one-one + onto

Bijective fn.

(d) $f(x) = |x| \operatorname{sgn} x$

$$f(x) = \begin{cases} x \cdot 1 & x > 0 \\ 0 & x = 0 \\ -x \cdot -1 & x < 0 \end{cases}$$

$$f(x) = \begin{cases} x & x > 0 \\ 0 & x = 0 \\ x & x < 0 \end{cases} \Rightarrow f(x) = x$$ — odd degree poly

$\Downarrow$

Range = R = codomain $\Rightarrow$ onto.

$f'(x) = 1 > 0 \Rightarrow$ Inc $\Rightarrow$ one-me.

(e) $f(x) = x^3 - 2x^2 + 5x + 13$

odd degree poly

$\Downarrow$

Range $= R \Rightarrow$ onto

Non Singular

OR

Bijective fn.

$y = x^3 - 2x^2 + 5x + 13.$

$\frac{dy}{dx} = 3x^2 - 4x + 5 > 0 \ \forall x \in R \Rightarrow$ inc $\Rightarrow$ one-one

$a = 3 > 0$

$D = 16 - 4 \cdot 5 \cdot 3 < 0$

$\Downarrow$

always +ve

(f) $f(x) = 2x^3 - 6x^2 - 18x + 17$

odd degree poly

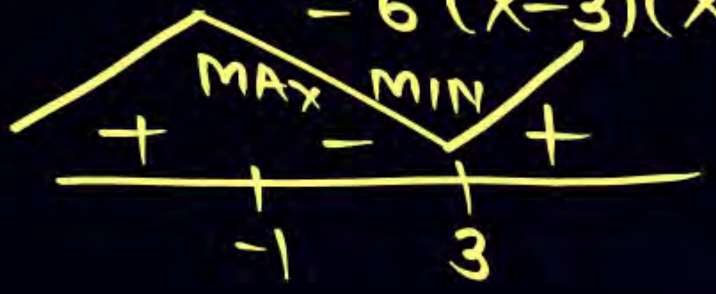
$\Downarrow$

Range $= R \Rightarrow$ onto fn

$y = 2x^3 - 6x^2 - 18x + 17$

$\frac{dy}{dx} = 6x^2 - 12x - 18 = 6(x^2 - 2x - 3)$

$= 6(x-3)(x+1) \Rightarrow \underbrace{\text{many-one}}$



(9) $f(x) = \dfrac{x^2+4x+30}{x^2-8x+18}$

Numerator: $D<0$, $a>0$ → always +ve

Denominator: $D<0$, $a>0$ → always +ve

Range of $f \neq R$

$\Downarrow$

In-to

Sign $\frac{dy}{dx}$:

-ve → 0 → -ve
+ve → 0 → +ve
} Neither MAX/MIN

-ve → 0 → +ve → MIN

+ve → 0 → -ve → MAX

$\phi(x) = \dfrac{f(x)}{g(x)}$ — $f(x)$ polynomial, $g(x)$ polynomial

If $g(x) \neq 0$ on R then ϕ is continuous on R

M(1)

$\dfrac{x^2+4x+30}{x^2-8x+18} = \dfrac{\cancel{30}\ 5}{\cancel{18}\ 3}$

$3x^2+12x+90 = 5x^2-40x+90$.

$2x^2-52x=0$

$x=0, 26$. $f(x) = 5/3 \begin{cases} x_1 = 0 \\ x_2 = 26 \end{cases} \Rightarrow$ Many one.

M(2)

$$f(x)=\frac{x^2+4x+30}{x^2-8x+18}$$ — is continuous for

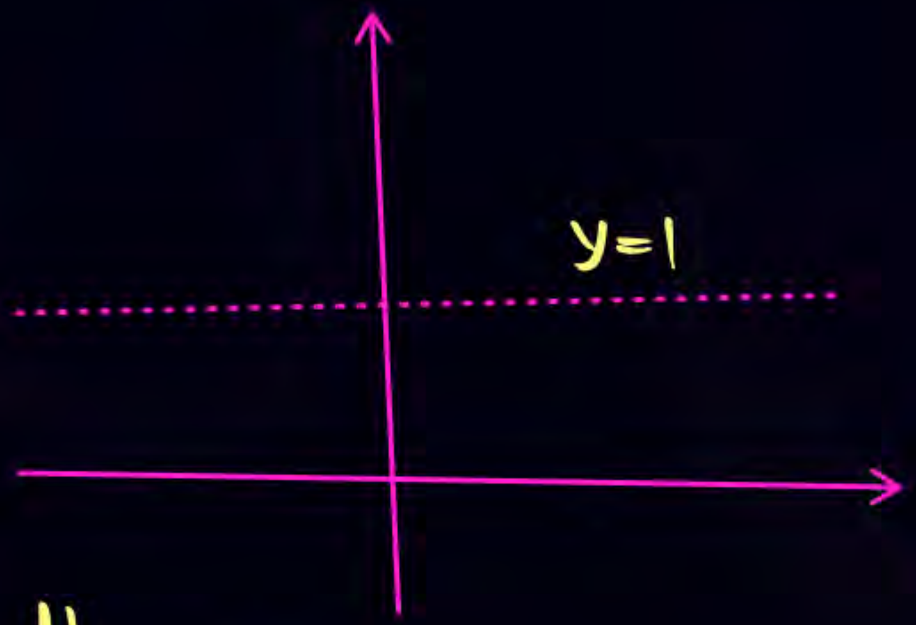
$\frac{1}{-\infty}, \frac{1}{\infty} \rightarrow 0$

$x^2-8x+18 \neq 0$

— $D<0$

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{1+\frac{4}{x}+\frac{30}{x^2}}{1-8/x+18/x^2} = 1$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{1+\frac{4}{x}+\frac{30}{x^2}}{1-8/x+18/x^2} = 1$$



$\Downarrow$

Many one

Sabse Important Baat Yaad Rahe

Sabhi Class Illustrations Retry Karnay hai...

Bumper Practice Questions

Find the Domain of Definition of the Given Functions

(i) $y = \sqrt{-px}(p > 0)$

(ii) $y = \frac{1}{x^2+1}$

(iii) $y = \frac{1}{x^3-x}$

(iv) $y = \frac{1}{\sqrt{x^2-4x}}$

(v) $y = \sqrt{x^2 - 4x + 3}$

(vi) $y = \frac{x}{\sqrt{x^2-3x+2}}$

(vii) $y = \sqrt{1 - |x|}$

(viii) $y = \log_x 2$

(ix) $y = \frac{1}{\log_{10}(1-x)} + \sqrt{x + 2}$

(x) $y = \sqrt{x} + \sqrt[3]{\frac{1}{x-2}} - \log_{10}(2x - 3)$

(xi) $y = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$

(xii) $y = \frac{1}{\sqrt{\sin x}} + \sqrt[3]{\sin x}$

(xiii) $y = \log_{10}(\sqrt{x - 4} + \sqrt{6 - x})$

(xiv) $y = \log_{10}[1 - \log_{10}(x^2 - 5x + 16)]$

Answers

(i) $-\infty < x \leq 0$

(ii) $x \in R$

(iii) $x \in R - \{-1, 0, 1\}$

(iv) $-\infty < x < 0 \;\&\; 4 < x < \infty$

(v) $-\infty < x \leq 1$ and $3 \leq x < \infty$

(vi) $-\infty < x < 1$ and $2 < x < \infty$

(vii) $-1 \leq x \leq 1$

(viii) $0 < x < 1$ and $1 < x < \infty$

(ix) $-2 \leq x < 0$ and $0 < x < 1$

(x) $\frac{3}{2} < x < 2$ and $2 < x < \infty$

(xi) $-1 < x < 0$ and $1 < x < 2; 2 < x < \infty$

(xii) $2k\pi < x < (2k+1)\pi$, where k is an integer.

(xiii) $4 \leq x \leq 6$

(xiv) $2 < x < 3$

Bumper Practice Questions

Find the range of the following functions :

(i) $f(x) = \frac{x-1}{x+2}$

(ii) $f(x) = \frac{2}{x}$

(iii) $f(x) = \frac{1}{x^2-x+1}$

(iv) $f(x) = \frac{x^2-x+1}{x^2+x+1}$

(v) $f(x) = e^{(x-1)^2}$

(vi) $f(x) = x^3 - x^2 + x + 1$

(vii) $f(x) = \log(x^8 + x^4 + x^2 + 1)$

(viii) $f(x) = \sin^2 x - 2\sin x + 4$

(ix) $f(x) = \sin(\log_2 x)$

(x) $f(x) = 2^{x^2} + 1$

(xi) $f(x) = \frac{e^{2x}-e^x+1}{e^{2x}+e^x+1}$

(xii) $f(x) = \frac{1}{8-3\sin x}$

Answers

(i) $R-\{1\}$

(ii) $R-\{0\}$

(iii) $\left(0,\frac{4}{3}\right]$

(iv) $\left[\frac{1}{3},3\right]$

(v) $[1,\infty)$

(vi) R

(vii) $[0,\infty)$

(viii) $[3,7]$

(ix) $[-1,1]$

(x) $[2,\infty)$

(xi) $\left[\frac{1}{3},1\right)$

(xii) $\left[\frac{1}{11},\frac{1}{5}\right]$

Previous TAH

Solutions

QUESTION

TAH 1

(b) Let $g(x) = \begin{cases} x^2 - 2, & -\infty < x < 0 \\ x, & 0 \le x < 2 \\ (x-2)^2, & 2 \le x < 4 \\ x - 4, & 4 \le x < \infty \end{cases}$.

If the equation $g(x) = k$ has four real and distinct roots, then find the sum of all possible integral values of k.

Tah-01

Kalpana
From Bihar...

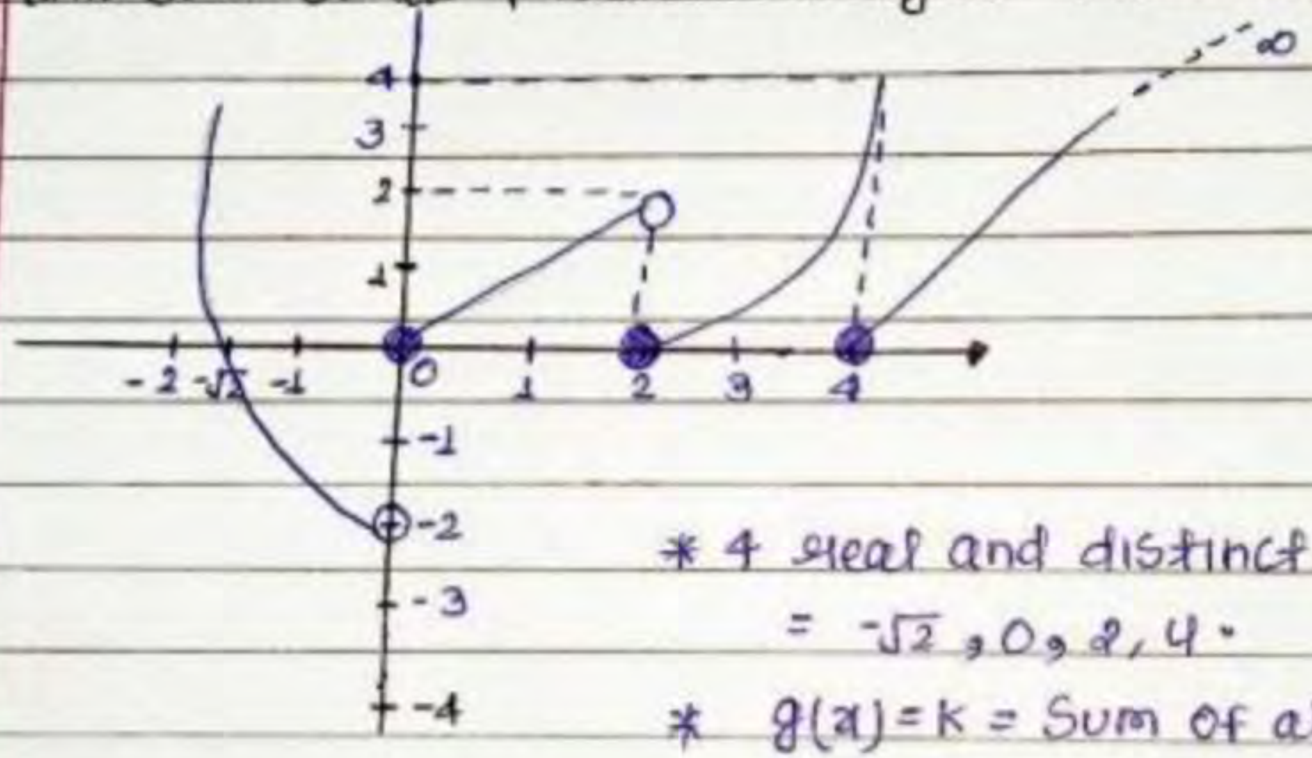
11th July

Tah-01

Let $g(x) = \begin{cases} x^2-2 & -\infty < x < 0 \\ x & 0 \le x < 2 \\ (x-2)^2 & 2 \le x < 4 \\ x-4 & 4 \le x < \infty \end{cases}$

If the eqn $g(x)=K$ has 4 real and distinct roots, then find the sum of all possible integral values of K.

Soln:-



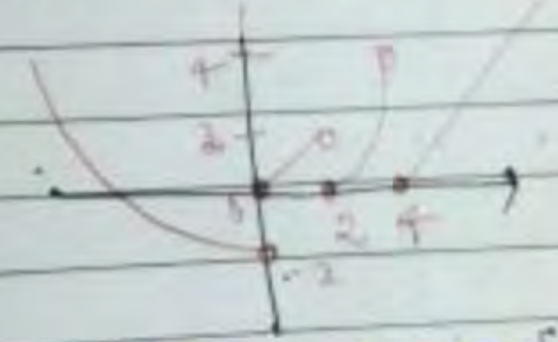
* 4 real and distinct roots are $= -\sqrt{2}, 0, 2, 4$.
* $g(x)=K$ = Sum of all integral values

$g(x) = K \in [0,2]$

So, Sum is $K = 0+1 = 1$ Ans

Tah-1) $g(x) = \begin{cases} x^2-2 & -\infty < x < 0 \\ x & 0 \le x < 2 \\ (x-2)^2 & 2 \le x < 4 \\ x-4 & 4 \le x < \infty \end{cases}$

$g(x)=k$ has 4 Real & dis Roots then sum of all values of k



Himanshu Saini
Sirsa Haryana
tah 2

for 4 Real soln $k \in [0,2)$

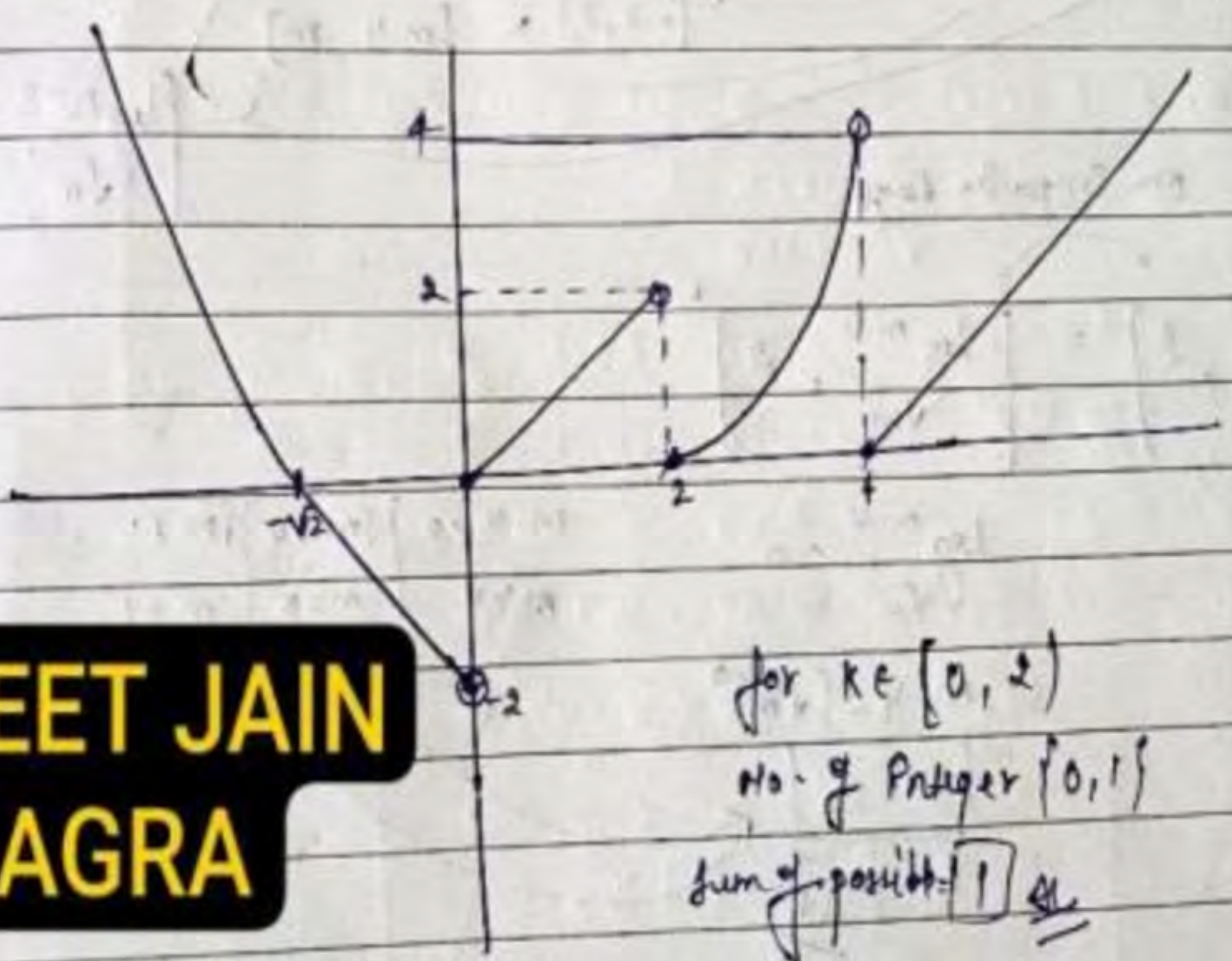
Integeral values of $K = 0, 1$

Sum of values of $K = 1$

TAH-1

Que let $g(x) = \begin{cases} x^2-2 & -\infty < x < 0 \\ x & 0 \le x < 2 \\ (x-2)^2 & 2 \le x < 4 \\ (x-4) & 4 \le x < \infty \end{cases}$ TAH-1

If the Equ^n $g(x) = k$ has four Real and Distinct Roots, then find the sum of all possible values of it.



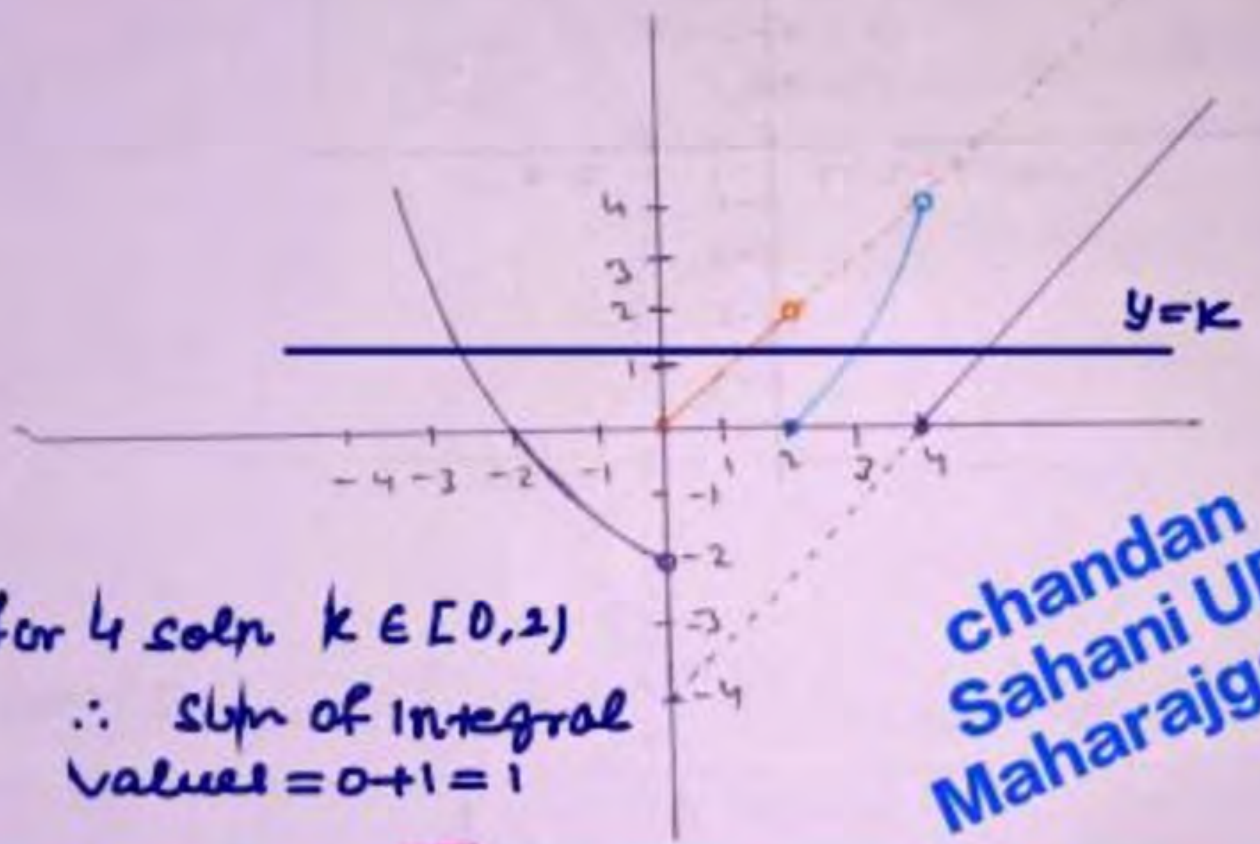
for $k \in [0, 2)$

No. of Integer $\{0, 1\}$

Sum of possible $= \boxed{1}$ Ans

AJEET JAIN AGRA

$g(x) = \begin{cases} x^2-2 & -\infty < x < 0 \\ x & 0 \le x < 2 \\ (x-2)^2 & 2 \le x < 4 \\ x-4 & 4 \le x < \infty \end{cases}$ Tah01



for 4 soln $k \in [0, 2)$

$\therefore$ sum of Integral values $= 0 + 1 = 1$

chandan Sahani UP Maharajganj

Q find the Range of K for which

$$||x^2-6x+8|-12| = K \quad \text{has}$$

Tah2

(1) No soln

(2) Exactly 2 real soln.

(3) Exactly 4 real solns.

(4) Exactly 6 real solutions.

Tah-02

Tah-(02)

Find the range of K for which $||x^2-6x+8|-12| = k$ has

(1) No solution

(2) Exactly 2 real solutions

(3) Exactly 4 soln

(4) Exactly 6 real soln.

$x^2-6x+8=0$

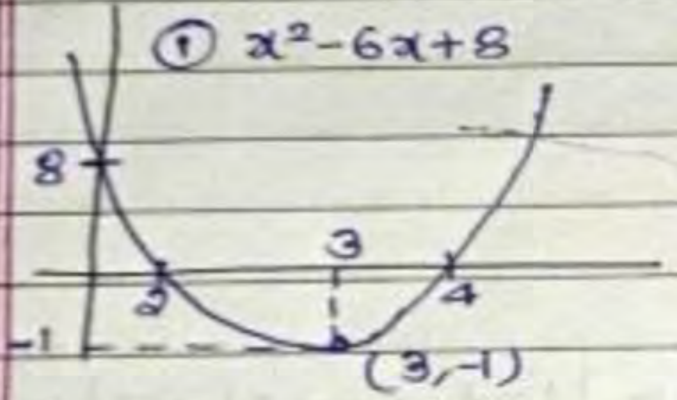
$(x-4)(x-2)=0$

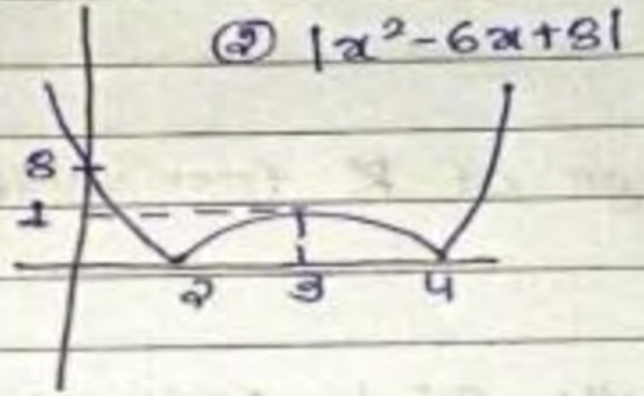
$x=4,2$ (Roots)

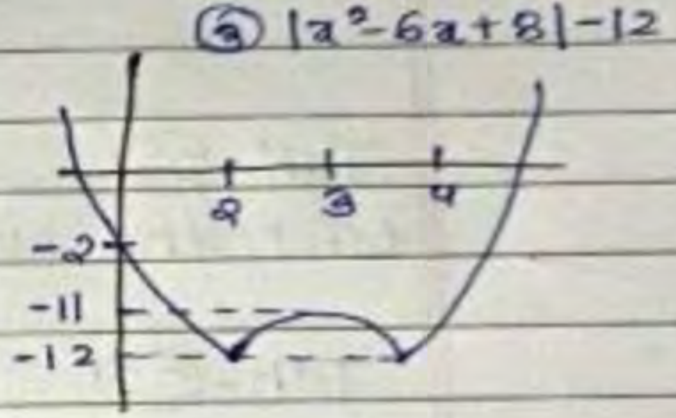
and vertex $= \left(\frac{-b}{2a}, \frac{-D}{4a}\right)$

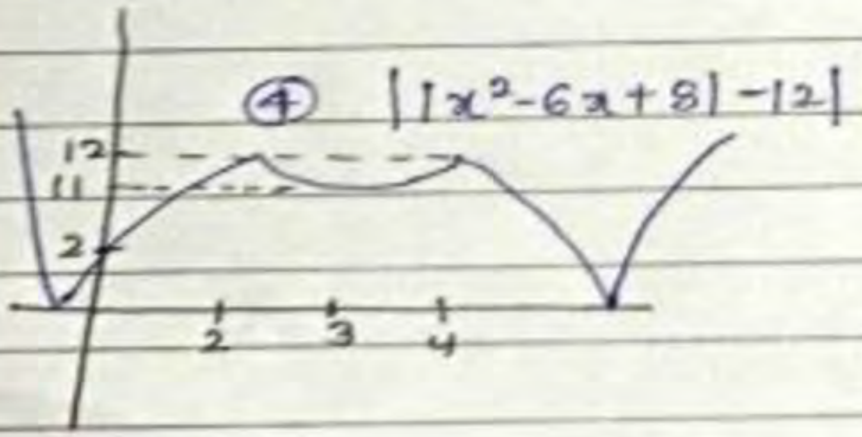
$= \frac{6}{2}, \frac{-4}{4}$

$= (3,-1)$









(i) No soln :- $(-\infty, 0)$

(ii) Exactly 2 soln :- $(12, \infty) \cup \{0\}$

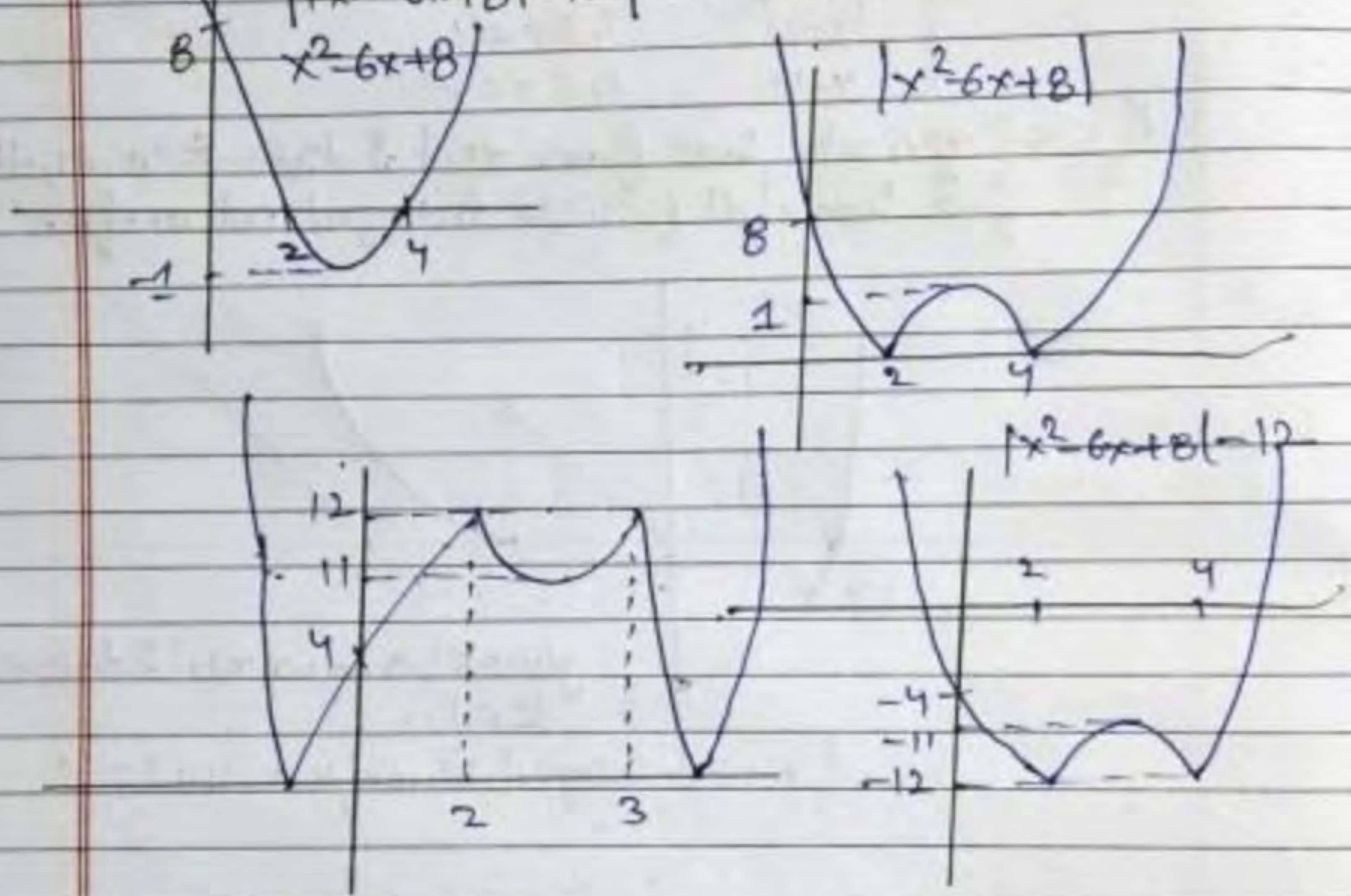
(iii) Exactly 4 soln : $k \in (0, 11)$

(iv) 6 soln : $(11, 12)$

The

TAH 2

Q. find the Range of k for which $||x^2-6x+8|-12| = k$ has.



(a) No sol^n ⇒ $k \in (-\infty, 0)$

(b) Exactly 2 real sol^n ⇒ $k \in (12, \infty) \cup \{0\}$

(c) Exactly 4 real sol^n ⇒ $k \in (0, 11)$

(d) Exactly 6 real sol^n ⇒ $k \in (11, 12)$

Deepak from Batala punjab

Tah : 02

$||x^2-6x+8|-12| = k$

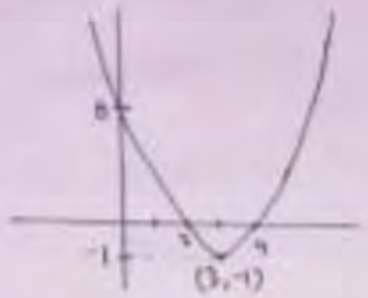
$y = x^2-6x+8$

coordinate

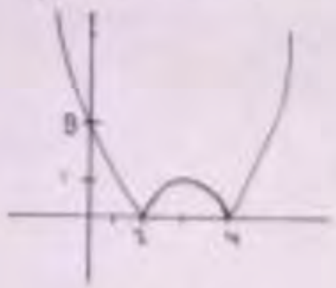
$\left(-\frac{b}{2a}, -\frac{D}{4a}\right)$

$\left(\frac{-(-3)}{2(1)}, -\frac{4}{4(1)}\right)$

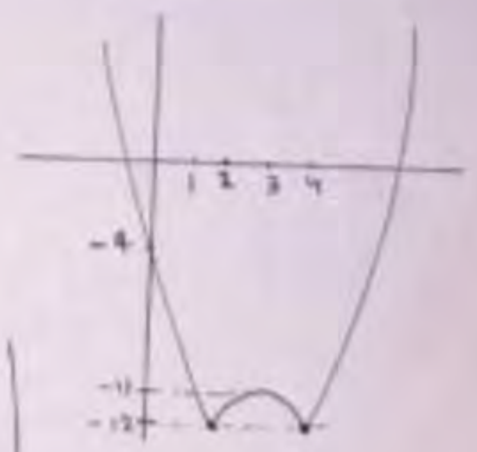
$(3, -1)$



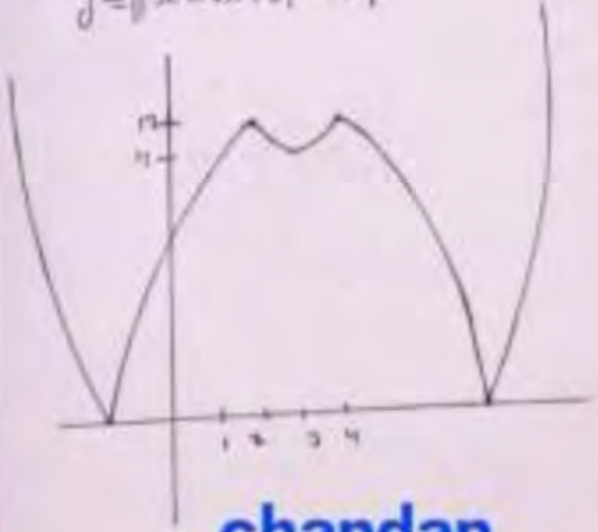
$y = |x^2-6x+8|$



$y = |x^2-6x+8|-12$



$y = ||x^2-6x+8|-12|$



① No solution $k \in (-\infty, 0)$

② Exactly 2 real solution $k \in (12, \infty) \cup \{0\}$

③ Exactly 4 real solutions $k \in (0, 11) \cup \{12\}$

④ Exactly 6 real solutions $k \in (11, 12)$

chandan Sahani UP Maharajganj

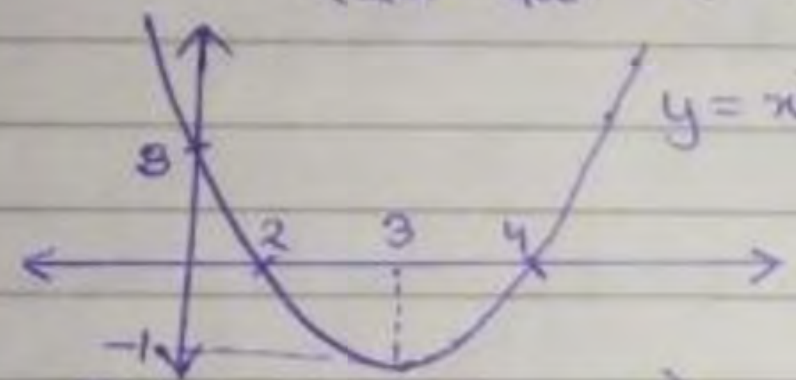
TAH 2
MOHIT SINGH
GHAZIABAD, U. P.

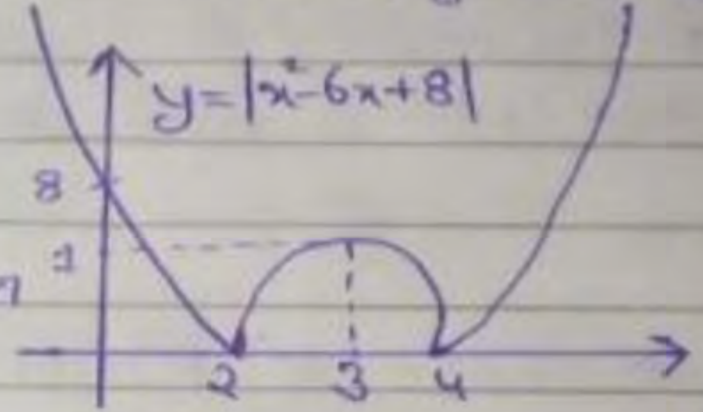
Ques find the range of k for which $||x^2-6x+8|-12| = k$ has

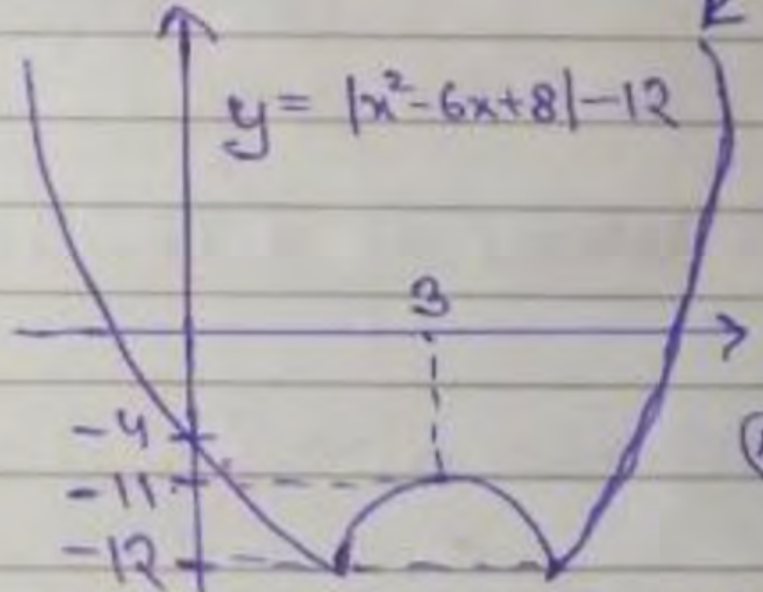
(1) No solution (2) Exactly 2 real solⁿ (3) Exactly 4 real solⁿ (4) Exactly 6 real solⁿ

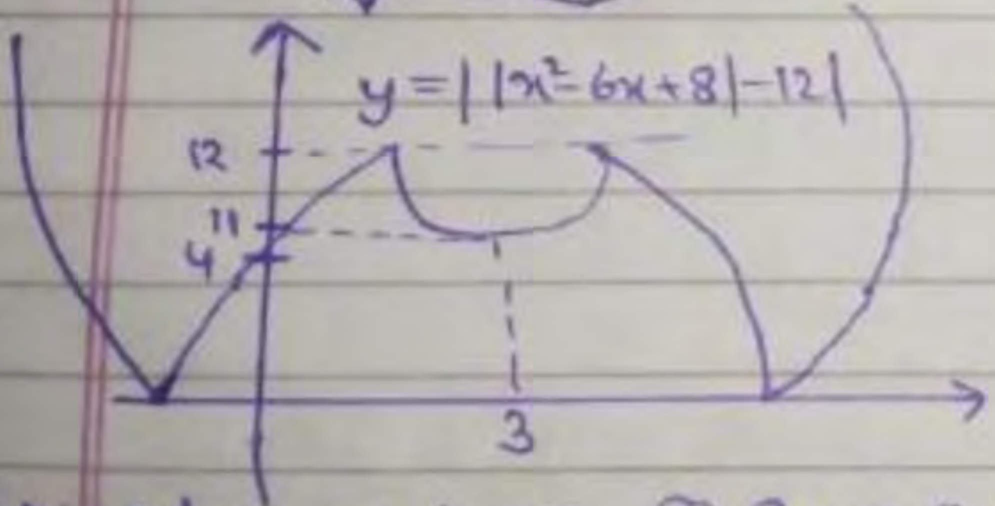
Ans $x^2-6x+8 \rightsquigarrow$ Root : 2, 4

vertex $\left(\frac{-b}{2a}, \frac{-D}{4a}\right) = (3, -1)$









(4) Exactly 6 solⁿ : $k \in \phi$

(1) No solution : $k<0$ (2) Exact 2 solution : $k>12$ (3) Exact 4 solⁿ : $k \in (0, 12) - \{11\}$

TAH-2

Q Find the range of K for which $||x^2-6x+8|-12| = k$ has

(1) no solution $\rightarrow (-\infty, 0) \in K$

(2) exactly 2 real solution $\rightarrow K \in (12, \infty) \cup \{0\}$

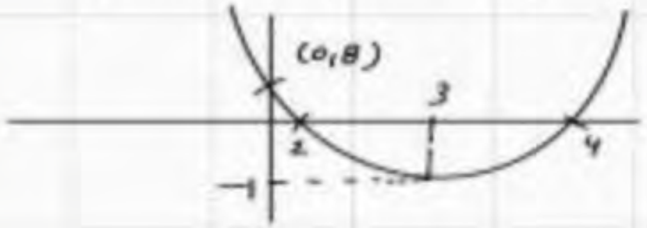
(3) Exactly 4 real solution $\rightarrow K \in (0, 11]$

(4) exactly 6 real solution $\rightarrow K \in (11, 12)$

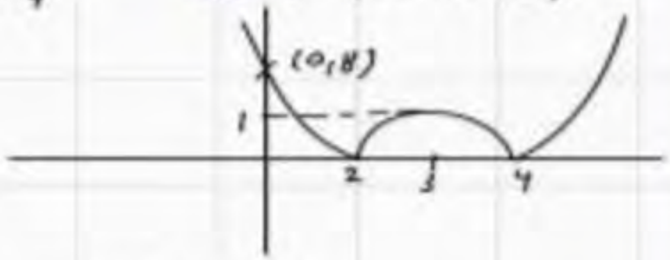
Solve step (1) $||x^2-6x+8|-12| = k$ $\qquad D = 36-32 = 4$

$y = x^2-6x+8 \rightarrow$ root $x=4$, $x=2$ $\qquad$ if $x=0$, $y=8$

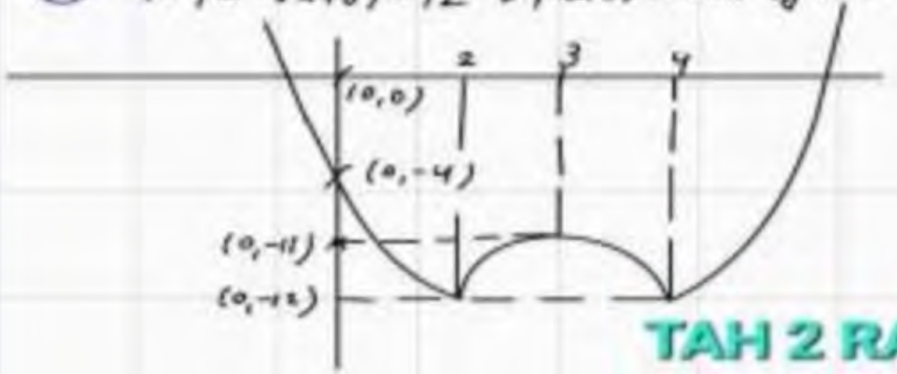
vertex $\Rightarrow (3, -1)$



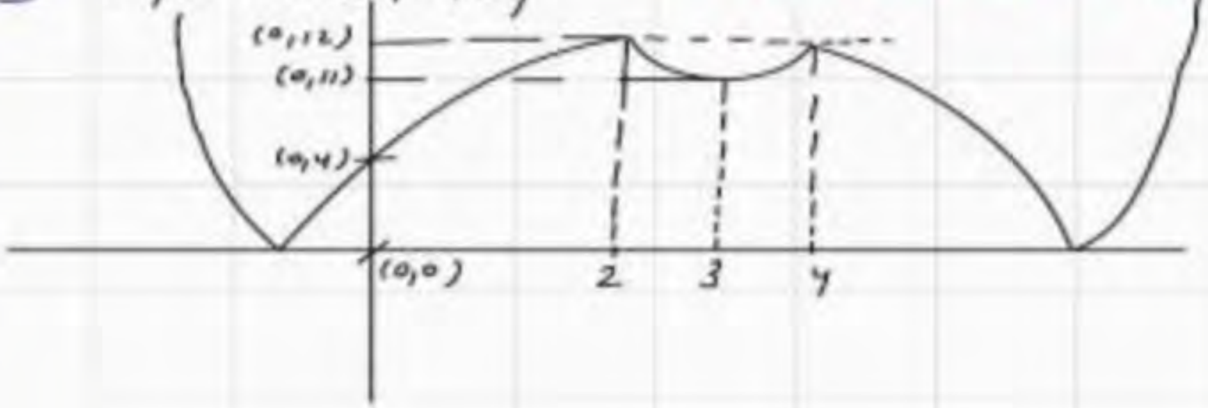
step (2) $y = |x^2-6x+8|$



step (3) $y = |x^2-6x+8| - 12$ → pull up x-axis by 12 unit



step (4) $y = ||x^2-6x+8|-12|$



QUESTION

TAH 3

Find Range of following functions:

(a) $f(x) = e^{(x-1)^2}$

(b) $f(x) = 2^{x^2} + 1$

(c) $f(x) = \frac{e^{2x}-e^x+1}{e^{2x}+e^x+1}$

Tah-03

Kalpana
From Bihar...

find range of following functions

(a) $f(x) = e^{(x-1)^2}$

$e^{(x-1)^2}$, $(x-1)^2 \to [0,\infty)$

$[e^0, e^\infty) = [1,\infty)$ Ans

Range = $[1,\infty)$

(b) $f(x) = 2^{x^2}+1$

$x^2 \to [0,\infty) \to [2^0, 2^\infty)$

$2^{x^2}+1$ = $[1,\infty)$ + 1

$[2,\infty)$

Range = $[2,\infty)$

(3) (a) $f(x) = e^{(x-1)^2}$

We know, $(x-1)^2 \geq 0$

Let $y = f(x)$

$y = e^{(x-1)^2} \Rightarrow \ln y = (x-1)^2$

$\frac{dy}{dx} = e^{(x-1)^2} \cdot 2(x-1) = 0$

$e^{(x-1)^2} \neq 0 \Rightarrow (x-1) = 0$ $\quad$ $x = 1 \rightarrow$ Minima Point

$e^{(x-1)^2}\big|_{min} = e^0 = 1$

$e^{(x-1)^2}\big|_{max} = \infty$

Range $= [1, \infty)$

AMAN SINGH

KUSHINAGAR

tah 3[a,b]

(2) $f(x) = 2^{x^2} + 1$

Let $y = 2^{x^2} + 1$

$\frac{dy}{dx} = 2^{x^2} 2x = 0$ $\quad$ $x = 0$

At $x = 0$ $\quad$ $y = 2^0 + 1 = 2$

Range of $y = [2, \infty)$

TAH-3

a) $f(x) = e^{(x-1)^2}$

Khushi

Sol^n: $e^{(x-1)^2}$

$(x-1)^2 \rightarrow [0, \infty)$

$\rightarrow [1, \infty)$

$R = [1, \infty)$ Ans

b) $f(x) = 2^{x^2} + 1$

Sol^n: $2^{x^2} + 1$

$x^2 \rightarrow \geq 0 : [0, \infty)$

$\rightarrow [1, \infty)$

$\rightarrow [2, \infty)$

$R = [2, \infty)$ Ans

QUESTION

TAH 5

Find the domain & range of the following functions :

$y = \sqrt{2 - x} + \sqrt{1 + x}$

TAH:-5

Find the domain and range of the foll. fn

$y=\sqrt{2-x}+\sqrt{1+x}$

$x\in[-1,2]$

$y^2=2-x+1+x+2(\sqrt{2-x}\cdot\sqrt{1+x})$

$y^2=3+2\sqrt{2-(x^2-x-1/4+1/4)}$

$y^2=3+2\sqrt{2-(x-1/2)^2+1/4}$

$y^2=3+\underbrace{2\,[0,3/2]}_{[0,3]}$

$y^2=[3,6]$ $\quad y\in[\sqrt{3},\sqrt{6}]$

$x-1/2\in\left[\frac{-3}{2},\frac{3}{2}\right]=\left[\frac{-3}{2},0\right]\cup\left[0,\frac{3}{2}\right]$

$(x-\frac{1}{2})^2\in[0,9/4]$

$9/4-(x-1/2)^2\in[0,9/4]$

$\sqrt{9/4-(x-1/2)^2}\in[0,3/2]$

HW

Tah-5

Manish Kumar
Bihar ❤️❤️

Q. Find the domain & range of the following functions:

$y = \sqrt{2-x} + \sqrt{1+x}$

M-1

Domain.

$\left.\begin{matrix} 2-x \geq 0 \\ 1+x \geq 0 \end{matrix}\right\} \Rightarrow [-1,2]$

$\Rightarrow y = \sqrt{2-x} + \sqrt{1+x}$

SBS

$y^2 = 2-x+1+x+2\sqrt{2-x}\sqrt{1+x}$

$\Rightarrow 3+2\sqrt{2+x-x^2}$

$\Rightarrow 3+2\sqrt{2-(x^2-x-\frac{1}{4}+\frac{1}{4})}$

$\Rightarrow 3+2\sqrt{2-(x-\frac{1}{2})^2+\frac{1}{4}}$

$y^2 \Rightarrow 3+2\sqrt{\frac{9}{4}-(x-\frac{1}{2})^2}$

$x \to [-1,2]$

$\Rightarrow x-\frac{1}{2} \in [\frac{-3}{2}, \frac{3}{2}]$

$\Rightarrow [\frac{-3}{2},0] \cup [0,\frac{3}{2}]$

$\Rightarrow (x-\frac{1}{2})^2 \in [0,\frac{9}{4}]$

$\Rightarrow \frac{9}{4}-(x-\frac{1}{2})^2 \in [0,\frac{9}{4}]$

$\Rightarrow \sqrt{\frac{9}{4}-(x-\frac{1}{2})^2} \in [0,\frac{3}{2}]$

$\Rightarrow 3+2\sqrt{\frac{9}{4}-(x-\frac{1}{2})^2}$

$[0,\frac{3}{2}]$

$[0,3]$

$[3,6]$

$y^2 = [3,6]$

$\boxed{y \in [\sqrt{3},\sqrt{6}]}$

M-2

$y = \sqrt{2-x} + \sqrt{1+x}$

Domain

$x \in [-1,2]$

$y^2 = 2-x+1+x+2\sqrt{2-x}\sqrt{1+x}$

$= 3+2\sqrt{2-x}\sqrt{1+x}$

AM GM Inequality

$\Rightarrow \frac{\overbrace{2-x + 1+x}^{\text{Non -ve}}}{2} \geq \sqrt{(2-x)(1+x)}$

$\Rightarrow \sqrt{2-x}\cdot\sqrt{1+x} \leq \frac{3}{2}$

$\Rightarrow \sqrt{2-x}\sqrt{1+x}\,\Big|_{Max} = \frac{3}{2}$

$\Rightarrow \sqrt{2-x}\sqrt{1+x}\,\Big|_{Min} = 0$

$y^2 = 3+2\sqrt{2-x}\sqrt{1+x}$

Max $= \frac{3}{2}$ Min $= 0$

$[0,3]$

$[3,6]$

$y^2 = [3,6]$

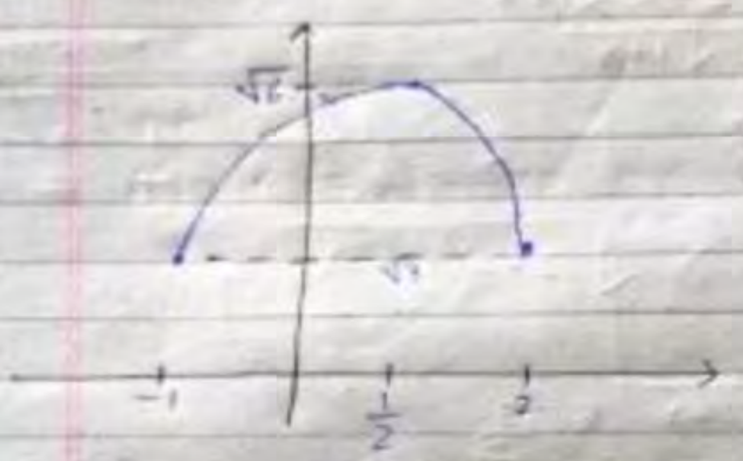
$y \in [\sqrt{3},\sqrt{6}]$ = Range//

Manish Kumar
Bihar ❤️❤️

(M-3) $y = \sqrt{2-x} + \sqrt{1+x}$

Domain

$\left.\begin{matrix} 2-x \geq 0 \\ 1+x \geq 0 \end{matrix}\right\rangle x \in [-1, 2]$



$\frac{dy}{dx} = \frac{-1}{2\sqrt{2-x}} + \frac{1}{2\sqrt{1+x}}$

$= \frac{\sqrt{2-x} - \sqrt{1+x}}{2\sqrt{2-x}\sqrt{1+x}} = 0$

$\Rightarrow \sqrt{2-x} = \sqrt{1+x}$

$2 - x = 1 + x$

$\boxed{x = \frac{1}{2}}$

$\boxed{\text{Range} \Rightarrow x \in [\sqrt{3}, \sqrt{6}]}$

(M-4) $y = \sqrt{2-x} + \sqrt{1+x}$

Domain

$\left.\begin{matrix} 2-x \geq 0 \\ 1+x \geq 0 \end{matrix}\right\rangle x \in [-1, 2]$

Domain $\Rightarrow [-1, 2]$

Manish Kumar
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Let $x = 2\sin^2\theta - \cos^2\theta$

$y = \sqrt{2 - (2\sin^2\theta - \cos^2\theta)} + \sqrt{1 + \overset{2\sin^2\theta}{\cancel{2\cos^2\theta}} - \cos^2\theta}$

$\Rightarrow \sqrt{2(1-\sin^2\theta) + \cos^2\theta} + \sqrt{1(1-\cos^2\theta) + \cancel{2\cos^2\theta}\, 2\sin^2\theta}$

$\Rightarrow \sqrt{3\cos^2\theta} + \sqrt{3\sin^2\theta}$

$\Rightarrow \sqrt{3}\,(\underbrace{|\sin\theta| + |\cos\theta|}_{[1, \sqrt{2}]})$

$[\sqrt{3}, \sqrt{6}] = \text{Range}$

Domain :→ $2 - x \geq 0$ and $1 + x \geq 0$

$\boxed{x \leq 2}$ $\boxed{x \geq -1}$

$\boxed{x \in [-1, 2]}$

Range :- $y = \sqrt{2-x} + \sqrt{1+x}$

TAH-5

$y^2 = 2 - \cancel{x} + 1 + \cancel{x} + 2\sqrt{2-x}\cdot\sqrt{1+x}$

$\Rightarrow 3 + 2\sqrt{2 + x - x^2}$

$\Rightarrow 3 + 2\sqrt{2 - (x^2 - x + \frac{1}{4} - \frac{1}{4})}$

$\Rightarrow 3 + 2\sqrt{2 - [(x - \frac{1}{2})^2 - \frac{1}{4}]}$

$\Rightarrow 3 + 2\sqrt{\frac{9}{4} - (x - \frac{1}{2})^2}$

$x \in [-1, 2] \rightarrow x - \frac{1}{2} \in [-\frac{3}{2}, \frac{3}{2}] \rightarrow [0, \frac{9}{4}] \rightarrow [0, \frac{9}{4}] \rightarrow [0, 3] \rightarrow [3, 6]$

Divyanshu Sagar
Bihar

∴ Range : $[3, 6]$

TA4-5

Find the Domain & range of the following

$y = \sqrt{2-x} + \sqrt{1+x}$

(m-1) clearly $y > 0$

Domain: $2-x \geq 0 \wedge 1+x \geq 0$

$x \leq 2 \wedge x \geq -1$

$x \in [-1, 2]$

S.O.S

$y^2 = 2-\cancel{x}+1+\cancel{x}+2\sqrt{(2-x)(1+x)}$

$y^2 = 3 + 2\sqrt{-x^2+x+2}$

$y^2 = 3 + 2\sqrt{-(x^2-x+\frac{1}{4}-\frac{1}{4})+2}$

$y^2 = 3 + 2\sqrt{-(x-\frac{1}{2})^2+\frac{9}{4}}$

$x \to [-1,2]$, $x-\frac{1}{2} \to [-\frac{3}{2},\frac{3}{2}]$, $(x-\frac{1}{2})^2 \to [0,\frac{9}{4}]$, $-(x-\frac{1}{2})^2 \to [-\frac{9}{4},0]$, $-(x-\frac{1}{2})^2+\frac{9}{4} \to [0,\frac{9}{4}]$, $\sqrt{-(x-\frac{1}{2})^2+\frac{9}{4}} \to [0,\frac{3}{2}]$

$y^2 = 3 + 2\sqrt{-(x-\frac{1}{2})^2+\frac{9}{4}}$

$\sqrt{-(x-\frac{1}{2})^2+\frac{9}{4}} \to [0,\frac{3}{2}]$, $2\sqrt{\dots} \to [0,3]$, $3+2\sqrt{\dots} \to [3,6]$

$y^2 \in [3,6]$

$\boxed{y \in [\sqrt{3}, \sqrt{6}]}$

TAH5 MRTHODE 1

RAHUL DHAKAD❤️❤️
FROM AGRA UP

(m-2) $y^2 = 3 + 2\sqrt{(2-x)(1+x)}$

clearly y +ve

A.M $\geq$ G.M

$\frac{2-\cancel{x}+1+\cancel{x}}{2} \geq \sqrt{(2-x)(1+x)}$

$\frac{3}{2} \geq \sqrt{(2-x)(1+x)}$

$\sqrt{(2-x)(1+x)}\Big|_{max} = \frac{3}{2}$, $\sqrt{(2-x)(1+x)}\Big|_{min} = 0$

$\sqrt{(2-x)(1+x)} \in [0, \frac{3}{2}]$

$y^2 = 3 + 2\sqrt{(2-x)(1+x)}$

$\sqrt{(2-x)(1+x)} \to [0,\frac{3}{2}]$, $2\sqrt{\dots} \to [0,3]$, $3+2\sqrt{\dots} \to [3,6]$

TAH5 METHODE 2 ,3

$y^2 \in [3,6] \Rightarrow \boxed{y \in [\sqrt{3}, \sqrt{6}]}$

(m-3) $y = \sqrt{2-x} + \sqrt{1+x}$ → Domain $[-1, 2]$

$\boxed{\text{Range: } [\sqrt{3}, \sqrt{6}]}$

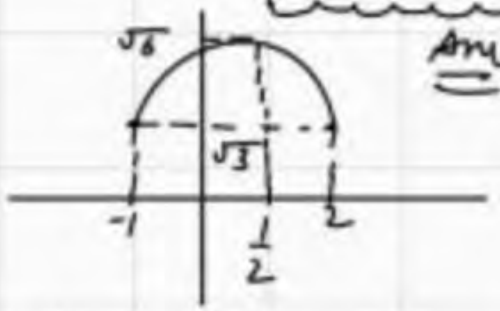
Ans

$\frac{dy}{dx} = \frac{-1}{2\sqrt{2-x}} + \frac{1}{2\sqrt{1+x}}$

$= \frac{-\sqrt{1+x} + \sqrt{2-x}}{2\sqrt{1+x}\sqrt{2-x}} = 0$

$2-x = 1+x$

$\boxed{x = \frac{1}{2}}$



at $x = \frac{1}{2}$, $y = \sqrt{2-\frac{1}{2}} + \sqrt{\frac{3}{2}} = \frac{2\sqrt{3}}{\sqrt{2}} = \sqrt{6}$

M-4 $y = \sqrt{2-x} + \sqrt{1+x}$, Domain $[-1, 2]$

Let $x = 2\cos^2\theta - \sin^2\theta$

$x = -1$ if $\cos^2\theta = 0$, $\sin^2\theta = 1$

$x = 2$ if $\cos^2\theta = 1$, $\sin^2\theta = 0$

$$y = \sqrt{2 - (2\cos^2\theta - \sin^2\theta)} + \sqrt{1 + 2\cos^2\theta - \sin^2\theta}$$

$$= \sqrt{3\sin^2\theta} + \sqrt{3\cos^2\theta}$$

$$= \sqrt{3}\,(\underbrace{|\sin\theta| + |\cos\theta|}_{[1, \sqrt{2}]}) \rightarrow [\sqrt{3}, \sqrt{6}] \quad \text{Ans}$$

TAH 5 METHODE 4

QUESTION [JEE Mains 2023 (25 Jan)]

TAH 6

Let $f : \mathbb{R} \to \mathbb{R}$ be a function defined by $f(x) = \log_{\sqrt{m}}\{\sqrt{2}(\sin x - \cos x) + m - 2\}$, for some m, such that the range of f is [0, 2]. Then the value of m is

(A) 4

(B) 3

(C) 5

(D) 2

Kalpana From Bihar

Tah-(06) [Mains 2023]

Let $f: R \to R$ be a fn. defined by $f(x) = \log_{\sqrt{m}}\{\sqrt{2}(\sin x - \cos x) + m - 2\}$ for some m, such that the range of f is $[0, 2]$. Then the value of m is.

Solution - $f(x) = \log_{\sqrt{m}}\{\sqrt{2}(\sin x - \cos x) + m - 2\}$

$[-\sqrt{2}, \sqrt{2}]$

$[-2, 2]$

$[-2 + m, 2 + m]$

$[m - 4, m]$

Range =

$[\log_{\sqrt{m}} m - 4, \log_{\sqrt{m}} m]$

But the range is $[0, 2]$ (given)

$\log_{\sqrt{m}} m - 4 = 0$ & $\log_{\sqrt{m}} m = 2$

$m - 4 = (\sqrt{m})^0$ $\quad$ $m = (\sqrt{m})^2$

$m - 4 = 1$ $\quad$ $m = m$

$m = 5$

Value of m is 05.

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Qn-8) $f: R \to R$. $f(x) = \log_{\sqrt{m}}\{\sqrt{2}(\sin x - \cos x) + m - 2\}$
for some m such that f has range $[0, 2]$ — value of m is

$$f(x) = \log_{\sqrt{m}}\left(\underbrace{\sqrt{2}(\sin x - \cos x)}_{[-\sqrt{2},\sqrt{2}]} + m - 2\right)$$

$[-2, 2]$

$[m-4, m]$

$$[\log_{\sqrt{m}} m-4, \log_{\sqrt{m}} m]$$

$$[\log_{\sqrt{m}} m-4, 2]$$

Given Range $[0, 2]$

$\boxed{m > 4}$

$m > 0$
$m \neq 1$

for $\log_{\sqrt{m}} m-4 = 0$

$m-4 = 1 \Rightarrow \underline{m = 5}$

$m = 5$ Ans

Himanshu Saini
Sirsa Haryana
tah 6

$$f(x) = \log_{\sqrt{m}}\{\sqrt{2}\,((\sin x - \cos x)) + m - 2\}$$

$$\sin x - \cos x \to [-\sqrt{2}, \sqrt{2}]$$

$$\to [-2, 2]$$

$$\to [-2+m, 2+m]$$

$$\to [m-4, m]$$

$$\Rightarrow \left[\log_{\sqrt{m}} m-4 , \log_{\sqrt{m}} m\right]$$

$$\Rightarrow \left[\log_{\sqrt{m}}(m-4), 2\right] \equiv [0, 2]$$

$\Downarrow$ on comparing

TAH-6

Divyanshu Sagar
Bihar

$$\log_{\sqrt{m}} m-4 = 0$$

$$m-4 = (\sqrt{m})^0$$

$$m-4 = 1$$

$$\boxed{m = 5}$$ Ans

QUESTION [JEE Mains 2023 (31 Jan)]

ASRQ TAH 7a

If the domain of the function $f(x) = \frac{[x]}{1+x^2}$, where $[x]$ is greatest integer $\le x$, is $[2, 6)$. then its range is

(A) $\left(\frac{5}{37}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$

(B) $\left(\frac{5}{37}, \frac{2}{5}\right]$

(C) $\left(\frac{5}{26}, \frac{2}{5}\right]$

(D) $\left(\frac{5}{26}, \frac{2}{5}\right] - \left\{\frac{9}{29}, \frac{27}{109}, \frac{18}{89}, \frac{9}{53}\right\}$

Tah-07(a) Pg-01

[Mains 2023]

If the domain of the fn $f(x)=\frac{[x]}{1+x^2}$, where $[x]$ is greatest integer $\le x$ is $[2, 6)$ then its range is

$$f(x)=\frac{[x]}{1+x^2}=\begin{cases}\frac{2}{1+x^2} & 2\le x<3\\ \frac{3}{1+x^2} & 3\le x<4\\ \frac{4}{1+x^2} & 4\le x<5\\ \frac{5}{1+x^2} & 5\le x<6\end{cases}$$

Kalpana
From Bihar....

* If $x \in [2,3)$ $\quad y = \dfrac{2}{1+x^2}$

$[2,3) \rightarrow [4,9) \rightarrow [5,10) \rightarrow \left(\frac{1}{10}, \frac{1}{5}\right] \rightarrow \left(\frac{1}{5}, \frac{2}{5}\right]$ — (I)

Then Range $= \left(\frac{1}{5}, \frac{2}{5}\right)$

* If $x \in [3,4)$ $\quad y = \dfrac{3}{1+x^2}$

$[3,4) \rightarrow [9,16) \rightarrow [10,17) \rightarrow \left(\frac{3}{17}, \frac{3}{10}\right]$ — (II)

* If $x \in [4,5)$ $\quad y = \dfrac{4}{1+x^2}$

$[4,5) \rightarrow [16,25) \rightarrow [17,26) \rightarrow \left[\frac{4}{26}, \frac{4}{17}\right)$ — (III)

* If $x \in [5,6)$

$y = \dfrac{5}{1+x^2}$

$[5,6) \rightarrow [25,36) \rightarrow [26,37) \rightarrow \left(\frac{5}{37}, \frac{5}{26}\right]$ — (IV)

Now (I) ∪ (II) ∪ (III) ∪ (IV)

Range of $f(x) = \left(\frac{5}{37}, \frac{2}{5}\right]$ Ans

QUESTION

ASRQ

TAH 7b

Find the domain & range of the following functions :

$$f(x) = \frac{x}{1+|x|}$$

$$f(x) = \frac{\sqrt{x+4}-3}{x-5}$$

TAH-7 (b)

Q $f(x) = \frac{x}{1+|x|}$

if $x \geq 0$

$f(x) = \frac{x}{1+x} = \frac{x+1-1}{1+x}$

$= 1 - \frac{1}{1+x}$

$[0, \infty)$

$[1, \infty)$

$(0, 1]$

$[0, 1) \rightarrow (1)$

if $x < 0$

$f(x) = \frac{x}{1-x}$

$= \frac{x}{-(x-1)} = \frac{x-1+1}{-(x-1)}$

$= -1 - \frac{1}{x-1}$

$x \in (-\infty, 0)$

$x-1 \in (-\infty, -1)$

$\frac{1}{x-1} \in (-1, 0)$

$-\frac{1}{x-1} \in (0, 1)$

$-1 - \frac{1}{x-1} \in (-1, 0) \rightarrow (2)$

$(1) \cup (2)$

$(-1, 0) \cup [0, 1) \Rightarrow \boxed{(-1, 1)}$ Ans

QUESTION [JEE Mains 2024 (6 April)]

TAH 7c

Let $f(x) = \frac{1}{7-\sin 5x}$ be a function defined on R. Then the range of the function $f(x)$ is equal to :

(A) $\left[\frac{1}{8}, \frac{1}{5}\right]$

(B) $\left[\frac{1}{7}, \frac{1}{6}\right]$

(C) $\left[\frac{1}{7}, \frac{1}{5}\right]$

(D) $\left[\frac{1}{8}, \frac{1}{6}\right]$

Ans. D

TAH-7 C

Kripa Shankar maurya varanasi

$$y = f(x) = \frac{1}{7 - \sin 5x}$$

$\sin 5x \in [-1, 1]$

$7 - \sin 5x \in [6, 8]$

$\left[\frac{1}{8}, \frac{1}{6}\right]$

$y \in \left[\frac{1}{8}, \frac{1}{6}\right]$ = Range A

7 (c) ⇒ Range of $f(x)$, $f(x) = \dfrac{1}{7 - \sin 5x}$

AMAN SINGH

KUSHINAGAR TAH 7[c]

$\sin 5x \to [-1, 1] \to [6, 8]$

$\to \left[\frac{1}{8}, \frac{1}{6}\right]$ (D) Ans

QUESTION

[redacted]

TAH 8

The range of the function $y = \frac{8}{9-x^2}$ is

(A) $(-\infty, \infty) - \{\pm 3\}$

(B) $\left[\frac{8}{9}, \infty\right)$

(C) $\left(0, \frac{8}{9}\right)$

(D) $(-\infty, 0) \cup \left[\frac{8}{9}, \infty\right)$

Ans. D

Que The Range of the func^n $y = \dfrac{8}{9-x^2}$ is. TAH 8

$$y = \frac{8}{9-x^2}$$

$$9y - yx^2 = 8$$

$$yx^2 - 9y = -8$$

$$x^2 = \frac{9y-8}{y}$$

$$x = \sqrt{\frac{9y-8}{y}}$$

$$\frac{9y-8}{y} \geq 0, \quad y \neq 0$$

$+ \quad 0 \quad - \quad 8/9 \quad +$

Range $= (-\infty, 0) \cup [8/9, \infty)$

Ans

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Tab-8) $y = \dfrac{8}{9 - x^2}$ Range.

$$9y - 8 - x^2 y = 0 \Rightarrow 9y - 8 = x^2 y$$

$$x = \sqrt{\dfrac{9y-8}{y}}$$

$$\dfrac{9y-8}{y} \geq 0$$

$$x \in (-\infty, 0) \cup [8/9, \infty)$$

TAH 8. The range of the fn $y = \frac{8}{9-x^2}$ is —

TAH 8

$$y = \frac{8}{9-x^2}$$

$$9 - x^2 \neq 0$$

$$\boxed{x \neq \pm 3}$$

$\therefore$ Domain: $R - \{\pm 3\}$.

$\therefore x \in (-\infty, \infty) - \{\pm 3\}$

$\therefore x^2 \in [0, 9) \cup (9, \infty)$

Now, $y = \frac{8}{9 - x^2}$

x^2: $[0,9) \cup (9,\infty)$

$9 - x^2$: $(-\infty, 0) \cup (0, 9]$

$(-\infty, 0) \cup [8/9, \infty)$

$\boxed{y \in (-\infty, 0) \cup [8/9, \infty)}$ Ans

Sourik Maiti
West Bengal

$y = \dfrac{8}{9 - x^2}$ Domain : $R - \{-3, 3\}$

$x^2 \rightarrow [0, \infty) - \{9\}$

$(-\infty, 9] - \{0\} = (-\infty, 0) \cup (0, 9]$

$8 \cdot (-\infty, 0) \cup [\frac{1}{9}, \infty) : (-\infty, 0) \cup [8/9, \infty)$

QUESTION

TAH 9

Find range of :

(1) $f(x) = \frac{2x-3}{x-1}$

(2) $f(x) = \frac{x+3}{2-5x}$

(3) $f(x) = \frac{6}{4x+7}$

(4) $f(x) = \frac{7x+5}{3}$

TAH:-9

Find range of:-

(1) $f(x) = \frac{2x-3}{x-1}$

$y \in R - \{2\}$

(2) $f(x) = \frac{x+3}{2-5x}$

$y = \frac{x+3}{-5x+2}$

$y \in R - \{-\frac{1}{5}\}$

(3) $f(x) = \frac{6}{4x+7}$

$f(x) = \frac{x0+6}{4x+7}$

$y \in R - \{0\}$

(4) $f(x) = \frac{7x+5}{3}$

$3y = 7x+5$

$3y-5 = 7x$

$\frac{3y-5}{7} = x$

$x \in R$

$y \in R$

TAH-④

Find range

① $f(x) = \frac{2x-3}{x-1}$

$y = \frac{2x-3}{x-1}$

$xy - y = 2x - 3$

$xy - 2x = y - 3$

$x(y-2) = y-3$

$x = \frac{y-3}{y-2}$

Range $\Rightarrow R - \{2\}$ Ans:

② $f(x) = \frac{x+3}{2-5x}$

$2y - 5xy = 3 + x$

$-x - 5xy = 3 - 2y$

$x(1+5y) = 2y - 3$

$x = \frac{2y-3}{1+5y}$

Range $\Rightarrow R - \{\frac{-1}{5}\}$

③ $f(x) = \frac{6}{4x+7}$

$4xy + 7y = 6$

$4xy = 6 - 7y$

$x = \frac{6-7y}{4y}$

Range $\Rightarrow R - \{0\}$

④ $f(x) = \frac{7x+5}{3}$

$3y = 7x + 5$

$x = \frac{3y-5}{7}$

Range $= R$

Name - Ankit kumar
from UP. (VNS)

QUESTION [JEE Mains 2023 (31 Jan)]

TAH 10

Let $f : \mathbb{R} - \{2, 6\} \rightarrow \mathbb{R}$ be real valued function defined as $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$. Then range of f is

(A) $\left(-\infty, -\frac{21}{4}\right] \cup [1, \infty)$

(B) $\left(-\infty, -\frac{21}{4}\right) \cup (0, \infty)$

(C) $\left(-\infty, -\frac{21}{4}\right] \cup [0, \infty)$

(D) $\left(-\infty, -\frac{21}{4}\right] \cup \left[\frac{21}{4}, \infty\right)$

TAH-10

$f(x) = \frac{x^2+2x+1}{x^2-8x+12}$, $f: R - \{2,6\} \to R$

$yx^2 - 8yx + 12y = x^2+2x+1$

$x^2(y-1) - x(8y+2) + (12y-1) = 0$

Case 1: $y - 1 \neq 0 \Rightarrow y \neq 1$

$x \in R$. So, $D \geq 0$

$(8y+2)^2 - 4(y-1)(12y-1) \geq 0$

$64y^2+4+32y-4(12y^2-12y-y+1) \geq 0$

$64y^2+\cancel{4}+32y-48y^2+48y+4y-\cancel{4} \geq 0$

$16y^2+84y \geq 0$

$4y(4y+21) \geq 0$

$+ \quad -\frac{21}{4} \quad - \quad 0 \quad +$

$y \in \left(-\infty, -\frac{21}{4}\right] \cup [0,\infty) - \{1\}$

Case 2: $y-1=0 \Rightarrow y=1$

$0x^2 - x(10) + (12-1) = 0$

$-10x+11=0$

$x = \frac{-11}{10}$

$y = 1$ also possible

(Case 1) $\cup$ (Case 2)

$y \in \left(-\infty, -\frac{21}{4}\right] \cup [0,\infty)$

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TAH-10. Let $f: R - \{2,6\} \to R$ be real valued fn defined as $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$. Then range of f is—

TAH 10

$\Rightarrow$ let, $\frac{x^2+2x+1}{x^2-8x+12} = y$

$\Rightarrow x^2+2x+1 = x^2y - 8xy + 12y$

$\Rightarrow (1-y)x^2 + (8y+2)x + (1-12y) = 0$

Case-1 If $1-y \neq 0 \Rightarrow y \neq 1$

Since, $x \in R - \{2,6\}$, $D \geq 0$

$\Rightarrow (8y+2)^2 - 4(1-y)(1-12y) \geq 0$

$\Rightarrow 64y^2+32y+4-4[1-13y+12y^2] \geq 0$

$\Rightarrow 64y^2+32y+\cancel{4}-\cancel{4}+52y-48y^2 \geq 0$

$\Rightarrow 16y^2+84y \geq 0$

$\Rightarrow 4y^2+21y \geq 0$

$\Rightarrow y(4y+21) \geq 0$

$\boxed{y \in \left(-\infty, -\frac{21}{4}\right] \cup [0,\infty) - \{1\}}$

If $y=1$

$10x - 11 = 0$

$x = \frac{11}{10} \in R - \{2,6\}$.

$y=1$ also possible.

$\boxed{y \in \left(-\infty, -\frac{21}{4}\right] \cup [0,\infty)}$ Ans

Sourik Maiti
West Bengal

Que Let $f : R - \{2, 6\} \to R$ be a real valued funⁿ defined as $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$ (TAH-10)

Solⁿ
$$y = \frac{x^2+2x+1}{x^2-8x+12}$$
$$x^2y - 8xy + 12y - x^2 - 2x - 1 = 0$$
$$(y-1)x^2 - (8y+2)x + 12y - 1 = 0$$

Case 1 $(y \neq 1)$ $\quad D \geq 0$
$$(8y+2)^2 - 4(y-1)(12y-1) \geq 0$$
$$64y^2 + 4 + 32y - 4(12y^2 - y - 12y + 1) \geq 0$$
$$16y^2 + 84y \geq 0$$
$$4y(4y+21) \geq 0$$

$+ \quad -\frac{21}{4} \quad - \quad 0 \quad +$

$$y \in \left(-\infty, -\frac{21}{4}\right] \cup [0, \infty) - \{1\}$$

Case 2 $(y = 1)$
$$-10x + 11 = 0$$
$$x = \frac{11}{10} \checkmark$$

from case ① ∪ case ②
$$\boxed{y \in \left(-\infty, -\frac{21}{4}\right] \cup [0, \infty)}$$
Ans

QUESTION

TAH 11

Read the symbols [ ] and { } as greatest integer function less than or equal to x and fractional part function..

(i) Find the number of real values of x, satisfying the equation $(x-2)[x] = \{x\} - 1$.

(ii) Find the number of solutions of the equation, $x^2 - 3x + [x] = 0$ in the interval $[0, 3]$.

(iii) If $[x]^2 + 3[x] - 10 \geq 0$, then find the range of x.

(iv) If $y = \sqrt{\operatorname{sgn}(x^2 - 2(k+1)x + 4)}$ is defined for all $x \in R$ then find number of integral values of k.

[Note: $\operatorname{sgn}(k)$ denotes signum function of k]

TAN.11

(i) $(x-2)[x] = \{x\}-1$

$(x-2)[x] = x-[x]-1$

$x[x]-2[x] = x-[x]-1$

$x[x]-[x] = x-1$

$(x-1)[x] = x-1$

$(x-1)[x]-(x-1) = 0$

$(x-1)([x]-1) = 0$

$x=1 \qquad [x]=1$

$x \in [1,2)$

$x \in [1,2)$ Ans

Pawan from UK

(ii) $x^2-3x+[x]=0 \qquad x \in [0,3]$

for $x \in [0,1)$

$x^2-3x+0=0$

$x(x-3)=0$

$x=0, 3 \to$ Rejected

for $x \in [2,3)$

$x^2-3x+2=0$

$(x-2)(x-1)=0$

$x=2, 1 \to$ Rejected

for $x \in [1,2)$

$x^2-3x+1=0$

$x=\frac{3\pm\sqrt{5}}{2} \to$ Rejected

for $x=3$

$x^2-3x+3=0$

$\hookrightarrow D<0$

$\hookrightarrow$ no real sol^n

$\therefore x=0,2$ i.e. there are two sol^{n}s

TAN.11

(iii) $[x]^2+3[x]-10 \geq 0$

let $[x]=t$

$t^2+3t-10 \geq 0 \Rightarrow (t+5)(t-2) \geq 0$

$t \in (-\infty,-5] \cup [2,\infty)$

~~for this to be true~~

~~$D \leq 0$~~

~~$9-4(-10$~~

$[x] \in (-\infty,-5] \cup [2,\infty)$

$\therefore x \in (-\infty,-4) \cup [2,\infty)$

Pawan from UK

(iv) $y=\sqrt{sgn(x^2-2(k+1)x+4)}$

$sgn(x^2-2(k+1)x+4) \geq 0$

for this; $x^2-2(k+1)x+4 \geq 0$

$\hookrightarrow D \leq 0$

$4(k+1)^2-4(4) \leq 0$

$k^2+2k+1-4 \leq 0$

$k^2+2k-3 \leq 0$

$(k+3)(k-1) \leq 0$

$k \in [-3,1]$

$\hookrightarrow$ No. of integral values: $\underbrace{-3,-2,-1,0,1}_{5}$

(i) $(x-2)[x] = \{x\} - 1$

$(x-2)(x-\{x\}) = \{x\} - 1$

$(x^2-2x-\{x\}(x-2) = \{x\} - 1$

$x^2-2x+1 \Rightarrow \{x\}(x-1)$

$\cancel{(x-1)}(x-1) = \{x\}\cancel{(x-1)}$

& $x=1$, $\{x\} = x-1$

$\Downarrow$

$0 \leq \{x\} < 1$

$0 \leq x-1 < 1$

$1 \leq x < 2$ & $x \in [1,2)$ Ans

(ii) $x^2-3x+[x]=0$, $x \in [0,3]$

$x \in [0,1)$, $x \in [1,2)$, $x \in [2,3)$

&

$x^2-3x=0$ $\quad$ $x^2-3x+1=0$ $\quad$ $x^2-3x+2=0$

$x \Rightarrow 0$, $x=3$ $\quad$ $x \Rightarrow \frac{3 \pm \sqrt{5}}{2}$ $\quad$ $(x-2)(x-1)=0$

$\quad$ $x \Rightarrow 2$, $x=1$

$x \Rightarrow \frac{3+\sqrt{5}}{2}, \frac{3-\sqrt{5}}{2}$

Na $x \in [3,4)$

$x^2-3x+3=0$

$x \Rightarrow \frac{3 \pm \sqrt{9-12}}{2}$, $\because a>0$ & $D<0$,

& $x \in \phi$

$\therefore$ No of Solⁿ $\Rightarrow 2$ Ans

(iii) $[x]^2+3[x]-10 \geq 0$ $\quad$ Let $[x] \to t$

$t^2+3t-10 \geq 0$

$(t+5)(t-2) \geq 0$

$t \in (-\infty,-5] \cup [2,\infty)$

$\because t \to [x]$

& $[x] \in (-\infty,-5] \cup [2,\infty)$

& $x \in (-\infty,-4) \cup [2,\infty)$ Ans

kamran Ashraf
Muzaffarpur

(iv) $y \Rightarrow \sqrt{\operatorname{Sgn}(x^2 - 2(K+1)x+4}$ $\forall x \in R$

& $y = \sqrt{\operatorname{Sgn}(x^2 - 2(K+1)x+4}$ is defn $\forall x \in R$

$x^2 - 2(K+1)x + 4 \geq 0$

So $\Delta \leq 0$

$4(K+1)^2 - 16 \leq 0$

$K^2 + 1 + 2K - 4 \leq 0$

$K^2 + 2K - 3 \leq 0$

$K^2 + 3K - K - 3 \leq 0$

$(K+3)(K-1) \leq 0$

So $K \in [-3, 1]$ Ans

No. of I value of $K \Rightarrow 5$ Ans

(Solution to RPP)

QUESTION [JEE Mains 2024 (6 April)]

RPP 1

If A is a square matrix of order 3 such that $\det(A) = 3$ and $\det\left(\operatorname{adj}\left(-4\operatorname{adj}\left(-3\operatorname{adj}\left(3\operatorname{adj}\left((2A)^{-1}\right)\right)\right)\right)\right) = 2^m 3^n$, then $m + 2n$ is equal to :

(A) 2

(B) 4

(C) 3

(D) 6

Ans. B

RPP-1

$O(A) = 3\times3$ & $\det(A) = 3$

$\det(\text{adj}(-4\,\text{adj}(-3\,\text{adj}(3\,\text{adj}(2A)^{-1})))) = 2^m \cdot 3^n$

Sol^n

$|\text{adj}(-4\,\text{adj}(-3\,\text{adj}(3\,\text{adj}(2A)^{-1})))|$

$|-4\,\text{adj}(-3\,\text{adj}(3\,\text{adj}(2A)^{-1}))|^2$

$= (-4)^6\,|\text{adj}(-3\,\text{adj}(3\,\text{adj}(2A)^{-1}))|^2$

$= (-4)^6\,|-3\,\text{adj}(3\,\text{adj}(2A)^{-1})|^4$

$(-4)^6(-3)^{12}\,|\text{adj}(3\,\text{adj}(2A)^{-1})|^4$

$(-4)^6(-3)^{12}\,|3\,\text{adj}(2A)^{-1}|^8$

$(-4)^6(-3)^{12}(3)^{24}\,|\text{adj}(2A)^{-1}|^8$

$= (2)^{12}(3)^{36}\,|(2A)^{-1}|^{16}$

$= (2)^{12}(3)^{36}\dfrac{1}{|2A|^{16}}$

$= (2)^{-36}\cdot(3)^{20}$

$m = -36 \mid n = 20 \mid m + 2n = 4$

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RPP 1

Q) If A is a sq matrix of order 3 s.t. $\det A = 3$ &

$\det(\text{adj}(-4\,\text{adj}(-3\,\text{adj}(3\,\text{adj}(2A)^{-1})))) = 2^m 3^n$

then $m + 2n = ?$

Sol^n $\det\left(\text{adj}\left(-4\,\text{adj}\left(-3\,\text{adj}\left(3\,\text{adj}\,\frac{1}{2}A^{-1}\right)\right)\right)\right)$

$\det\left(\text{adj}\left(-4\,\text{adj}\left(-3\,\text{adj}\left(3\cdot\frac{1}{4}\,\text{adj}\,A^{-1}\right)\right)\right)\right)$

$\det\left(\text{adj}\left(-4\,\text{adj}\left(\frac{-3\cdot3^2}{4^2}\,\text{adj}\,\text{adj}\,A^{-1}\right)\right)\right)$

$\det\left(\text{adj}\left(\frac{-4\cdot3^6}{4^4}\cdot\text{adj}(\text{adj}(\text{adj}\,A^{-1}))\right)\right)$

$\det\left(\frac{3^{12}}{4^6}\,\text{adj}(\text{adj}(\text{adj}(\text{adj}\,A^{-1})))\right)$

$\frac{3^{36}}{4^{18}}\det(\text{adj}(\text{adj}(\text{adj}(\text{adj}\,A^{-1}))))$

$\frac{3^{36}}{2^{36}}\,|A^{-1}|^{(3-1)^4} = 3^{36}\cdot2^{-36}\cdot\frac{1}{|A|^{16}} = \frac{3^{36}\cdot2^{-36}}{3^{16}}$

$= 3^{20}\cdot2^{-36}$

$m = -36,\ n = 20$

$m + 2n = -36 + 40 = 4$

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RPP| $|A| = 3$.

$\det[\operatorname{adj}(-4\operatorname{adj}(-3\operatorname{adj}(3\operatorname{adj}((2A)^{-1})))] = 2^m 3^n$

$m + 2n = ?$

$\det(\operatorname{adj}(-4(\operatorname{adj}(-3(\operatorname{adj}(3(\tfrac{1}{2})^2(\operatorname{adj}A^{-1}))))$

$\det(\operatorname{adj}(-4(3)^2\operatorname{adj}((3)^2(\tfrac{1}{2})^4\operatorname{adj}(\operatorname{adj}A^{-1})))$

$\det(\operatorname{adj}((-4)(3)^2(3)^4(\tfrac{1}{2})^8\operatorname{adj}\operatorname{adj}\operatorname{adj}A^{-1}))$

$\det(4^2 3^{12}(\tfrac{1}{2})^{16}(\operatorname{adj}(\operatorname{adj}(\operatorname{adj}(\operatorname{adj}A^{-1})))))$

$(4^6 3^{36}(\tfrac{1}{2})^{48}\det(\operatorname{adj}(\operatorname{adj}(\operatorname{adj}(\operatorname{adj}A^{-1})))))$

$\frac{2^{12}}{2^{48}}3^{36}(|A|^{16})^{-1} = 2^{-36}\frac{3^{36}}{3^{16}} = 3^{+20}2^{-36}$

$n = 20$

$m + 2n = -36 + 40 = 4$ $\qquad m = -36$

$= \text{Ans}$

Ques For $\alpha, \beta \in R$ and a natural number n, let $A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$

Then $2A_{10} - A_8$ is

(A) $4\alpha+2\beta$ (B) 0 (C) $2n$ (D) $2\alpha+4\beta$

Ans $A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix} \xrightarrow{R_2 \to R_2 - 2R_1} A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 0 & 0 & -(2\alpha+\beta) \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$

$$A_r = (2\alpha+\beta)\left(3r-(3r-2)\right)$$
$$A_r = 2(2\alpha+\beta)$$
$$A_r = 4\alpha+2\beta$$

$$2A_{10} = 8\alpha+4\beta$$
$$A_8 = 4\alpha+2\beta$$
$$2A_{10} - A_8 = 4\alpha+2\beta \text{ Ans}$$

RPP 1

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GHAZIABAD, U. P.

QUESTION [JEE Mains 2024 (6 April)]

RPP2

For $\alpha, \beta \in \mathbb{R}$ and a natural number n, let $A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$. Then $2A_{10} - A_8$ is

(A) $4\alpha + 2\beta$

(B) 0

(C) $2n$

(D) $2\alpha + 4\beta$

Ans. A

Pg

$$A_x = \begin{vmatrix} x & 1 & \frac{n^2}{2}+\alpha \\ 2x & 2 & n^2-\beta \\ 3x-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$$ Then $2A_{10} - A_8$ is

$$A_{10} = \begin{vmatrix} 10 & 1 & \frac{n^2}{2}+\alpha \\ 20 & 2 & n^2-\beta \\ 28 & 3 & n\frac{(3n-1)}{2} \end{vmatrix}$$

$$A_8 = \begin{vmatrix} 8 & 1 & \frac{n^2}{2}+\alpha \\ 16 & 2 & n^2-\beta \\ 22 & 3 & n\frac{(3n-1)}{2} \end{vmatrix}$$

$$2A_{10} - A_8 \Rightarrow \begin{vmatrix} 12 & 1 & \frac{n^2}{2}+\alpha \\ 24 & 2 & n^2-\beta \\ 34 & 3 & n\frac{(3n-1)}{2} \end{vmatrix}$$

$C_1 \to C_1 - 12C_2$

$$\Rightarrow \begin{vmatrix} 0 & 1 & \frac{n^2}{2}+\alpha \\ 0 & 2 & n^2-\beta \\ -2 & 3 & n\frac{(3n-1)}{2} \end{vmatrix}$$

$$\Rightarrow -2(\cancel{n^2}-\beta-\cancel{n^2}-2\alpha)$$

$$\Rightarrow 4\alpha+2\beta$$

RPP 2

Page No.:
Date:

Q) For $\alpha, \beta \in R$ & a natural no. n, let $A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & n\frac{(3n-1)}{2} \end{vmatrix}$

Then $2A_{10} - A_8$ is

Sol^n $A_{10} = \begin{vmatrix} 10 & 1 & n^2/2+\alpha \\ 20 & 2 & n^2-\beta \\ 28 & 3 & n\frac{(3n-1)}{2} \end{vmatrix} = \begin{vmatrix} 10 & 1 & n^2/2+\alpha \\ 0 & 0 & -2\alpha-\beta \\ 28 & 3 & n\left(\frac{3n-1}{2}\right) \end{vmatrix}$

$= (2\alpha+\beta)(30-28) = 2(2\alpha+\beta)$

$A_8 = \begin{vmatrix} 8 & 1 & n^2/2+\alpha \\ 16 & 2 & n^2-\beta \\ 22 & 3 & n\frac{(3n-1)}{2} \end{vmatrix} = \begin{vmatrix} 8 & 1 & n^2/2+\alpha \\ 0 & 0 & -2\alpha-\beta \\ 22 & 3 & n\left(\frac{3n-1}{2}\right) \end{vmatrix}$

$= (2\alpha+\beta)(24-22) = 2(2\alpha+\beta)$

$2(A_{10}) - A_8 = 4(2\alpha+\beta) - 2(2\alpha+\beta)$
$= 2(2\alpha+\beta) = 4\alpha+2\beta$

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RPP-3

Let $A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$

Sol^n $R_2 \to R_2 - 2R_1$

$R_3 \to R_3 - 3R_1$

$A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 0 & 0 & -2\alpha-\beta \\ 2 & 0 & -\frac{n}{2}-3\alpha \end{vmatrix}$

$2A_{10} = 8\alpha + 4\beta$

$A_8 = 4\alpha + 2\beta$

$2A_{10} - A_8 = 4\alpha + 2\beta$ (Ans)

ADRISH SIL FROM WEST BENGAL HOOGHLY

Let $\alpha\beta \neq 0$ and $A = \begin{bmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\alpha \end{bmatrix}$. If $\begin{bmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{bmatrix} = B$ (cofactors)

is the matrix of cofactors of the elements of A, then det (AB) is equal to:

RPP 3

$\det(AB) = \det(A) \cdot \det B$ $\quad B = \begin{vmatrix} \text{Det of} \\ \text{Co-factors} \end{vmatrix}_{3\times 3} = A^2$

$= \det(A)\,(\det(A))^2$

$= (\det A)^3$

Cofactors of $a_{11} = 2\alpha^2 - \alpha\beta = 3\alpha \Rightarrow 2\alpha - \beta = 3$ — (i)

Co-factors of $a_{12} = -(2\alpha^2+\beta^2) = -9 \Rightarrow 2\alpha^2+\beta^2 = 9$ — (ii)

from eq^n (ii) $\Rightarrow 2\alpha^2 + (2\alpha-3)^2 = 9$

$\Rightarrow 2\alpha^2 + 4\alpha^2 - 12\alpha + 9 = 9$

$\Rightarrow 6\alpha^2 - 12\alpha = 0$

$\Rightarrow \alpha(\alpha-2) = 0$

$\boxed{\therefore \alpha = 0}$, $\boxed{\alpha = 2}$

$\alpha = 0$ not possible, as $\alpha\beta \neq 0$.

If $\alpha = 2$, from eq^n (i) $\Rightarrow$ $4 - \beta = 3$ $\therefore \beta = 1$.

Now, $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ -1 & 2 & 4 \end{vmatrix} = 2\begin{vmatrix} 1 & 1 & 3 \\ 2 & 1 & 1 \\ -1 & 1 & 4 \end{vmatrix}$

$= 2\begin{vmatrix} -1 & 0 & 2 \\ 3 & 0 & -3 \\ -1 & 1 & 4 \end{vmatrix}$ $\begin{bmatrix} R_1' = R_1 - R_2 \\ R_2' = R_2 - R_3 \end{bmatrix}$

$= -2(3-6) = 6$.

$\therefore \det(A) = 6$.

Now, $\det(AB) = (\det A)^3 = (6)^3 = (216)$ Ans.

sourik Maiti West Bengal

QUESTION [JEE Mains 2024 (5 April)]

RPP 3

Let $\alpha\beta \neq 0$ and $A = \begin{bmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\alpha \end{bmatrix}$. If $B = \begin{bmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{bmatrix}$ is the matrix of cofactors of the elements of A, then $\det(AB)$ is equal to:

(A) 64

(B) 343

(C) 125

(D) 216

Ans. D

RPP-3

Let $\alpha\beta \neq 0$ & $A = \begin{bmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\alpha \end{bmatrix}$. If $B = \begin{bmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{bmatrix}$ is the matrix cofactor of the element A, then det (AB) is equal to

Sol^n (C)$a_{13} = b_{13}$

$\alpha^2 + \alpha\beta = 3\alpha$

$\alpha + \beta = 3$ — (i)

(C)$a_{32} = b_{32}$

$-(\beta^2 - 3\alpha) = 5$

$(C)_{23} = (b)_{23}$

$0 = -2\alpha$ (N.P)

$C_{a11} = b_{11}$

$2\alpha^2 - \alpha\beta = 3\alpha$

$2\alpha - 2\beta = 2$ (ii)

$\alpha = 2, \beta = 1$

$\det(AB) = \det(A)^3$

$\begin{vmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ -1 & 2 & 4 \end{vmatrix}^3 = (6 - 2(9) + 3(6))^3$

$= (6)^3$

$= 216$ (Ans)

ADRISH SIL FROM WEST BENGAL HOOGHLY

RPP 3

Q Let $\alpha\beta \neq 0$ & $A = \begin{bmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\alpha \end{bmatrix}$, if $B = \begin{bmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{bmatrix}$

is the matrix of cofactors of elements of A, then det(AB) is equal to.

Sol^n $\det(AB) = \det A \cdot \det B = (\det A)^3$

Now, Cofactor of a_{11}

$2\alpha^2 - \alpha\beta = 3\alpha$

Cofactor of a_{13} $\alpha^2 + \alpha\beta = 3\alpha$

$3\alpha^2 = 6\alpha$

$3\alpha^2 - 6\alpha = 0$

$3\alpha(\alpha - 2) = 0$

$\alpha = 0$, $\boxed{\alpha = 2}$

$\alpha\beta \neq 0$ $\boxed{\beta = 1}$

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ -1 & 2 & 4 \end{bmatrix}$ $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ -1 & 2 & 4 \end{vmatrix} = \begin{vmatrix} 0 & 4 & 7 \\ 2 & 2 & 1 \\ -1 & 2 & 4 \end{vmatrix}$

$= \begin{vmatrix} 0 & 4 & 7 \\ 0 & 6 & 9 \\ -1 & 2 & 4 \end{vmatrix}$

$|A| = -1(36 - 42) = 6$

$|AB| = 216$ Ans

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Let $\alpha\beta \neq 0$ and $A = \begin{bmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\alpha \end{bmatrix}$. If $\begin{bmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{bmatrix} = B$ (cofactors)

is the matrix of co-factors of the elements of A, then $\det(AB)$ is equal to:

RPP 3

$$\det(AB) = \det(A) \cdot \det B$$
$$= \det(A) \cdot (\det(A))^2$$
$$= (\det A)^3$$

$\Delta' = \left| \begin{matrix} \text{Det of} \\ \text{Co-factors} \end{matrix} \right|_{3\times 3} = \Delta^2$

Cofactors of $a_{11} = 2\alpha^2 - \alpha\beta = 3\alpha \Rightarrow 2\alpha - \beta = 3$ — (i).

Co-factors of $a_{12} = -(2\alpha^2 + \beta^2) = -9 \Rightarrow 2\alpha^2 + \beta^2 = 9$ — (ii).

from eqn (ii) $\Rightarrow$ $2\alpha^2 + (2\alpha - 3)^2 = 9$

$\Rightarrow 2\alpha^2 + 4\alpha^2 - 12\alpha + 9 = 9$

$\Rightarrow 6\alpha^2 - 12\alpha = 0$

$\Rightarrow \alpha(\alpha - 2) = 0$

$\boxed{\therefore \alpha = 0}$, $\boxed{\alpha = 2}$

$\alpha = 0$: Not possible, as $\alpha\beta \neq 0$.

If $\alpha = 2$.
from eqn (i) $\Rightarrow$
$4 - \beta = 3$
$\therefore \beta = 1$.

Now, $|A| = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 2 & 1 \\ -1 & 2 & 4 \end{vmatrix} = 2\begin{vmatrix} 1 & 1 & 3 \\ 2 & 1 & 1 \\ -1 & 1 & 4 \end{vmatrix}$

$= 2\begin{vmatrix} -1 & 0 & 2 \\ 3 & 0 & -3 \\ -1 & 1 & 4 \end{vmatrix}$ $\begin{bmatrix} R_1' = R_1 - R_2 \\ R_2' = R_2 - R_3 \end{bmatrix}$

$= -2(3 - 6) = 6$.

$\therefore \det(A) = 6$.

Now, $\det(AB) = (\det A)^3 = (6)^3 = (216)$ Ans.

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THANK
YOU

PRAYAS

JEE 2025

Lecture- 10 | Mathematics

Relation & Functions

By- Ashish Agarwal Sir (IIT Kanpur)

Topics *to be covered*

1. Even and Odd Functions
2. Periodic Functions

Recap *of previous lecture*

1. A continuous function which is increasing or decreasing on its domain is one-one/Injective

2. If for a function Co-domain = Range, then the function is onto/Surjective

3. A function which is not one-one is Many one

4. A function which is not onto Into.

5. If the derivative of a function $dy/dx = f'(x) \geq 0$ on an interval where equality to 0 holds at some discrete points of the interval not forming a subinterval then the function is Increasing on the interval.

Recap of previous lecture

6. If the derivative of a function $dy/dx = f'(x) \leq 0$ on an interval where equality to 0 holds at some discrete points of the interval not forming a subinterval then the function is Decreasing on the interval.

7. If for the function $f(x_1) = f(x_2)$ where $x_1 \neq x_2$ then the function is Many one

8. Even & periodic functions are Many one.

9. Number of one-one functions from A to B + number of many one functions from A to B = Total no: of fns from A to B.

Discussion: Homework of Previous Class

Bumper Practice Questions

Find the Domain of Definition of the Given Functions

(i) $y = \sqrt{-px}(p > 0)$

(ii) $y = \frac{1}{x^2+1}$

(iii) $y = \frac{1}{x^3-x}$

(iv) $y = \frac{1}{\sqrt{x^2-4x}}$

(v) $y = \sqrt{x^2 - 4x + 3}$

(vi) $y = \frac{x}{\sqrt{x^2-3x+2}}$

(vii) $y = \sqrt{1 - |x|}$

(viii) $y = \log_x 2$

(ix) $y = \frac{1}{\log_{10}(1-x)} + \sqrt{x + 2}$

(x) $y = \sqrt{x} + \sqrt[3]{\frac{1}{x-2}} - \log_{10}(2x - 3)$

(xi) $y = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$

(xii) $y = \frac{1}{\sqrt{\sin x}} + \sqrt[3]{\sin x}$

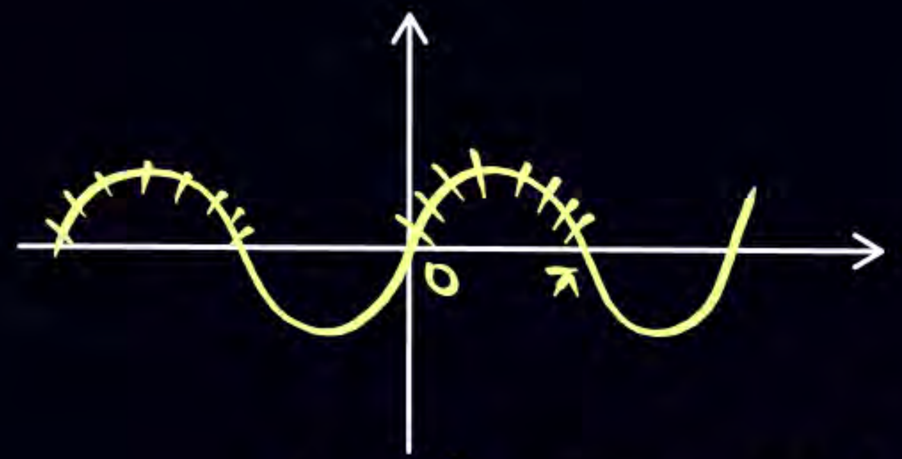
(xiii) $y = \log_{10}(\sqrt{x - 4} + \sqrt{6 - x})$

(xiv) $y = \log_{10}[1 - \log_{10}(x^2 - 5x + 16)]$

(xii) $f(x) = \sqrt[3]{\sin x} + \dfrac{1}{\sqrt{\sin x}}$

$\sqrt[3]{\sin x} \downarrow x \in R$

$\dfrac{1}{\sqrt{\sin x}} \downarrow \sin x > 0$



$x \in (2n\pi + 0, 2n\pi + \pi)$

$x \in (2n\pi, (2n+1)\pi), n \in I$

Bumper Practice Questions

Find the range of the following functions :

(i) $f(x) = \frac{x-1}{x+2}$

(ii) $f(x) = \frac{2}{x}$

(iii) $f(x) = \frac{1}{x^2-x+1}$

(iv) $f(x) = \frac{x^2-x+1}{x^2+x+1}$

(v) $f(x) = e^{(x-1)^2}$

(vi) $f(x) = x^3 - x^2 + x + 1$

(vii) $f(x) = \log\left(x^8 + x^4 + x^2 + 1\right)$

(viii) $f(x) = \sin^2 x - 2\sin x + 4$

(ix) $f(x) = \sin(\log_2 x)$

(x) $f(x) = 2^{x^2} + 1$

(xi) $f(x) = \frac{e^{2x}-e^x+1}{e^{2x}+e^x+1}$

(xii) $f(x) = \frac{1}{8-3\sin x}$

(vii) $f(x) = \log_{10}(x^8 + x^4 + x^2 + 1)$

$\log_{10}(\underbrace{\underbrace{\underset{\geq 0}{x^2} + \underset{\geq 0}{x^4} + \underset{\geq 0}{x^8}}_{[0,\infty)} + 1}_{[1,\infty)})$ — $[0,\infty)$

Gola method should be applied when the expression contains a single fn

Ex: $f(x) = \underbrace{\sin^4 x}_{[0,1]} + \underbrace{\cos^4 x}_{[0,1]}$ — $[0,2]$

Gadho|Gadhiyoo aisaa naa karo

$\phi(x) = \underbrace{f(x)}_{[a,b]} + \underbrace{g(x)}_{[c,d]}$

if $f(x_1) = a = g(x_1)$

$f(x_2) = b = g(x_2)$

then

$\phi(x) = \underbrace{f(x) + g(x)}_{[a+c,\ b+d]}$

Range of Rational fns.

Type (1) $f(x)=\dfrac{ax+b}{cx+d}$ Range : $R-\left\{\dfrac{a}{c}\right\}$ provided : $\dfrac{a}{c}\neq\dfrac{b}{d}$

Ex: $f(x)=\dfrac{7-5x}{2x-3}$ Range : $R-\left\{\dfrac{-5}{2}\right\}$

Ex: $f(x)=\dfrac{2x+4}{x+2}$ $\dfrac{2}{1}=\dfrac{4}{2}$

$f(x)=\dfrac{2(x+2)}{x+2}$

$f(x)=2$, $x\neq-2$ Range : $\{2\}$.

Type ②

$$f(x)=\frac{\cancel{(ax+b)}(cx+d)}{\cancel{(ax+b)}(ex+f)}$$

When Nr & Den have a common factor

$$f(x)=\frac{cx+d}{ex+f} \qquad ax+b\neq 0 \Rightarrow x\neq -\frac{b}{a}$$

$$\text{Range} = R-\left\{\frac{c}{e}, f\left(-\frac{b}{a}\right)\right\}.$$

Ex: $f(x)=\dfrac{\cancel{(2x-1)}(3x-2)}{(5x-1)\cancel{(2x-1)}}$

$$f(x)=\frac{3x-2}{5x-1},\ x\neq 1/2.$$

Range: $R-\{3/5, f(1/2)\} = R-\left\{3/5, \dfrac{-1/2}{3/2}\right\} = R-\{3/5, -1/3\}.$

Type ③ When num & Den contain no common factor

applicable $\frac{\text{quad}}{\text{quad}}$, $\frac{\text{linear}}{\text{quad}}$, $\frac{\text{quad}}{\text{linear}}$.

Ex: $f(x) = \frac{x^2 - x + 1}{x^2 + x + 1} = y$

$$x^2 - x + 1 = x^2 y + xy + y$$

$$x^2(y-1) + x(y+1) + y - 1 = 0 \quad \text{— (A)}$$

Case ① $y \neq 1$

Since $x \in R$ $D \geq 0$

$$D = (y+1)^2 - 4(y-1)\cdot(y-1) \geq 0$$

$$(y+1)^2 - (2(y-1))^2 \geq 0$$

$$(y+1-2(y-1))\,(y+1+2(y-1)) \geq 0$$

$$(3-y)(3y-1) \geq 0$$

$$(y-3)(3y-1) \leq 0$$

$+ \quad - \quad +$

$\frac{1}{3} \quad 3$

$y \in \left[\frac{1}{3}, 3\right] - \{1\}$

Case (II) If $y = 1$

from (A) $2x = 0$

$x = 0 \in R$

$\Downarrow$

$y = 1$ is possible.

Ans: $\left[1/3, 3\right] - \{1\} \cup \{1\}$

$\parallel$

$y \in \left[1/3, 3\right] =$ Range

Ex: $f(x) = \dfrac{x^2 - x + 1}{x^2 + x + 1} = y$

M② $y = \dfrac{x^2 + x + 1 - 2x}{x^2 + x + 1}$

$y = 1 - \dfrac{2x}{x^2 + x + 1}$

Case① $x \neq 0$ $\quad y = 1 - \dfrac{2}{x + \frac{1}{x} + 1}$

$(-\infty, -2] \cup [2, \infty)$

$(-\infty, -1] \cup [3, \infty)$

$2\,[-1, 0) \cup (0, 1/3]$

$\| $

$[-2, 0) \cup (0, 2/3]$

$(1, 3] \cup [1/3, 1)$

$[1/3, 1) \cup (1, 3]$

Case⑪ if $x = 0 \Rightarrow y = 1$

$\cup$

$y \in [1/3, 3]$ Ans.

QUESTION

Read the symbols [ ] and { } as greatest integer function less than or equal to x and fractional part function..

(i) Find the number of real values of x, satisfying the equation $(x-2)[x] = \{x\} - 1$.

(ii) Find the number of solutions of the equation, $x^2 - 3x + [x] = 0$ in the interval $[0, 3]$.

(iii) If $[x]^2 + 3[x] - 10 \geq 0$, then find the range of x.

(iv) If $y = \sqrt{\operatorname{sgn}(x^2 - 2(k+1)x + 4)}$ is defined for all $x \in R$ then find number of integral values of k.
[Note: $\operatorname{sgn}(k)$ denotes signum function of k]

(1) $(x-2)[x] = \{x\} - 1$

$x[x] - 2[x] = x - [x] - 1$

$x[x] - [x] = x - 1$

$[x](x-1) = (x-1)$

$[x](x-1) - (x-1) = 0$

$(x-1)([x]-1) = 0$

$x = 1$ or $[x] = 1$

$x = 1$ or $x \in [1,2) \Rightarrow$ Ans: $[1,2)$

$$\operatorname{sgn} x = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

(4) $y = \sqrt{\operatorname{sgn}(x^2 - 2(k+1)x + 4)}$

Domain $= R$

$\operatorname{sgn}(x^2 - 2(k+1)x + 4) \geq 0 \ \forall x \in R$

$x^2 - 2(k+1)x + 4 \geq 0 \ \forall x \in R$

$\Downarrow$

$D \leq 0, \ a = 1 > 0$

$\Downarrow$

$4(k+1)^2 - 4 \cdot 4 \leq 0$

$(k+1+2)(k+1-2) \leq 0$

$(k+3)(k-1) \leq 0$

$k \in [-3, 1]$ Ans

QUESTION

(ASRQ)

$\{x\} = x - [x]$

$x = \{x\} + [x]$

Number of real roots of the equation $6x - 7[x] = 2$ is

[Note: [k] denotes greatest integer function less than or equal to k]

(A) 6

(B) 5

(C) 8

(D) 7

$6x - 7[x] = 2$

$6(\{x\} + [x]) - 7[x] = 2$

$6\{x\} + 6[x] - 7[x] = 2$

$0 \le \{x\} = \dfrac{2+[x]}{6} < 1$

$0 \le \dfrac{2+[x]}{6} < 1$

$0 \le 2+[x] < 6 \Rightarrow -2 \le [x] < 4$

$0 \le \{x\} < 1$

$[x]$ is an Integer

$[x] = -2, -1, 0, 1, 2, 3$

$\{x\} = 0, \frac{1}{6}, \frac{2}{6}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}.$

$x = -2, -5/6, 1/3, 3/2, 8/3, \frac{23}{6}$

Ans. A

Aao Machay Dhmaal Deh Swaal pe Deh Swaal

QUESTION [JEE Mains 2024 (27 Jan)]

The function $f : N - \{1\} \to N$; defined by $f(n)$ = the highest prime factor of n, is

(A) one-one only

(B) neither one-one nor onto

(C) onto only

(D) both one-one and onto

$f : \{2, 3, 4, ---\} \longrightarrow \{1, 2, 3, 4 ---\}$

$f(n)$ = highest prime factor of n.

$f(33) = 11 = f(44)$

$\Downarrow$

f is many one.

clearly Range of f = prime no:s

$\downarrow$

$R_f \neq N$

$\Downarrow$

Into

Ans. B

QUESTION [JEE Mains 2024 (6 April)]

The function $f(x) = \frac{x^2+2x-15}{x^2-4x+9}, x \in \mathbb{R}$ is

(A) both one-one and onto.

(B) onto but not one-one.

(C) neither one-one nor onto.

(D) one-one but not onto.

$f(x) = \frac{x^2+2x-15}{x^2-4x+9}, x \in R$

$\Downarrow$

Many one i.e not one-one

codomain = R

$f(x)\ y = \frac{x^2+2x-15}{\underbrace{x^2-4x+9}_{D<0 \text{ (no real roots)}}}$ is continuous

$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{1+2/x-15/x^2}{1-4/x+9/x^2} = 1$

$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{1+2/x-15/x^2}{1-4/x+9/x^2} = 1$

Range $\neq R$

$\Downarrow$

Into fn.

Ans. C

QUESTION [JEE Mains 2023 (29 Jan)]

TAHI

Let $f: R \to R$ be a function such that $f(x) = \frac{x^2+2x+1}{x^2+1}$. Then

(A) $f(x)$ is many-one in $(-\infty, -1)$

(B) $f(x)$ is one-one in $(-\infty, \infty)$

(C) $f(x)$ is one-one in $[1, \infty)$ but not in $(-\infty, \infty)$

(D) $f(x)$ is many-one in $(1, \infty)$

Ans. C

QUESTION

The function $f : [2, \infty) \to Y$ defined by $f(x) = x^2 - 4x + 5$ is both one-one and onto if:

(A) $Y = R$

(B) $Y = [1, \infty)$

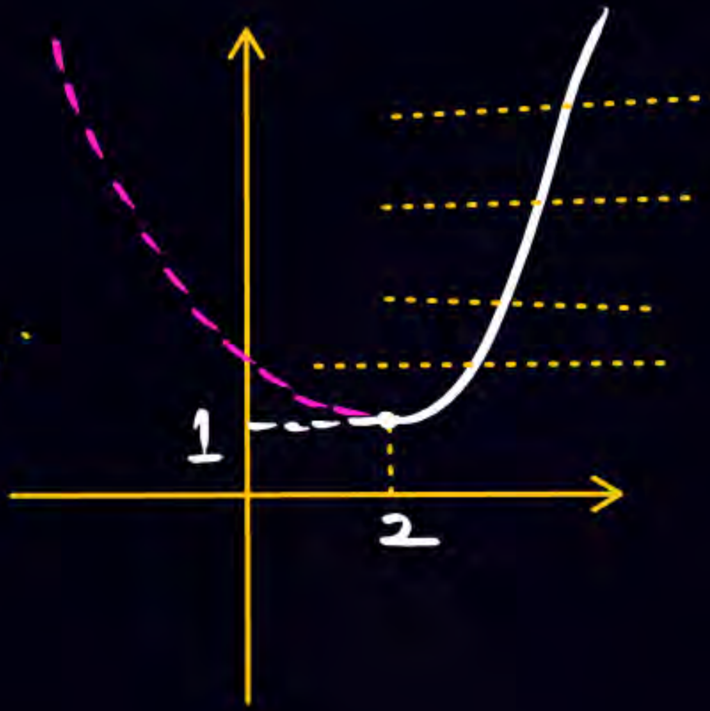
(C) $Y = [4, \infty)$

(D) $[5, \infty)$

$f : [2, \infty) \longrightarrow Y$ — one-one + onto.

Domain $= [2, \infty)$

codomain $= Y =$ Range $\because f$ is onto.



$f(x) = x^2 - 4x + 4 + 1$

$= (x-2)^2 + 1$

$[2, \infty)$

$[0, \infty) \to [0, \infty) \to [1, \infty) =$ Range of $f = Y$

$y = f(x) = \dfrac{ax+b}{cx+d}$ $\left(\dfrac{a}{c} \neq \dfrac{b}{d}\right)$ is always monotonic

Yaani always Inc or dec

$\Downarrow$

one-one

$$\frac{dy}{dx} = \frac{(cx+d)\cdot a - (ax+b)\cdot c}{(cx+d)^2}$$

$$= \frac{(ad-bc)}{(cx+d)^2}$$

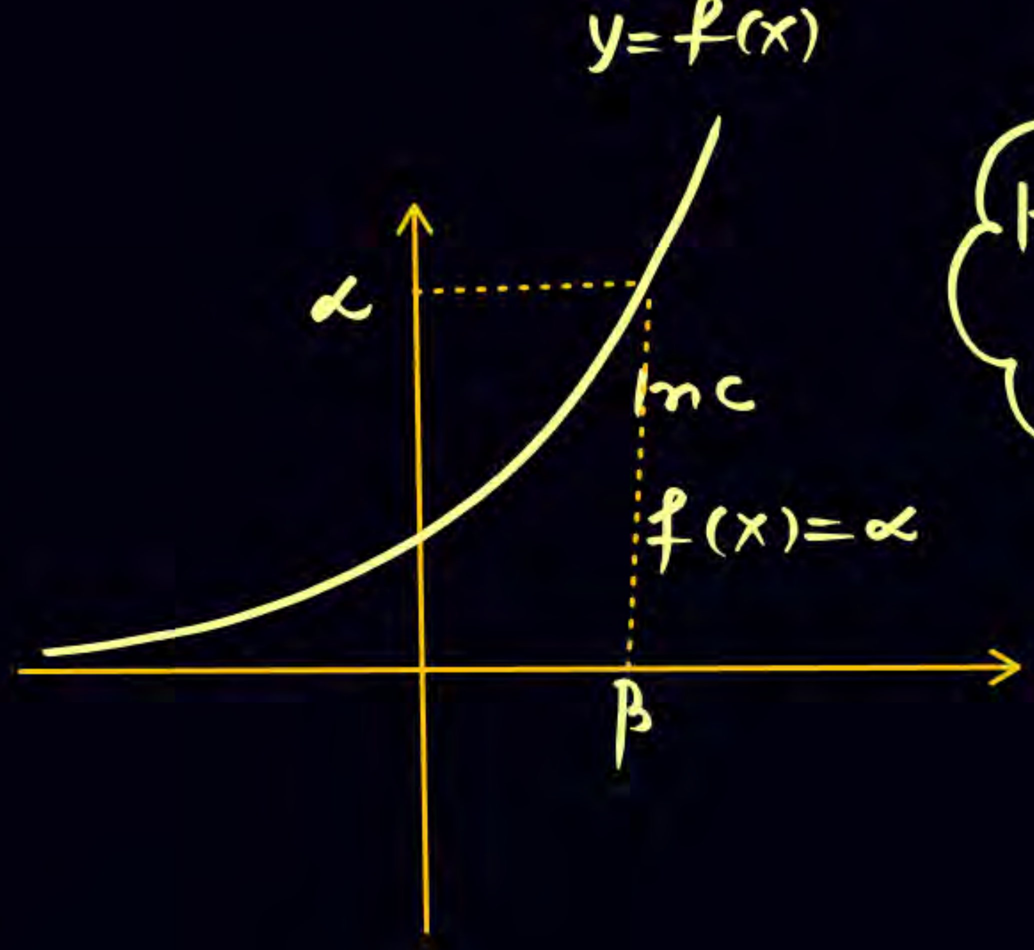
$ad-bc>0 \to$ Inc.

$ad-bc<0 \to$ dec.

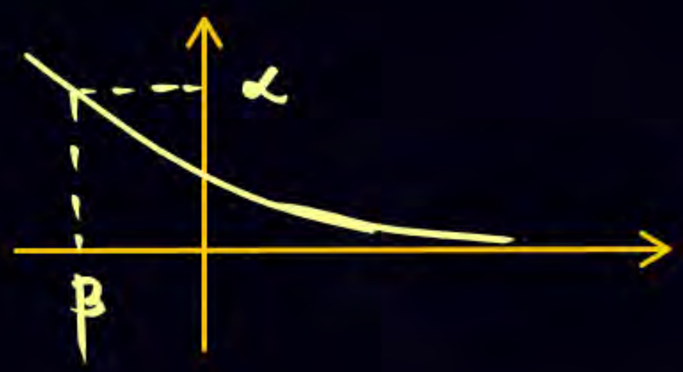
$ad-bc=0$

$ad=bc$.

$\dfrac{a}{c} = \dfrac{b}{d}$ (N.P)



Inc/Dec, fn takes a value at atmost one point



QUESTION [JEE Mains 2019]

Let $A = \{x \in R : x \text{ is not a positive integer}\}$. Define a function $f : A \rightarrow R$ as $f(x) = \frac{2x}{x-1}$, then f is :

(A) neither injective nor surjective

(B) not injective

(C) injective but not surjective

(D) surjective but not injective

$f(x) = \frac{2x}{x-1}$ $\Downarrow$ one-one

$D_f : R - I^+$ $\Downarrow$ Range $\neq R$ $\Downarrow$ Into fn.

QUESTION [JEE Mains 2022 (28 June)]

Tan 2

Let a function $f: N \rightarrow N$ be defined by $f(n) = \begin{bmatrix} 2n, & n = 2, 4, 6, 8, \ldots \\ n-1, & n = 3, 7, 11, 15, \ldots \\ \frac{n+1}{2}, & n = 1, 5, 9, 13, \ldots \end{bmatrix}$ then, f is

(A) one-one but not onto

(B) onto but not one-one

(C) neither one-one nor onto

(D) one-one and onto

QUESTION [IIT-JEE Mains 2012 (Paper 1)]

The function $f : [0, 3] \to [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is

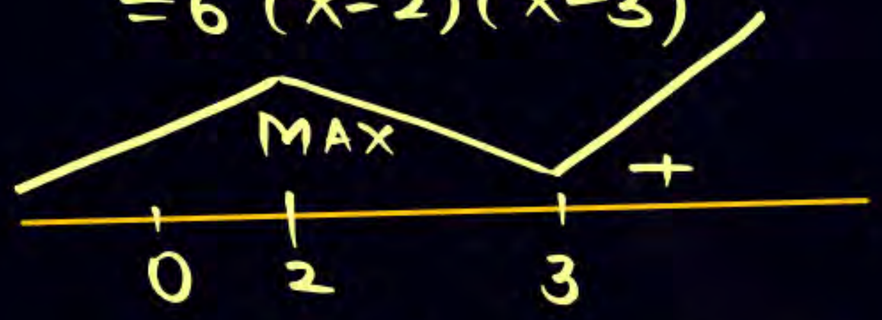
(A) one-one and onto.

(B) onto but not one-one.

(C) one-one but not onto.

(D) neither one-one nor onto.

$$f'(x) = 6x^2 - 30x + 36$$
$$= 6(x^2 - 5x + 6)$$
$$= 6(x-2)(x-3)$$



In $[0,3]$ f has a local MAX at $x=2$

hence is many one

$f(2) = 16 - 60 + 72 + 1 = 29$

$f(0) = 1$, $f(3) = 54 - 135 + 108 + 1 = 28 \Rightarrow f_{min} = 1$

Range $[1, 29]$ = onto

Ans. B

QUESTION

(ASRQ)

Let $f : R \to R$ such that $f(x) = x^3 + 2x^2 + 7x + 5 + 3\sin x - 4\cos x$ be a function then f(x) is

(A) one-one and onto

(B) one-one but not onto

(C) onto but not one-one

(D) neither one-one nor onto

$$f(x) = x^3+2x^2+7x+5+3\sin x-4\cos x$$

$\Downarrow$

Range $= R \Rightarrow$ onto

$$\lim_{x\to\infty} f(x) = \infty, \quad \lim_{x\to-\infty} f(x) = -\infty \quad R_f = (-\infty, \infty)$$

$$f'(x) = \underbrace{3x^2+4x+7}_{\text{MIN Value}} + \underbrace{3\cos x+4\sin x}_{[-5,5]}$$

MIN Value $= -\dfrac{(4^2-4\cdot3\cdot7)}{4\cdot3} = 17/3 = 5.\ldots$

$$f'(x) = \underbrace{3x^2+4x+7}_{>5} + \underbrace{3\cos x + 4\sin x}_{[-5,5]} > 0$$

$$\Downarrow$$

f is one-one.

1) $y = ax^2+bx+c \quad , a>0$

$y_{min} = \frac{-D}{4a}$

2) $y = ax^2+bx+c \, , a<0$

$y_{max} = \frac{-D}{4a}$

QUESTION

(AsRQ)

If numbers of ordered pairs (p, q) from the set S = {1, 2, 3, 4, 5} such that the function $f(x)=\frac{x^3}{3}+\frac{p}{2}x^2+qx+10$ defined from R to R is injective, is n then n is divisible by

(A) ~~3~~

(B) ~~5~~

(C) 7

(D) 11

$\underbrace{f: R\to R}$: $f(x)=\frac{x^3}{3}+\frac{p}{2}x^2+qx+10$ is one-one $\begin{cases}\text{Inc}\\ \text{or}\\ \text{dec}\end{cases}$ on R

$p, q \in \{1,2,3,4,5\}$

$f'(x)=x^2+px+q \begin{cases} \geq 0 \text{ on } R \\ \text{or} \\ \leq 0 \text{ on } R \end{cases}$ — $\underbrace{\text{N.P}}$

$\Downarrow$

$x^2+px+q \geq 0$ on R

$D \leq 0,\ a=1>0$

$p^2-4q \leq 0 \Rightarrow p^2 \leq 4q.$

$$p^2 \le 4q$$

$q=1,\ p=1,2$ — 2 pair $(1,1)(2,1)$

$q=2,\ p=1,2$ — 2 pair $(1,2)(2,2)$

$q=3,\ p=1,2,3$ — 3 pair $(1,3)(2,3)(3,3)$

$q=4,\ p=1,2,3,4$ — 4 pair $(1,4)(2,4)(3,4)(4,4)$

$q=5,\ p=1,2,3,4$ — 4 pair $(1,5)(2,5)(3,5)(4,5)$

15 pairs

QUESTION

(ASRQ)

If functions f(x) and g(x) are defined on R → R such that

$f(x)=\begin{cases}x+3, & x\in \text{rational}\\ 4x, & x\in \text{irrational}\end{cases}$, $g(x)=\begin{cases}x+\sqrt{5}, & x\in \text{irrational}\\ -x, & x\in \text{rational}\end{cases}$, then $(f-g)(x)$ is

A. one-one and onto

B. ~~neither one-one nor onto~~

C. one-one but not onto

D. onto but not one-one

$$(f-g)(x)=\begin{cases}4x-(x+\sqrt{5}) & x\in \text{irrational}\\ x+3-(-x) & x\in \text{rational}\end{cases}$$

$$(f-g)(x)=\begin{cases}3x-\sqrt{5}. & x\in \text{irrational}\\ 2x+3 & x\in \text{rational}\end{cases}$$

$$(f-g)\left(\frac{\sqrt{5}}{3}\right)=3\cdot\left(\frac{\sqrt{5}}{3}\right)-\sqrt{5}=0$$

$$(f-g)(-3/2)=2(-3/2)+3=0$$

$$(f-g)\left(\frac{-3}{2}\right)=(f-g)\left(\frac{\sqrt{5}}{3}\right)$$

$\Downarrow$

Many one.

$$(f-g)(x)=\begin{cases} 3x-\sqrt{5} & x\in \text{irrational} \\ 2x+3 & x\in \text{rational} \end{cases}$$

$2x+3 \longrightarrow$ rational no: as output

clearly $-\sqrt{5}\notin R_{f-g}$

$3x-\sqrt{5}=-\sqrt{5}$

$x=0$ but x should be irrational

(N.P)

$R_{f-g}\neq R$.

QUESTION

Let $f : R \to R$ be defined by

$f(x) = (a^2-1)(a^2-4)x^3 + (a^2-1)(a+2)x^2 + (a+1)(a+2)x + a + 5.$

If $f(x)$ is into then number of possible values of 'a' are

(A) 1

(B) 2

(C) 3

(D) more than 3

$f(x)$ is into $\Rightarrow R_f \neq R$

Every odd degree polynomial on R has Range R

$\Downarrow$

f should not be an odd degree polynomial

$(a^2-1)(a^2-4)=0$

$a=-1,1,2,-2$

$a=-1 \quad f(x)=4$ ✓

$a=1 \quad f(x)=6x+6$ ✗

$a=2 \quad f(x)=12x^2+12x+7$ ✓

$a=-2 \quad f(x)=3$ ✓

Even fn & odd fn

A fn 'f' defined on a symmetric Domain D_f

(i.e if $x \in D_f$ then $-x \in D_f$) is said to be

(i) Even fn : If $f(-x) = f(x) \ \forall \ x \in D_f$

i.e $f(x) - f(-x) = 0 \ \forall \ x \in D_f$

(ii) odd fn : If $f(-x) = -f(x) \ \forall \ x \in D_f$

i.e $f(x) + f(-x) = 0 \ \forall \ x \in D_f$

★ $f(x) = x^2 \quad x \in [-2, 4]$

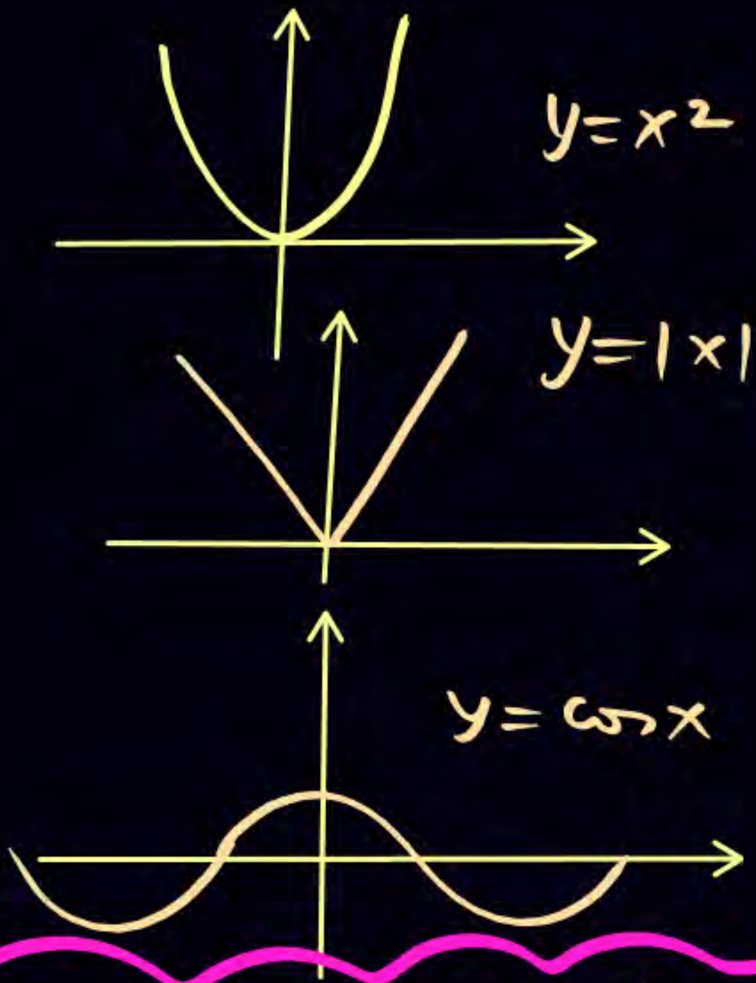
Not even fn.

Ex: $f(x)=x^2,\ x\in R$ (Even fn)

$f(x)=|x|,\ x\in R$, (Even fn)

$f(x)=\cos x,\ x\in R$ (Even fn)

$f(x)=e^x+e^{-x},\ x\in R$ (Even)



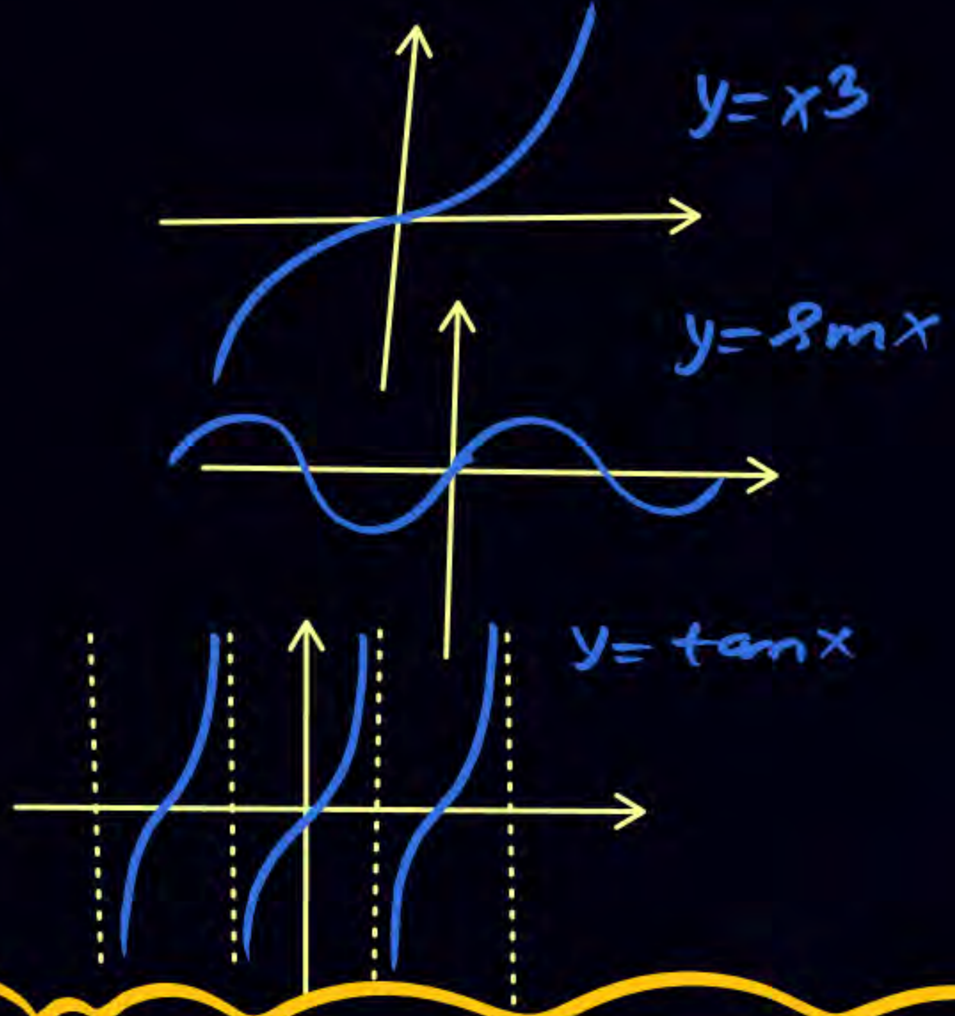
Graphs of Even fn are symmt about y axis.

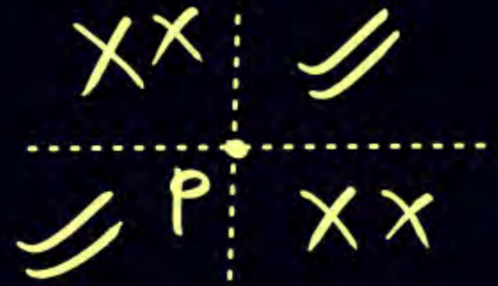
Even fn $\Rightarrow$ Many one

Ex: $f(x) = x^3$ (odd fn)

$f(x) = \sin x$

$f(x) = \tan x$





Graphs of odd fns are symmetric about origin

If f is an odd fn & $0 \in D_f$ then $f(0) = 0$ $\longrightarrow$ $f(-x) = -f(x)$

$f(-0) = -f(0)$

$f(0) = -f(0)$

$2f(0) = 0 \Rightarrow f(0) = 0$

If f is a derivable Even fn then its derivative fn f' is odd fn.

If f is a derivable odd fn then it's derivative f' is Even fn

$f(-x) = -f(x)$ odd fn

Diff. bothsides

$-f'(-x) = -f'(x)$

$f'(-x) = f'(x)$

$\Downarrow$

Even fn.

$f(-x) = f(x)$ Even fn

$-f'(-x) = f'(x)$

$f'(-x) = -f'(x)$ odd fn.

Every fn is not odd or even i.e there are fns which are neither odd nor even Ex: $f(x) = x^3 - x^2$

A fn defined on R which is both odd & even is only zero fn i.e $f(x) = 0$

Odd $\Rightarrow$ $f(-x) = -f(x) \longrightarrow f(x) = -f(-x)$

Even $\Rightarrow$ $f(-x) = f(x) \longrightarrow f(x) = f(-x)$

$2f(x) = 0$ — $f(x) = 0$

Every constant fn defined on a symmetric domain is even.

$f(x)=\lambda$, $f(-x)=\lambda$ $\Rightarrow$ $f(x)=f(-x)$

$\Downarrow$

Even fn

Ex: $f(x)=\operatorname{sgn}(x^2+x+1)=1$ $x\in R$

$\Downarrow$

Even fn

Ex: $f(x)=\dfrac{x-1}{x-1}$ is neither odd nor Even

$f(x)=1$ $x\neq 1$

QUESTION

Tan 4

Find whether the following functions are even or odd or none

(a) $f(x) = \log\left(x + \sqrt{1+x^2}\right)$ $\longrightarrow$ $f(-x) = \ln\left(-x + \sqrt{1+x^2}\right)$

$= \ln\left(\sqrt{1+x^2} - x\right) = \ln\left(\frac{(\sqrt{1+x^2} - x)\cdot(\sqrt{1+x^2} + x)}{(\sqrt{1+x^2} + x)}\right)$

$= \ln\left(\frac{1}{x + \sqrt{1+x^2}}\right) = \ln\left(x + \sqrt{1+x^2}\right)^{-1}$

$= -\ln\left(x + \sqrt{1+x^2}\right) = -f(x)$

$f(-x) = -f(x) \Rightarrow$ odd fn

(b) $f(x) = \frac{x(a^x+1)}{a^x-1}$

(c) $f(x) = \sin x + \cos x$

(d) $f(x) = x\sin^2 x - x^3$

(e) $f(x) = \sin x - \cos x$

(f) $f(x) = \frac{(1+2^x)^2}{2^x}$

(g) $f(x) = \frac{x}{e^x-1} + \frac{x}{2} + 1$

(h) $f(x) = \left[(x+1)^2\right]^{1/3} + \left[(x-1)^2\right]^{1/3}$

(g) $f(x) = \frac{x}{e^x-1} + \frac{x}{2} + 1$

$$f(-x) = \frac{-x}{e^{-x}-1} - \frac{x}{2} + 1 = \frac{-xe^x}{1-e^x} - \frac{x}{2} + 1$$

$$f(x) - f(-x) = \frac{x}{e^x-1} + \frac{x}{2} + \cancel{1} + \frac{xe^x}{1-e^x} + \frac{x}{2} - \cancel{1}$$

$$f(x) - f(-x) = \frac{x}{\cancel{e^x-1}}\left(\cancel{1-e^x}\right) + x = -x + x$$

$$f(x) - f(-x) = 0$$

$f(-x) = f(x)$ — even fn

Any fn f defined on a symmetric Domain can be uniquely written as a sum of an odd fn & an even fn.

Proof: $f(x) = \underbrace{\dfrac{f(x)-f(-x)}{2}}_{g(x)} + \underbrace{\dfrac{f(x)+f(-x)}{2}}_{h(x)}$.

odd fn.

$g(x) = \dfrac{f(x)-f(-x)}{2}$ — $g(-x) = \dfrac{f(-x)-f(x)}{2} = -\left(\dfrac{f(x)-f(-x)}{2}\right) = -g(x)$

$h(x) = \dfrac{f(x)+f(-x)}{2}$ — $h(-x) = \dfrac{f(-x)+f(x)}{2} = h(x)$ — Even fn

QUESTION

$R_f : (0, \infty)$

If $h(x) = Ax^5 + B\sin x + C\ln\left(\frac{1+x}{1-x}\right) + 7$, where A, B, C are non-zero real constants and $h\left(\frac{-1}{2}\right) = 6$, then find the vale of $h\left(\frac{\text{sgn}(e^{-x})}{2}\right)$. $= h\left(\frac{1}{2}\right)$

$h(x) = \cancel{Ax^5} + \cancel{B\sin x} + \cancel{C\ln\left(\frac{1+x}{1-x}\right)} + 7$

$h(-x) = -Ax^5 - B\sin x + C\ln\left(\frac{1-x}{1+x}\right) + 7$

$h(-x) = -Ax^5 - B\sin x + C\ln\left(\frac{1+x}{1-x}\right)^{-1} + 7$

Add

$h(-x) = \cancel{-Ax^5} - \cancel{B\sin x} - \cancel{C\ln\left(\frac{1+x}{1-x}\right)} + 7$

$h\left(\frac{-1}{2}\right) = 6$

$h(1/2) = ?$

$h(x) + h(-x) = 14$

put $x = \frac{1}{2}$, $h(1/2) + h(-1/2) = 14 \Rightarrow h(1/2) = 8$

QUESTION

Jan 5

Suppose that $f(x)$ is a function of the form $f(x) = \frac{ax^8+bx^6+cx^4+dx^2+15x+1}{x} (x \neq 0)$.
If $f(5) = 2$ then the value of $f(-5)$ is equal to

(A) -2

(B) 28

(C) 13

(D) -13

Ans. B

QUESTION

The smallest natural number k for which $f(x) = \ln\left(x^3 + \sqrt{x^6 + 1}\right) + \sin 5x + \left[\frac{x^2}{k}\right]$ is an odd function $\forall\, x \in [-2\pi, 2\pi]$, is ([y] denotes largest integer $\leq$ y)

(A) 38

(B) 39

(C) 40

(D) 41

$f(-x) = -f(x) \quad \forall\, x \in [-2\pi, 2\pi]$

$f(-x) = \ln\left(\sqrt{x^6+1} - x^3\right) - \sin 5x + \left[\frac{x^2}{k}\right]$

$f(-x) = -\ln\left(x^3 + \sqrt{x^6+1}\right) - \sin 5x + \left[\frac{x^2}{k}\right]$

Now $f(-x) = -f(x) \quad \forall\, x \in [-2\pi, 2\pi]$

$\cancel{-\ln\left(x^3+\sqrt{x^6+1}\right)} \cancel{-\sin 5x} + \left[\frac{x^2}{k}\right] = \cancel{-\ln\left(x^3+\sqrt{x^6+1}\right)} \cancel{-\sin 5x} - \left[\frac{x^2}{k}\right]$

$2\left[\frac{x^2}{k}\right] = 0 \Rightarrow \left[\frac{x^2}{k}\right] = 0 \quad \forall\, x \in [-2\pi, 2\pi]$

$$\left[\frac{x^2}{k}\right]=0 \quad \forall \; x \in [-2\pi, 2\pi]$$

$$0 \le \frac{x^2}{k} < 1 \quad \forall \; x \in [-2\pi, 2\pi]$$

$$x^2 \in [0, 4\pi^2] \qquad \pi^2 \approx g$$

$$9.8 \times 4 = 39.2$$

$$k > 4\pi^2$$

$$k_{MIN} = 40$$

Periodic Functions

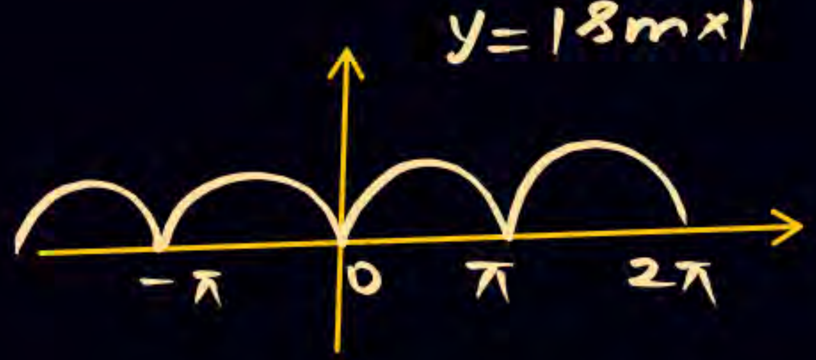
A fn f is said to be periodic if there exists a no: $T>0$ s.t $f(x+T)=f(x)\ \forall\ x\in D_f$

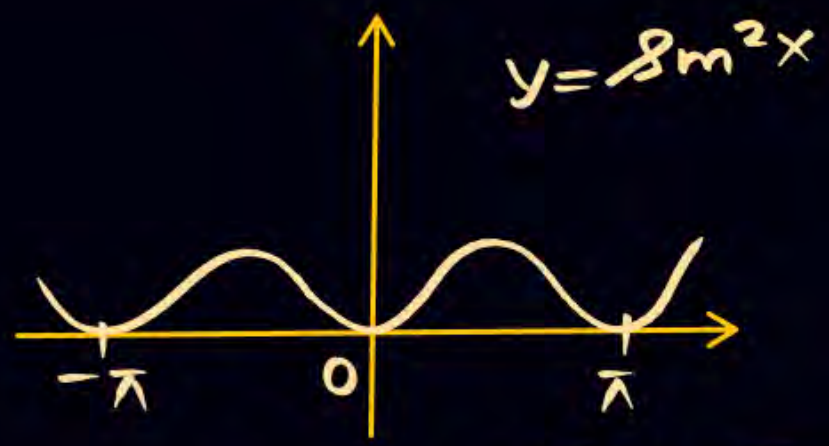
The smallest possible value of $T>0$ satisfying above definition is called fundamental period | period of fn.

Ex: $\sin(x+2\pi)=\sin x$
$\sin(x+4\pi)=\sin x$
$\sin(x+6\pi)=\sin x$
$\vdots \quad \vdots$

$\left.\right\} T=2\pi$

fn	Period
cosec x , sin x	2π
sec x, cos x	2π
cot x, tan x	π
$\{x\}$	1





$|\sin x|, |\cos x|, (\sin x)^{2n}, (\cos x)^{2n}$ have period π

$(\sin x)^{2n+1}, (\cos x)^{2n+1}$ have period 2π — $n \in N$

Graphical Pechaan

All periodic fn have repeating pattern in their Graphs

Sabse Important Baat Yaad Rahe

Sabhi Class Illustrations Retry Karnay hai...

Bumper Practice Problems

Find Range of following rational functions:

(1) $f(x)=\frac{3-2x}{5x-7}$

(2) $f(x)=\frac{x^2-6x+8}{x^2-5x+6}$

(3) $f(x)=\frac{3x-6}{5-2x}$

(4) $f(x)=\frac{(2x-1)(6x-3)}{(5x+2)(2x-1)}$

(5) $f(x)=\frac{x^2-2x+4}{x^2+2x+4}$

(6) $f(x)=\frac{x^2-6x+1}{x^2+6x+1}$

(7) $f(x)=\frac{x^2-5x+4}{x^2+2x-3}$

Answers

(1) $R-\left\{\frac{2}{5}\right\}$

(2) $R-\{1,2\}$

(3) $R-\left\{-\frac{3}{2}\right\}$

(4) $R-\left\{\frac{6}{5},0\right\}$

(5) $\left[\frac{1}{3},3\right]$

(6) $(-\infty,-2]\cup\left[-\frac{1}{2},\infty\right)$

(7) $R-\left\{1,-\frac{3}{4}\right\}$

Today's KTK

$$\text{No Selection} \xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}} \text{Selection with good Rank}$$

Class illustrations

Module,DPP

KTK,TAH CHALLENGER

QUESTION

(KTK 1)

If range of function f(x) whose domain is set of all real numbers is [−2, 4], then range of function $g(x) = \frac{1}{2}f(2x+1)$ is equal to :

(A) [−2, 4]

(B) [−1, 2]

(C) [−3, 9]

(D) [−2, 2]

Ans. B

QUESTION

(KTK 2)

Let two functions $f(x)$ and $g(x)$ are defined on $R \to R$ such that

$$f(x) = \begin{cases} x^2, & x \in \text{ irrational} \\ 2 - x^2, & x \in \text{ rational} \end{cases} \text{ and } g(x) = \begin{cases} 2 - x^2, & x \in \text{ irrational} \\ x^2, & x \in \text{ rational} \end{cases}.$$

Then the function $f + g : R \to R$ is

(A) injective as well as surjective.

(B) injective but not surjective.

(C) surjective but not injective.

(D) neither surjective nor injective.

Ans. D

QUESTION

(KTK 3)

Let $f(x)$ be a real valued function defined on $f : R \to R$ such that $f(x) = [x]^2 + [x+1] - 3$, where $[x] =$ the greatest integer $\leq x$. Then

(A) $f(x)$ is a many-one and into function

(B) $f(x) = 0$ for infinite number of values of x

(C) $f(x) = 0$ for only two real values

(D) none of these

Ans. A, B

QUESTION

(KTK 4)

Let $f(x) = \sqrt{\frac{1}{x^2 + 2\sqrt{c}x + 1}}$. If domain of $f(x)$ is $(-\infty, \infty)$, then the number of integers in the range of 'c' is

(A) 3

(B) 2

(C) 1

(D) 0

Ans. C

QUESTION

(KTK 5)

Classify the following functions as injective, surjective, both or none.

(a) $f : R \to R$, be a function defined by $f(x) = \frac{x^2+4x+30}{x^2-8x+18}$.

(b) $f : R \to R$, be a function defined by $f(x) = x^3 - 6x^2 + 11x - 6$

(c) $f : R \to R$, be a function defined by $f(x) = (x^2 + x + 5)(x^2 + x - 3)$

(d) $f : R \to \{x \in R : -1 < x < 1\}$, be a function defined by $f(x) = \frac{x}{1+|x|}$

(e) $f : [-1, 3] \to [-37, 27]$, be a function defined by $f(x) = 2x^3 - 6x^2 - 18x + 17$

Ans. (a) neither surjective nor injective;
(b) surjective but not injective;
(c) neither injective nor surjective;
(d) injective and surjective;
(e) injective and surjective

QUESTION

(KTK 6)

If domain of $y = f(x)$ is $[-3, 2]$, then domain of $f(|[x]|)$ is equal to
[Note: [k] denotes greatest integer function less than or equal to k]

(A) $[-3, 2]$

(B) $[-2, 3)$

(C) $[-3, 3]$

(D) $[-2, 3]$

Ans. B

QUESTION

(KTK 7)

Find domain of $f(x) = \sqrt{\log_{1/3}\left(\log_4([x]^2 - 5)\right)}$ (Where $[\cdot]$ denotes G.I.F.)

Ans. $[-3, -2) \cup [3, 4)$

QUESTION

(KTK 8)

Let $f(x) = \sqrt{\log_2\left(\frac{10x-4}{4-x^2}\right) - 1}$. Then sum of all integers in domain of $f(x)$ is

(A) -15

(B) -16

(C) -17

(D) -18

Ans. C

QUESTION

(KTK 9)

The domain of the function, $f(x) = \frac{\sqrt{\sin x}}{\sqrt{(x-2)(8-x)}}$ is

(A) $[0, \pi] \cup [2\pi, 8)$

(B) $(2, \pi] \cup [2\pi, 8)$

(C) $(2, 8)$

(D) $(0, 8)$

Ans. B

QUESTION

(KTK 10)

The domain of the function $f(x) = \sqrt{10 - \sqrt{x^4 - 21x^2}}$ is

(A) $[5, \infty)$

(B) $[-\sqrt{21}, \sqrt{21}]$

(C) $[-\sqrt{5}, \sqrt{21}] \cup [\sqrt{21}, \sqrt{5}] \cup \{0\}$

(D) $(-\infty, -5)$

QUESTION

(KTK 11)

Find the domain $f(x) = \dfrac{1}{\sqrt{|[|x|-5]|-11}}$ where [.] denotes greatest integer function.

Homework from Module

Chapter: FUNCTIONS

Prarambh: Try Domain & Range Problems.

Prabal :

(Revision Practice Problems)

QUESTION

RPPI

Let $\sum_{n=1}^{\infty}\left(\frac{n}{n^4+4}\right)=\frac{p}{q}$,

where p & q are coprime natural numbers then $|2p-q|$ is equal to

(A) 3

(B) 2

(C) 8

(D) 9

Ans. B

QUESTION

RPP2

For the series,

$$S = 1 + \frac{1}{(1+3)}(1+2)^2 + \frac{1}{(1+3+5)}(1+2+3)^2 + \frac{1}{(1+3+5+7)}(1+2+3+4)^2 + \cdots$$

(A) 7^{th} term is 16

(B) 7^{th} term is 18

(C) Sum of first 10 terms is $\frac{405}{4}$

(D) Sum of first 10 terms is $\frac{505}{4}$

Ans. A, D

QUESTION

RPP3

Let $a_n, n \geq 1$, be an arithmetic progression with first term 2 and common difference 4 , Let M_n be the average of the first n terms. Then the sum $\sum_{n=1}^{10} M_n$ is

(A) 110

(B) 335

(C) 770

(D) 1100

Ans. A

Solutions

Bumper Practice Questions

Find the Domain of Definition of the Given Functions

(i) $y = \sqrt{-px}(p > 0)$

(ii) $y = \frac{1}{x^2+1}$

(iii) $y = \frac{1}{x^3-x}$

(iv) $y = \frac{1}{\sqrt{x^2-4x}}$

(v) $y = \sqrt{x^2 - 4x + 3}$

(vi) $y = \frac{x}{\sqrt{x^2-3x+2}}$

(vii) $y = \sqrt{1 - |x|}$

(viii) $y = \log_x 2$

(ix) $y = \frac{1}{\log_{10}(1-x)} + \sqrt{x+2}$

(x) $y = \sqrt{x} + \sqrt[3]{\frac{1}{x-2}} - \log_{10}(2x - 3)$

(xi) $y = \frac{3}{4-x^2} + \log_{10}(x^3 - x)$

(xii) $y = \frac{1}{\sqrt{\sin x}} + \sqrt[3]{\sin x}$

(xiii) $y = \log_{10}(\sqrt{x-4} + \sqrt{6-x})$

(xiv) $y = \log_{10}[1 - \log_{10}(x^2 - 5x + 16)]$

Answers

(i) $-\infty < x \leq 0$

(ii) $x \in R$

(iii) $x \in R - \{-1, 0, 1\}$

(iv) $-\infty < x < 0 \;\&\; 4 < x < \infty$

(v) $-\infty < x \leq 1$ and $3 \leq x < \infty$

(vi) $-\infty < x < 1$ and $2 < x < \infty$

(vii) $-1 \leq x \leq 1$

(viii) $0 < x < 1$ and $1 < x < \infty$

(ix) $-2 \leq x < 0$ and $0 < x < 1$

(x) $\frac{3}{2} < x < 2$ and $2 < x < \infty$

(xi) $-1 < x < 0$ and $1 < x < 2; 2 < x < \infty$

(xii) $2k\pi < x < (2k+1)\pi$, where k is an integer.

(xiii) $4 \leq x \leq 6$

(xiv) $2 < x < 3$

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from
Madhya Pradesh

Bumper Practice Question.

Find the Domain of Definition of the Given functions.

(i) $y=\sqrt{-Px}$ $(P>0)$

⇒ $-Px \ge 0$

$x \le 0$

Domain : $(-\infty,0]$

(ii) $y=\frac{1}{x^2+1}$

Domain : R

(iii) $y=\frac{1}{x^3-x}$

$x^3-x\neq0$

$x(x^2-1)\neq0$

$x(x-1)(x+1)\neq0$

⇒ $x\neq0,\ x\neq-1,\ x\neq1$

Domain : $R-\{-1,0,1\}$

(iv) $y=\frac{1}{\sqrt{x^2-4x}}$

⇒ $x^2-4x>0$

$x(x-4)>0$

$x\in(-\infty,0)\cup(4,\infty)$

Domain : $(-\infty,0)\cup(4,\infty)$

(v) $y=\sqrt{x^2-4x+3}$

⇒ $x^2-4x+3\ge0$

$(x-1)(x-3)\ge0$

Domain : $[-\infty,1]\cup[3,\infty)$

(vi) $y=\frac{x}{\sqrt{x^2-3x+2}}$

⇒ $x^2-3x+2>0$

$(x-2)(x-1)>0$

$(-\infty,1)\cup(2,\infty)$

Domain : $(-\infty,1)\cup(2,\infty)$

(vii) $y=\sqrt{1-|x|}$

⇒ $1-|x|\ge0$

⇒ $|x|\le1$

$x\in[-1,1]$

Domain : $[-1,1]$

(viii) $y=\log_x 2$

⇒ $x>0,\ x\neq1$

Domain : $(0,\infty)-\{1\}$

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from
Madhya Pradesh

(ix) $y=\frac{1}{\log_{10}(1-x)}+\sqrt{x+2}$

⇒ $\log_{10}(1-x)\neq0$, $1-x>0$, $x+2\ge0$

$1-x\neq1$ $\quad x<1$ $\quad x\ge-2$

$x\neq0$

$x\in[-2,1)$

Domain : $[-2,1)$

(x) $y=\sqrt{x}+\sqrt[3]{\frac{1}{x-2}}-\log_{10}(2x-3)$

⇒ $x\ge0$, $x-2\neq0$ $\quad 2x-3>0$

$x\neq2$ $\quad x=3/2$

$x\in(\frac{3}{2},\infty)-\{2\}$

Domain : $(\frac{3}{2},\infty)-\{2\}$

(xi) $y=\frac{3}{4-x^2}+\log_{10}(x^3-x)$

⇒ $4-x^2\neq0$ $\quad x^3-x>0$

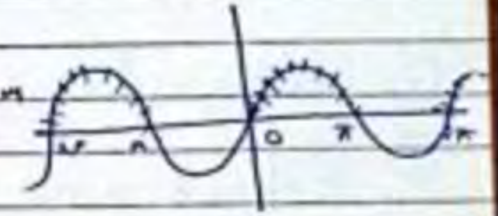
$x^2\neq4$ $\quad x(x^2-1)>0$

$x=\pm2$ $\quad x(x-1)(x+1)>0$

$x\in(-1,0)\cup(1,\infty)$

Domain : $(-1,0)\cup(1,\infty)-\{2\}$

(xii) $y=\frac{1}{\sqrt{\sin x}}+\sqrt[3]{\sin x}$



$\sin x>0$

$x\in(2n\pi,2n\pi+\pi)$

Domain : $(2n\pi,2n\pi+\pi)$

(xiii) $y=\log_{10}(\sqrt{x-4}+\sqrt{6-x})$

⇒ $\sqrt{x-4}\ge0$, $\sqrt{6-x}\ge0$

$x-4\ge0$ $\quad 6-x\ge0$

$x\ge4$ $\quad x\ge6$

$x\in[4,6]$

Domain : $[4,6]$

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PAGE NO.
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(xiv) $y = \log_{10}\left[1 - \log_{10}(x^2 - 5x + 16)\right]$

$x^2 - 5x + 16 \geq 0$

$a > 0,\ D < 0$

alway +ve.

$\log_{10}(x^2 - 5x + 16) > 0$

$\log_{10}(x^2 - 5x + 16) < 1$

$x^2 - 5x + 6 < 10$

$x^2 - 5x + 6 < 0$

$(x - 3)(x - 2) < 0$

$x \in (2, 3)$

Domain : $(2, 3)$

Bumper Practice Questions

Find the range of the following functions :

(i) $f(x) = \frac{x-1}{x+2}$

(ii) $f(x) = \frac{2}{x}$

(iii) $f(x) = \frac{1}{x^2-x+1}$

(iv) $f(x) = \frac{x^2-x+1}{x^2+x+1}$

(v) $f(x) = e^{(x-1)^2}$

(vi) $f(x) = x^3 - x^2 + x + 1$

(vii) $f(x) = \log\left(x^8 + x^4 + x^2 + 1\right)$

(viii) $f(x) = \sin^2 x - 2\sin x + 4$

(ix) $f(x) = \sin(\log_2 x)$

(x) $f(x) = 2^{x^2} + 1$

(xi) $f(x) = \frac{e^{2x}-e^{x}+1}{e^{2x}+e^{x}+1}$

(xii) $f(x) = \frac{1}{8-3\sin x}$

Answers

(i) $R - \{1\}$

(ii) $R - \{0\}$

(iii) $\left(0, \frac{4}{3}\right]$

(iv) $\left[\frac{1}{3}, 3\right]$

(v) $[1, \infty)$

(vi) R

(vii) $[0, \infty)$

(viii) $[3, 7]$

(ix) $[-1, 1]$

(x) $[2, \infty)$

(xi) $\left[\frac{1}{3}, 1\right)$

(xii) $\left[\frac{1}{11}, \frac{1}{5}\right]$

Kalpana

Bumper Practice Questions

Find the range of following functions.

(i) $f(x) = \frac{x-1}{x+2}$

Range $= R - \{1\}$

(ii) $f(x) = \frac{2}{x}$

$x \in R - \{0\}$

(iii) $f(x) = \frac{1}{x^2 - x + 1}$

$\frac{1}{x^2 - x + \frac{1}{4} - \frac{1}{4} + 1}$

$\frac{1}{\left(x - \frac{1}{2}\right)^2 + \frac{3}{4}}$

$[0, \infty)$

$\left[\frac{3}{4}, \infty\right) \to \left(0, \frac{4}{3}\right]$

$x \in \left(0, \frac{4}{3}\right]$

(iv) $f(x) = \frac{x^2 - x + 1}{x^2 + x + 1}$ → always +ve

$y = \frac{x^2 - x + 1}{x^2 + x + 1}$

$y \neq 1$

$x^2(y-1) + x(y+1) + y - 1 = 0$

$D \geq 0$

$y^2 + 2y + 1 - 4(y^2 - 2y + 1) \geq 0$

$-3y^2 + 10y - 3 \geq 0$

$3y^2 - 10y + 3 \cancel{\geq 0} \leq 0$

$3y^2 - 9y - y + 3 \cancel{\geq 0} \leq 0$

$(3y-1)(y-3) \cancel{\geq 0} \leq 0$

$y \in \left[\frac{1}{3}, 3\right] - \{1\}$, $y \in \left[\frac{1}{3}, 3\right]$

$y = 1$ $\quad 2x = 0$

$x = 0$ (possible)

(v) $f(x) = e^{(x-1)^2}$

$e^{(x-1)^2} \to [0,\infty)$

$[e^0, e^\infty)$

$\quad \shortparallel$

$[1, \infty)$

(vi) $f(x) = x^3 - x^2 + x + 1$

$x^3 - x^2 + x + 1$

(Odd degree Polynomial)

Range $= R$.

(vii) $f(x) = \log(x^6 + x^4 + x^2 + 1)$

$\Rightarrow y = \log(\underbrace{\underbrace{x^6+x^4+x^2}_{\geq 0}+1}_{\geq 1})$

$\Rightarrow R_f = [\log 1, \log \infty)$.

$\Rightarrow R_f = [0, \infty)$.

(viii) $f(x) = \sin^2 x - 2\sin x + 4$

$\Rightarrow y = \sin^2 x - 2\sin x + 4$

$y = (\sin x - 1)^2 + 3$.

$[-1, 1]$

$[-2, 0]^2$

$[0, 4] + 3$.

$\Rightarrow R_f = [3, 7]$.

(ix) $f(x) = \sin(\log_2 x)$

$\Rightarrow \therefore R_f = [-1, 1]$

(x) $f(x) = 2^{x^2} + 1$

$\Rightarrow y = 2^{x^2} + 1 \Rightarrow y - 1 = 2^{x^2}$

$= \log_2(y-1) = x^2 \geq 0$.

$\Rightarrow (y-1) > 0 \Rightarrow y > 1$.

$= \log_2(y-1) \geq 0 \Rightarrow y - 1 \geq 1$

$\Rightarrow y \in (2, \infty)$.

(xi) $f(x) = \dfrac{1}{8 - 3\sin x}$

$\Rightarrow y = \dfrac{1}{8 - 3\sin x}$

$[-1, 1]$

$[-3, 3]$

$[5, 11]$.

$\left[\frac{1}{11}, \frac{1}{5}\right]$ Ans

xi) $f(x) = \dfrac{e^{2x} - e^{x} + 1}{e^{2x} + e^{x} + 1}$

$= 1 - \dfrac{2}{e^{x} + \frac{1}{e^{x}} + 1}$

$e^{x} + \frac{1}{e^{x}} \to [2, \infty)$

$e^{x} + \frac{1}{e^{x}} + 1 \to [3, \infty)$

$\to \left(0, \frac{1}{3}\right]$

$\to \left(0, \frac{2}{3}\right]$

$= \cancel{\left(1, \frac{1}{3}\right]}$

$\left[\frac{1}{3}, 1\right)$

xii) $f(x) = \dfrac{1}{8 - 3\sin x}$

$8 - 3\sin x \to [5, 11]$

$\to \left[\frac{1}{11}, \frac{1}{5}\right]$

$\sin x \in [-1, 1]$

$3\sin x \in [-3, 3]$ $\quad -3\sin x \in [-3, 3]$

$8 - 3\sin x \in [5, 11]$


THANK
YOU

PRAYAS

JEE 2025

Lecture- 11 | **Mathematics**

Relation & Functions

By- Ashish Agarwal Sir (IIT Kanpur)

Topics *to be covered*

1. Periodic Functions
2. Functional Equations

QUESTION

(ASRQ)

$f : R \to R$ is defined as $f(x) = \begin{cases} x^2 + 2mx - 1 & \text{for } x \le 0 \\ mx - 1 & \text{for } x > 0 \end{cases}$.

If $f(x)$ is one-one then $\overset{m}{\wedge}$ must lies in the interval

parabola $x_v = -m$

$y = f(x)$: $y = x^2 + 2mx - 1$, $x \le 0$; $y = mx - 1$, $x > 0$.

(A) $(-\infty, 0)$

(B) $(-\infty, 0]$

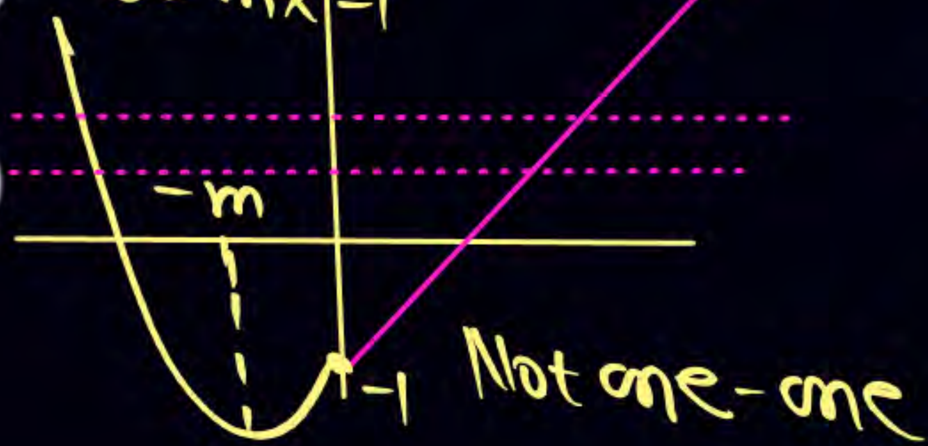
(C) $(0, \infty)$

(D) $[0, \infty)$

Case (I) $m > 0$ $\Rightarrow m < 0$

$y = x^2 + 2mx - 1$ $\quad y = mx - 1$

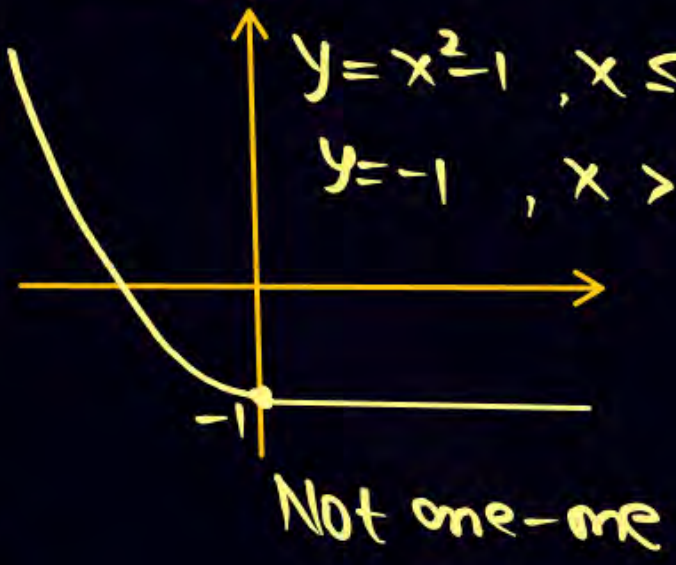
$y = mx - 1$



Case (II) $m = 0$

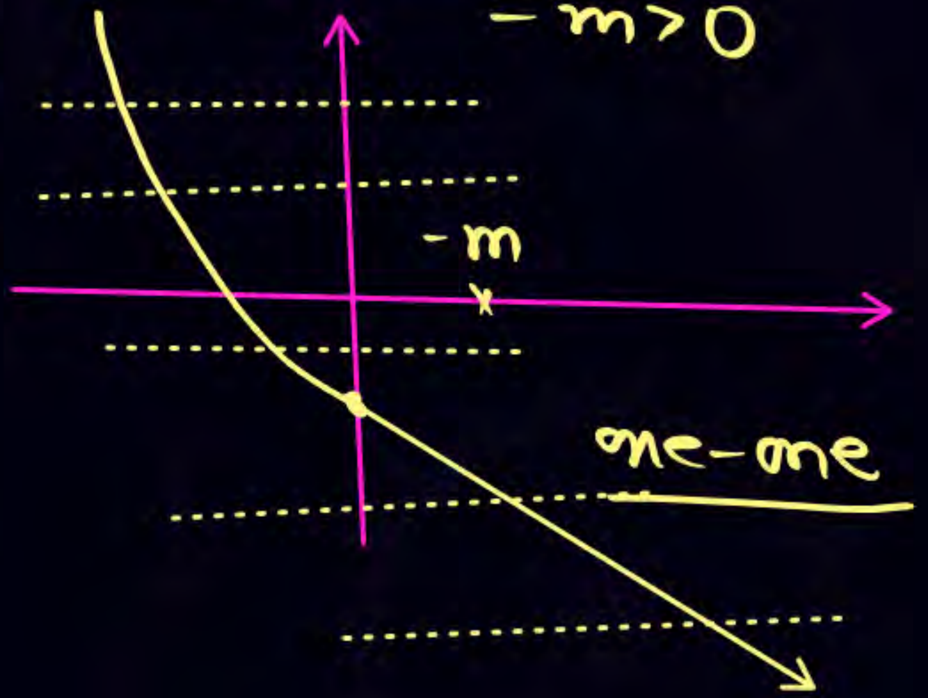
$y = x^2 - 1, \; x \le 0$

$y = -1, \; x > 0$



Case (III) $m < 0$

$-m > 0$



QUESTION

(KCLS)

codomain = Range b'coz f is onto

Let $f: R \to [1, \infty)$ be defined as $f(x) = \log_{10}\left(\sqrt{3x^2 - 4x + k + 1} + 10\right)$. If $f(x)$ is surjective, then

(A) $k = 1/3$

(B) $k < 1/3$

(C) $k > 1/3$

(D) $k = 1$

$\log_{10}\left(\sqrt{3x^2-4x+k+1}+10\right)$ has Range $[1, \infty)$

$\sqrt{3x^2-4x+k+1}+10$ has Range $[10^1, 10^\infty)$

$\sqrt{3x^2-4x+k+1}$ has Range $[0, \infty)$.

$y = ax^2+bx+c,\ a>0$

Range: $\left[-\frac{D}{4a}, \infty\right)$

$(3x^2-4x+k+1)$ has Range $[0, \infty)$

$\Downarrow$

$\left[-\frac{D}{4a}, \infty\right) = [0, \infty)$

$-\frac{D}{4\cdot 3} = 0 \Rightarrow D = 0 \Rightarrow 16 - 12(k+1) = 0 \Rightarrow k = 1/3.$

Gadho/Gadhiyo aisaa naa karo

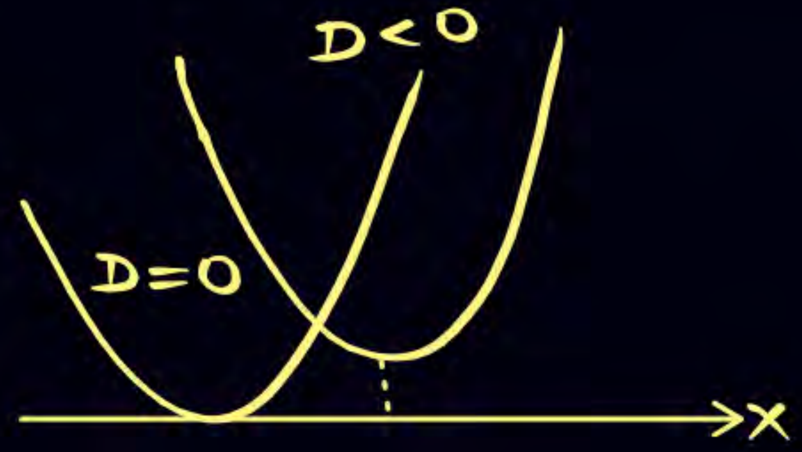
$3x^2-4x+1$ has Range $[0,\infty)$

$3x^2-4x+1\geq 0 \ \forall x\in R$

$D\leq 0$

$\Downarrow$

$k\in\left(-\infty,\frac{1}{3}\right]$



QUESTION

(ASRQ)

Prove that $f(x)=\frac{2x(\sin x+\tan x)}{2[2+(x/\pi)]-3}$ is always odd. ([.] denotes G.I.F)

$$f(x)=\frac{2x(\sin x+\tan x)}{2\left(2+\left[\frac{x}{\pi}\right]\right)-3}=\frac{2x(\sin x+\tan x)}{4-3+2\left[\frac{x}{\pi}\right]}=\frac{2x(\sin x+\tan x)}{1+2\left[\frac{x}{\pi}\right]}$$

$$f(x)=\frac{2x(\sin x+\tan x)}{1+2\left[\frac{x}{\pi}\right]}$$

$$f(-x)=\frac{-2x(-\sin x-\tan x)}{1+2\left[\frac{-x}{\pi}\right]}$$

$$f(-x)=\frac{2x(\sin x+\tan x)}{1+2[-x/\pi]}$$

Observe!! $f(x)\Big|_{x=n\pi}=0$

$$f(-x) = \frac{2x(\sin x + \tan x)}{1 + 2\left[-\frac{x}{\pi}\right]}$$

we know

$$\left[\frac{x}{\pi}\right] + \left[\frac{-x}{\pi}\right] = \begin{cases} -1 & \frac{x}{\pi} \neq n \\ 0 & \frac{x}{\pi} = n \end{cases}$$

$$\left[\frac{x}{\pi}\right] + \left[\frac{-x}{\pi}\right] = \begin{cases} -1 & x \neq n\pi \\ 0 & x = n\pi \end{cases}$$

$n \in I$

$$\left[\frac{-x}{\pi}\right] = \begin{cases} -1 - \left[\frac{x}{\pi}\right] & x \neq n\pi \\ -\left[\frac{x}{\pi}\right] & x = n\pi \end{cases}$$

$$f(-x) = \begin{cases} \dfrac{2x(\sin x + \tan x)}{1 + 2\left(-1 - \left[\frac{x}{\pi}\right]\right)} & x \neq n\pi \\ \dfrac{0}{1 - 2\left[\frac{x}{\pi}\right]} = 0 & x = n\pi \end{cases}$$

$$f(-x) = \begin{cases} \dfrac{2x(\sin x + \tan x)}{-\left(1 + 2\left[\frac{x}{\pi}\right]\right)} & x \neq n\pi \\ 0 & x = n\pi \end{cases} = \begin{cases} -f(x) & x \neq n\pi \\ -f(x) & x = n\pi \end{cases} = -f(x)\ \forall x \in R$$

$\Downarrow$

f is odd fn.

Periodic Functions

$\sec x, \operatorname{cosec} x, \cos x, \sin x, (\sin x)^{2n+1}, (\cos x)^{2n+1} \longrightarrow 2\pi$

$|\sin x|, |\cos x|, \tan x, \cot x$
$(\sin x)^{2n}, (\cos x)^{2n} \longrightarrow \pi$

If $0 \in D_f$ & f is peridic with period T then

$f(-T) = f(-T+T)$.

$f(-T) = f(0) = f(0+T)$

$f(-T) = f(0) = f(T)$

Discontinuities of a periodic fn also repeat periodically with same period as that of fn.

Ex: $f(x) = \tan x$ — $T = \pi$

discontinuous at $x = \ldots, \frac{-3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2} \ldots$

$\pi \quad \pi \quad \pi$

Ex: $f(x) = \{x\}$ — $T = 1$

discontinuous at $x = \ldots -2, -1, 0, 1, 2 \ldots$

$1, 1, 1, 1$

★ if $f(x), g(x)$ are periodic fns with period T_1 & T_2 resp.

then period of $f(x) \pm g(x)$, $f(x) \cdot g(x)$, $\frac{f(x)}{g(x)}$

is $T = \text{LCM}(T_1, T_2)$ provided there does not exist any +ve real no: $< T$ which acts as period.

$T_1 = 2$ sec $\quad$ 3 sec $= T_2$

Tillu Lallu kallu

$T = 6$ sec

$\Downarrow$

$T = \text{LCM}(2, 3) = 6$.

★ $\text{LCM}\left(\underbrace{\frac{p}{q}, \frac{r}{s}}_{\text{Rational No:s}}\right) = \frac{\text{LCM}(p, r)}{\text{HCF}(q, s)}$

★ LCM of Rational with irrational is not possible

★ LCM of Irrational with irrational may or may not be possible.

Ex: $\text{Lcm}(24, 56) = 7\times3\times8$ $\begin{cases} \dfrac{7\times3\times8}{24} = 7 \\ \dfrac{\cancel{7}\times3\times8}{56} = 3 \end{cases}$

sabse chhota +ve no: jo dono se divide hojayay.

Ex: $\text{Lcm}(3, \pi) = 3\pi$

$\dfrac{3\pi}{3} = \pi \notin I$

Ex: $\text{Lcm}(3\pi, 4\pi) = 12\pi$

$\dfrac{12\pi}{3\pi} = 4$, $\dfrac{12\pi}{4\pi} = 3$

Ex: $\text{Lcm}(\pi, e) = \pi e$

$e\pi/\pi = e \notin I$

Ex: $f(x) = |\sin x| + |\cos x|$

$T_1 = \pi$ $\quad$ $T_2 = \pi$

$\text{LCM}(T_1, T_2) = \pi$

$\Downarrow$

period $= \pi$

$$f(\pi/2 + x) = |\sin(\pi/2 + x)| + |\cos(\pi/2 + x)|$$

$$= |\cos x| + |-\sin x|$$

$$= |\cos x| + |\sin x| = f(x) \rightarrow T = \pi/2$$

Here fundamental period is not $\text{LCM}(T_1, T_2)$ b'coz we got a no: $\pi/2 < \text{LCM}$ which acts as period

★ If $f(x), g(x)$ are both periodic then $f(x)+g(x)$ may not be period.

Ex: $f(x) = \sin x$ — $T_1 = 2\pi$

$g(x) = \{x\}$ — $T_2 = 1$

$f(x)+g(x) = \sin x + \{x\}$ — Aperiodic OR non periodic

$\mathrm{lcm}(\pi, 1)$ is not possible

$\pi \notin Q$, $1 \in Q$

Golden point

★ If $y = f(x)$ has period T then $y = f(ax+b)$ has period $\frac{T}{|a|}$

proof: case① if $a>0$

$f(a(x+T')+b) = f(ax+b)$

$f(ax+b+aT') = f(ax+b)$

let $ax+b = t$

$f(aT'+t) = f(t)$ — period of $f = aT'$

$aT' = T$

$T' = T/a$

case⑪ if $a<0$.

$f(a(x-T')+b) = f(ax+b)$

$f(ax+b-aT') = f(ax+b)$

$f(t-aT') = f(t)$

$\Downarrow$

period of $f = -aT' = T$

$T' = \frac{T}{-a}$.

$T' = \frac{T}{a}, a>0$

$T' = \frac{T}{-a}, a<0$

$T' = \frac{T}{|a|}$

$$f(x) \rightarrow T$$

$$f(ax+b) \rightarrow \frac{T}{|a|}$$

$$\frac{T}{|\text{coeff of } x|}$$

$$f(x) = \sin(2x) \Rightarrow T = \pi$$

$$f(x) = \{3x\} \Rightarrow T = \frac{1}{3}$$

$$f(x) = \cos(3-5x) \Rightarrow T = \frac{2\pi}{5}$$

$$f(x) = |\sin(3x-7)| \Rightarrow T = \frac{\pi}{3}$$

$$f(x) = |\sin 2x| + |\cos 2x| \Rightarrow T = \pi/4$$

$$f(x+nT)=f(x) \quad , n\in I$$

Fundamental period T

Other possible periods

$2T, 3T, 4T, \ldots$

$\Downarrow$

If T is fundamental period then nT, $n\in N$ is also a period i.e natural multiples of T are also a period of T

$$f(x+T)=f(x)$$

$$f(x+2T)=f((x+T)+T)=f(x+T)=f(x)$$

$$f(x+3T)=f((x+2T)+T)=f(x+2T)=f(x)$$

$$f(x-T)=f(x-T+T)=f(x)$$

$$f(x-2T)=f(x-2T+2T)=f(x)$$

NOTE :

i. Odd powers of $\sin x, \cos x, \sec x, \operatorname{cosec} x$ are periodic with period 2π.

ii. None zero integral powers of $\tan x, \cot x$ are periodic with period π.

iii. None zero even powers or modulus of $\sin x, \cos x, \sec x, \operatorname{cosec} x$ are periodic with period π.

iv. $f(T) = f(0) = f(-T)$, where ' T ' is the period.
If $f(x)$ has a period T then $f(ax + b)$ has a period $T/|a| (a \neq 0)$.

v. If $f(x)$ & $g(x)$ are periodic with period T_1 & T_2 respectively, then period of $f(x) \pm g(x), f(x) \cdot g(x), f(x)/g(x)$ is L.C.M. of (T_1, T_2).

(a) LCM of T_1 & T_2 is defined when T_1/T_2 is rational.

(b) LCM of $\left\{\frac{a}{b}, \frac{p}{q}\right\} = \frac{\text{LCM of }(a,p)}{\text{HCF of }(b,q)}$

Kuch Kaam ki Baatien

- If $f(x)$ has a period T & $g(x)$ also has a period T then it does not mean that $f(x) + g(x)$ must have a period T.
  e.g. $f(x) = |\sin x| + |\cos x|; \sin^4 x + \cos^4 x$

- Every constant function which is continuous $\forall\, x \in R$ is always periodic, with no fundamental period.

- If $f(x)$ has a period p, then $\frac{1}{f(x)}$ and $\sqrt{f(x)}$ also has a period p.

- If $f(x)$ and $g(x)$ are periodic then $f(x) + g(x)$ need not be periodic.
  e.g. $f(x) = \cos x$ and $g(x) = \{x\}$ [T_1 = irrational, T_2 = rational

QUESTION

$\frac{2\pi}{2/3} = 3\pi$

Find the period of the following function.

(a) $f(x) = \cos\frac{2x}{3} - \sin\frac{4x}{5}$ $\quad \frac{2\pi}{4/5} = \frac{5\pi}{2}$ $\quad T = \text{lcm}\left(\frac{3\pi}{1}, \frac{5\pi}{2}\right)$, $T = \frac{15\pi}{1}$ — Ans. 15π

(b) $f(x) = \cos(\sin x)$ — Ans. π

(c) $f(x) = \sin(\cos x)$ — Ans. 2π

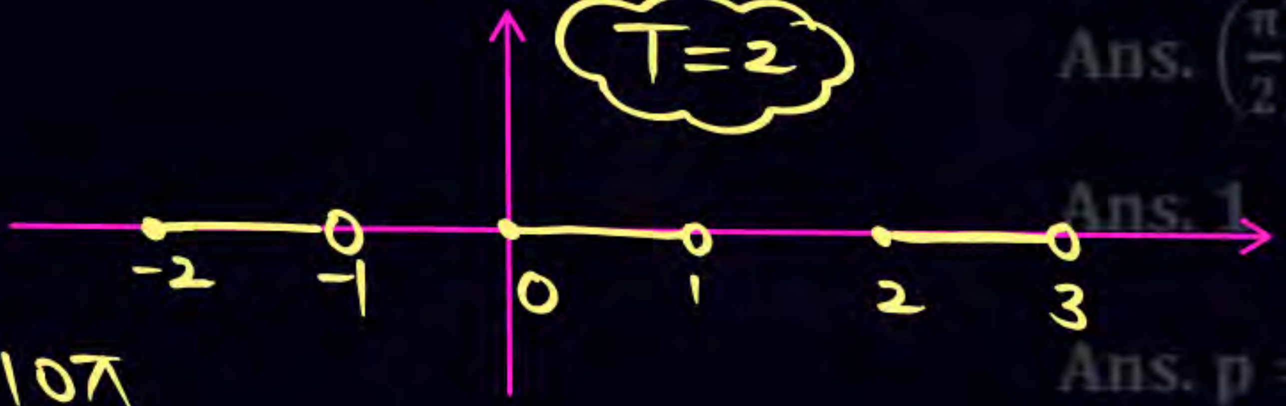
(d) $f(x) = \sin^4 x + \cos^4 x$ — Ans. $\left(\frac{\pi}{2}\right)$

(e) $f(x) = x - [x] = \{x\}$ $\quad T = 1$ — Ans. 1

(f) $f(x) = 2\cos\left(\frac{x-\pi}{5}\right)$ $\quad T = \frac{2\pi}{1/5} = 10\pi$ — Ans. $p = 10\pi$

(i) $y = \tan\left(\frac{\pi}{2}[x]\right)$

$T = 2$



(g) (i) $\tan\left(\frac{\pi}{2}[x]\right) \to 2$; (ii) $\tan\left(\frac{\pi}{4}[x]\right) \to 4$;

(iii) $\sin\left(\frac{\pi}{2}[x]\right) \to 4$; (iv) $\sin\left(\frac{\pi}{4}[x]\right) \to 8$

$f(x) = \cos(\sin x)$

clearly f is periodic with a period $= 2\pi$

Possible fundamental periods

$\frac{2\pi}{2}, \frac{2\pi}{3}, \frac{2\pi}{4} - - - -$

$\downarrow$

$\pi, 2\pi/3, 2\pi/4 - - -$

$f(x+\pi) = \cos(\sin(x+\pi))$

$= \cos(-\sin x) = \cos(\sin x) = f(x)$

$\Downarrow$

period $= \pi$ — possible fundamental periods $\pi/2, \pi/3, \pi/4 - - -$

$f(g(x))$ — if periodic then periodic

Every Period is a natural multiple of fundamental period

$\Downarrow$

fundamental period is obtained by dividing a Period by natural no:

$f(x) = \sin(\cos x)$ — Clearly a period is 2π

possible fundamental period

$$\frac{2\pi}{2}, \frac{2\pi}{3}, \frac{2\pi}{4} \text{---}$$

$$\downarrow$$

$$\cancel{\pi}, \cancel{\frac{2\pi}{3}}, \cancel{\frac{\pi}{2}} \text{---}$$

$$f(x+\pi) = \sin(\cos(x+\pi))$$
$$= \sin(-\cos x)$$
$$= -\sin\cos x \neq f(x)$$

$$\Downarrow$$

$$\boxed{T = 2\pi}$$

(d) $f(x) = \sin^4 x + \cos^4 x$

$$T_1 = \pi, \quad T_2 = \pi$$

$$\text{LCM}(T_1, T_2) = \pi$$

possible fundamental period

$$\frac{\pi}{2}, \frac{\pi}{3}, \pi/4 \, \ldots$$

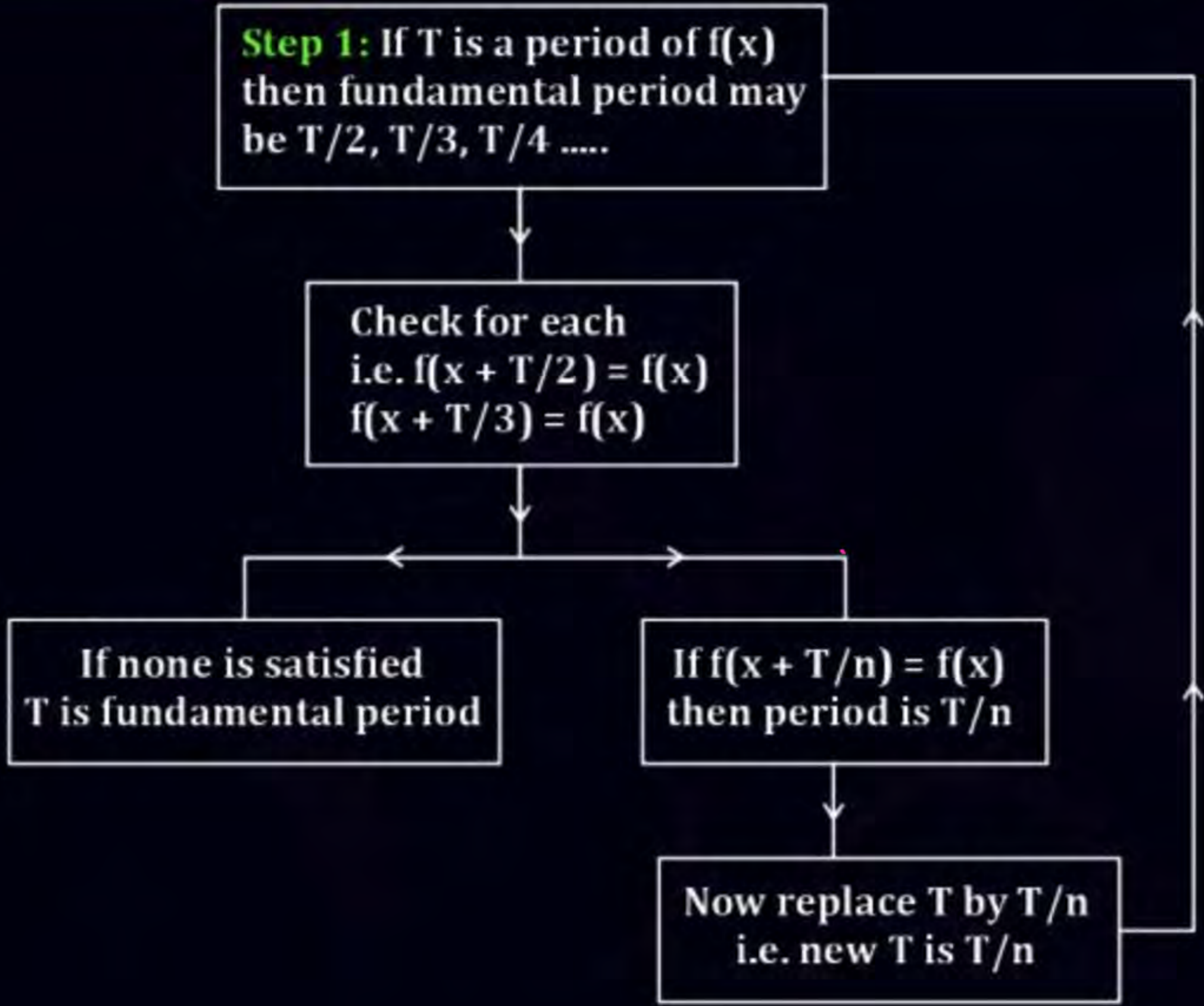
$$f(x + \pi/2) = \cos^4 x + \sin^4 x = f(x)$$

$$T = \pi/2$$

possible fundamental period $\Rightarrow$ $T = \pi/2$

$$\cancel{\pi/4}, \cancel{\pi/6}, \cancel{\frac{\pi}{12}}, \ldots$$

Finding Fundamental Period



Hermite's Identity ($[\cdot]$ denotes G.I.F)

★ $[x]+\left[x+\frac{1}{n}\right]+\left[x+\frac{2}{n}\right]+\cdots+\left[x+\frac{n-1}{n}\right]=[nx]\ \forall n\in N$

Ex: $[x]+\left[x+\frac{1}{100}\right]+\left[x+\frac{2}{100}\right]+\cdots+\left[x+\frac{99}{100}\right]=[100x]$

proof : let $f(x)=[x]+\left[x+\frac{1}{n}\right]+\left[x+\frac{2}{n}\right]+\cdots+\left[x+\frac{n-2}{n}\right]+\left[x+\frac{n-1}{n}\right]-[nx]$

$\Rightarrow f\left(x+\frac{1}{n}\right)=\left[x+\frac{1}{n}\right]+\left[x+\frac{2}{n}\right]+\left[x+\frac{3}{n}\right]+\cdots+\left[x+\frac{n-1}{n}\right]+[x+1]-\left[n\left(x+\frac{1}{n}\right)\right]$

$f\left(x+\frac{1}{n}\right)=\left[x+\frac{1}{n}\right]+\left[x+\frac{2}{n}\right]+\left[x+\frac{3}{n}\right]+\cdots+\left[x+\frac{n-1}{n}\right]+[x]+1-[nx+1]$

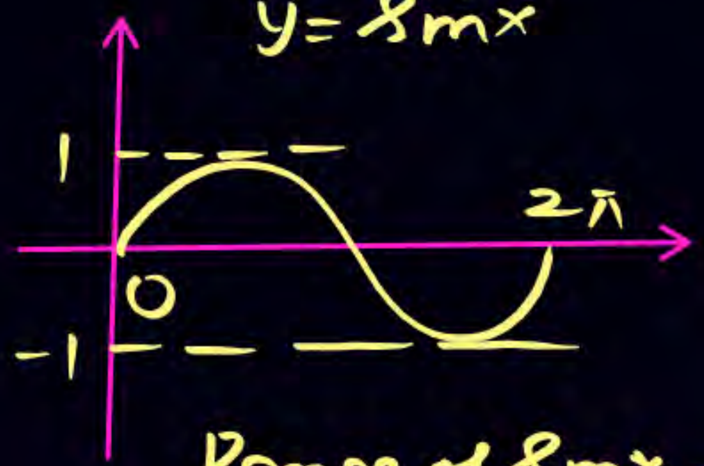
$f(x+1/n)=[x+1/n]+[x+2/n]+[x+3/n]+\cdots+\left[x+\frac{n-1}{n}\right]+[x]\cancel{+1}-[nx]\cancel{-1}$

$f(x+1/n)=[x]+[x+1/n]+[x+2/n]+[x+3/n]+\cdots+\left[x+\frac{n-1}{n}\right]-[nx]=f(x)$

$f(x+\frac{1}{n}) = f(x) \curvearrowright f$ is periodic. $\Rightarrow T = \frac{1}{n}$.

if $x \in [0, \frac{1}{n})$

$$f(x) = \underset{\overset{\shortparallel}{0}}{[x]} + \underset{\overset{\shortparallel}{0}}{[x+\tfrac{1}{n}]} + \underset{\overset{\shortparallel}{0}}{[x+\tfrac{2}{n}]} + \dots + \underset{\overset{\shortparallel}{0}}{[x+\tfrac{n-1}{n}]} - \underset{\overset{\shortparallel}{0}}{[nx]}$$



Range of $\sin x = [-1, 1]$ in $[0, 2\pi)$

$0 \le x < \frac{1}{n}$

$0 \le x + 1/n < 2/n$

$1 - \frac{1}{n} = \frac{n-1}{n} \le x + \frac{n-1}{n} < 1$

$f(x) = 0 \ \forall\ x \in [0, 1/n)$

$\Rightarrow f(x) = 0 \ \forall\ x \in R$

$\Rightarrow [x] + [x+1/n] + [x+2/n] + \dots + [x+\frac{n-1}{n}] - [nx] = 0 \ \forall\ x \in R$

$\Rightarrow [x] + [x+1/n] + [x+2/n] + \dots + [x+\frac{n-1}{n}] = [nx]$

Sabse Important Baat Yaad Rahe

Sabhi Class Illustrations Retry Karnay hai...

Today's KTK

No Selection $\xrightarrow[\text{Apnao IIT Jao}]{\text{TRISHUL}}$ Selection with good Rank

Class illustrations

Module,DPP

KTK,TAH
CHALLENGER

QUESTION

Let $\{x\}$ & $[x]$ denotes the fraction and integral part of a real number x respectively, then match the column.

Column-I		Column-I	
(A)	$[x^2] \geq 4$	(p)	$x \in [2, 4)$
(B)	$[x]^2 - 5[x] + 6 = 0$	(q)	$x \in (-\infty, -2] \cup [2, \infty)$
(C)	$x = \{x\}$	(r)	$x \in (-\infty, -5)$
(D)	$[x] < -5$	(s)	$x \in \{-2\}$
		(t)	$x \in [0, 1)$

Previous TAH

Solutions

QUESTION [JEE Mains 2023 (29 Jan)]

Let $f: R \to R$ be a function such that $f(x) = \frac{x^2+2x+1}{x^2+1}$. Then

(A) $f(x)$ is many-one in $(-\infty, -1)$

(B) $f(x)$ is one-one in $(-\infty, \infty)$

(C) $f(x)$ is one-one in $[1, \infty)$ but not in $(-\infty, \infty)$

(D) $f(x)$ is many-one in $(1, \infty)$

Ans. C

TAH-1

Let $f: R \to R$ be a fn such that $f(x) = \frac{x^2+2x+1}{x^2+1}$, Then

$$y = \frac{x^2+2x+1}{x^2+1}$$

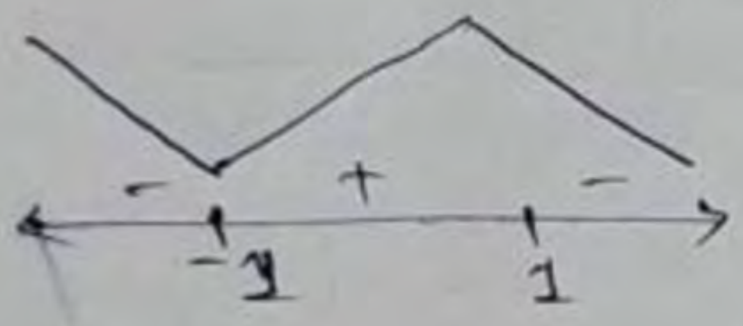
$$y' = \frac{(2x+2)(x^2+1) - (2x)(x^2+2x+1)}{(x^2+1)^2} = \frac{\cancel{2x^3}+\cancel{2x}+2x^2+2-\cancel{2x^3}-4x^2-\cancel{2x}}{(x^2+1)^2}$$

$$y' = \frac{2-2x^2}{(x^2+1)^2} = \frac{-2(x^2-1)}{(x^2+1)^2} = -\frac{2(x-1)(x+1)}{(x^2+1)^2}$$

clearly,

$y' \leq 0$ for $x \in (-\infty, -1] \cup [1, \infty)$

$\therefore f(x)$ is decreasing in $x \in (-\infty, -1] \cup [1, \infty)$



$\therefore f(x)$ is one-one in $[1, \infty)$ but not in $(-\infty, \infty)$

QUESTION [JEE Mains 2022 (28 June)]

Let a function $f: N \to N$ be defined by $f(n) = \begin{bmatrix} 2n, & n = 2, 4, 6, 8, \ldots \\ n-1, & n = 3, 7, 11, 15, \ldots \\ \frac{n+1}{2}, & n = 1, 5, 9, 13, \ldots \end{bmatrix}$ then, f is

(A) one-one but not onto

(B) onto but not one-one

(C) neither one-one nor onto

(D) one-one and onto

QUESTION [JEE Mains 2022 (28 June)]

Tah 2

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Let a function $f: N \to N$ be defined by $f(n) = \begin{cases} 2n, & n = 2, 4, 6, 8, \ldots \\ n-1, & n = 3, 7, 11, 15, \ldots \\ \frac{n+1}{2}, & n = 1, 5, 9, 13, \ldots \end{cases}$ then, f is

(A) one-one but not onto

(B) onto but not one-one

(C) neither one-one nor onto

(D) one-one and onto

TAH-2

$f(n) = \begin{cases} 4, 8, 12, 16, 20 \ldots \\ 2, 6, 10, 14, 18 \ldots \\ 1, 3, 5, 7, 9 \ldots \end{cases} = N = \text{Rang} = \text{codomain}$

$\therefore$ onto fn + one-one Ans

QUESTION

Find whether the following functions are even or odd or none

(a) $f(x) = \log\left(x + \sqrt{1 + x^2}\right)$

(b) $f(x) = \frac{x(a^x+1)}{a^x-1}$

(c) $f(x) = \sin x + \cos x$

(d) $f(x) = x\sin^2 x - x^3$

(e) $f(x) = \sin x - \cos x$

(f) $f(x) = \frac{(1+2^x)^2}{2^x}$

(g) $f(x) = \frac{x}{e^x-1} + \frac{x}{2} + 1$

(h) $f(x) = \left[(x + 1)^2\right]^{1/3} + \left[(x - 1)^2\right]^{1/3}$

b) $f(x) = \frac{x(a^x+1)}{a^x-1}$

$f(-x) = \frac{-x(a^{-x}+1)}{a^{-x}-1} = \frac{-x(1+a^x)}{(1-a^x)}$

$f(-x)=f(x) \rightarrow$ even fn $\quad = \frac{x(1+a^x)}{(a^x-1)} = f(x)$

Ans

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West Bengal

c) $f(x) = \sin x + \cos x$

$f(-x) = \cos x - \sin x \rightarrow$ Neither odd nor even Ans

d) $f(x) = x\sin^2 x - x^3$

$f(-x) = -x[\sin(-x)]^2 - (-x)^3$

$= -x\sin^2 x + x^3$

$= -(x\sin^2 x - x^3)$

$f(-x) = -f(x) \longrightarrow$ odd fn Ans

e) $f(x) = \sin x - \cos x$

$f(-x) = \sin(-x) - \cos(-x)$

$= -(\sin x + \cos x)$

$\downarrow$

Neither Even nor odd

Ans

f) $f(x) = \frac{(1+2^x)^2}{2^x}$

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$f(-x) = \frac{(1+2^{-x})^2}{2^{-x}} = \frac{(2^x+1)^2}{(2^x)^2 \cdot 2^{-x}}$

$= \frac{(2^x+1)^2}{2^x} = f(x)$

$f(-x) = f(x) \longrightarrow$ <u>even fn</u> Ans

h) $f(x) = [(x+1)^2]^{1/3} + [(x-1)^2]^{1/3}$

$f(-x) = [(-x+1)^2]^{1/3} + [(-x-1)^2]^{1/3}$

$= [(1-x)^2]^{1/3} + [(x+1)^2(-1)^2]^{1/3}$

$= [(x-1)^2]^{1/3} + [(x+1)^2]^{1/3}$

$f(-x) = f(x) \longrightarrow$ <u>even fn</u> Ans

QUESTION

Suppose that $f(x)$ is a function of the form $f(x)=\frac{ax^8+bx^6+cx^4+dx^2+15x+1}{x}(x\neq 0)$. If $f(5)=2$ then the value of $f(-5)$ is equal to

(A) -2

(B) 28

(C) 13

(D) -13

Ans. B

7

$$f(x) = \frac{ax^8 + bx^6 + cx^4 + dx^2 + 15x + 1}{x},$$

$f(5) = 2$, $f(-5)$

$$f(-x) = \frac{ax^8 + bx^6 + cx^4 + dx^2 - 15x + 1}{-x}$$

$$= \frac{-ax^8 - bx^6 - cx^4 - dx^2 + 15x - 1}{x}$$

$$f(x) + f(-x) = \frac{30x}{x} = 30$$

$x = 5$

$f(5) + f(-5) = 30$

$f(-5) = 28$

Tah-04
Sudesh Dhal
Odisha

TAH-5

Suppose that $f(x)$ is a fn of the form $f(x) = \frac{ax^8 + bx^6 + cx^4 + dx^2 + 15x + 1}{x}$ $(x \neq 0)$. If $f(5) = 2$ then the value of $f(-5)$ is equal to

$$f(x) = \frac{ax^8 + bx^6 + cx^4 + dx^2 + 15x + 1}{x}$$

$$f(-x) = \frac{+ax^8 + bx^6 + cx^4 + dx^2 - 15x + 1}{-x}$$

$$f(-x) = \frac{-ax^8 - bx^6 - cx^4 - dx^2 + 15x - 1}{x}$$

Now,

$$f(-x) + f(x) = \frac{30x}{x}$$

Put $x = 5$

$f(-5) + f(5) = 30$

$f(-5) + 2 = 30$

$f(-5) = 28$ Ans

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Bumper Practice Problems

Find Range of following rational functions:

(1) $f(x) = \frac{3-2x}{5x-7}$

(2) $f(x) = \frac{x^2-6x+8}{x^2-5x+6}$

(3) $f(x) = \frac{3x-6}{5-2x}$

(4) $f(x) = \frac{(2x-1)(6x-3)}{(5x+2)(2x-1)}$

(5) $f(x) = \frac{x^2-2x+4}{x^2+2x+4}$

(6) $f(x) = \frac{x^2-6x+1}{x^2+6x+1}$

(7) $f(x) = \frac{x^2-5x+4}{x^2+2x-3}$

Answers

(1) $R - \left\{\frac{2}{5}\right\}$

(2) $R - \{1, 2\}$

(3) $R - \left\{-\frac{3}{2}\right\}$

(4) $R - \left\{\frac{6}{5}, 0\right\}$

(5) $\left[\frac{1}{3}, 3\right]$

(6) $(-\infty, -2] \cup \left[-\frac{1}{2}, \infty\right)$

(7) $R - \left\{1, -\frac{3}{4}\right\}$

Bumper practice problem

(1) $f(x) = \frac{3-2x}{5x-7}$

$\boxed{y \in R - \{-2/5\}}$

BPP 01
Pg 1
DONE BY SUDESH DHAL
ODISHA

(2) $f(x) = \frac{x^2-6x+8}{x^2-5x+6}$

$f(x) = \frac{\cancel{(x-2)}(x-4)}{\cancel{(x-2)}(x-3)} = \frac{x-4}{x-3}$

$\boxed{y \in R - \{1,2\}}$

(3) $f(x) = \frac{3x-6}{5-2x}$

$y \in R - \{-3/2\}$

(4) $f(x) = \frac{(2x-1)(6x-3)}{(5x+2)(2x-1)}$

$f(x) = \frac{6x-3}{5x+2}$, $x \neq 1/2$

$y \in R - \{6/5, 0\}$

(5) $f(x) = \frac{x^2-2x+4}{x^2+2x+4}$

$f(x) = y = \frac{x^2-2x+4}{x^2+2x+4}$

$= (y-1)x^2 + (2y+2)x + 4(y-1) = 0$ $\quad \boxed{y \neq 1}$

$(y+1)^2 - 4(y-1)^2 \geq 0$

$(y+1+2y-2)(y+1-2y+2) \geq 0$

$(3y-1)(y-3) \leq 0$

$\boxed{y \in [1/3, 3]} - \{1\}$

if $y = 1$

$\boxed{x = 0}$

$\boxed{y \in [1/3, 3]}$

BPP
Pg 2
DONE BY SUDESH DHAL
ODISHA

(6) $f(x) = \frac{x^2-6x+1}{x^2+6x+1}$

$\Rightarrow (y-1)x^2 + 6(y+1)x + y - 1 = 0$

$\Rightarrow D \geq 0 \qquad y \neq 1$

$36(y+1)^2 - 4(y-1)^2 \geq 0$

$(3y+3)^2 - (y-1)^2 \geq 0$

$(4y-2)(2y+4) \geq 0$

$\boxed{y \in (-\infty, -2] \cup [1/2, \infty)} \boxed{-\{1\}}$

$\Rightarrow \boxed{y = 1}$

$\boxed{x = 0} \Rightarrow \boxed{y = 1}$

$y \in (-\infty,$

BPP

Find Range:

1) $f(x) = \frac{3-2x}{5x-7}$

Range of $f(x) \in R - \left\{-\frac{2}{5}\right\}$ Ans

2) $f(x) = \frac{x^2-6x+8}{x^2-5x+6}$

$= \frac{(x-4)\cancel{(x-2)}}{(x-3)\cancel{(x-2)}}$ $x \neq 2$

$f(x) = \frac{x-4}{x-3}$

Range: $R - \{1, 2\}$ Ans

3) $f(x) = \frac{3x-6}{5-2x}$

Range : $R - \left\{-\frac{3}{2}\right\}$ Ans

4) $f(x) = \frac{\cancel{(2x-1)}(6x-3)}{(5x+2)\cancel{(2x-1)}}$

$f(x) = \frac{6x-3}{5x+2}$; $x \neq \frac{1}{2}$

Range : $R - \left\{\frac{6}{5}, 0\right\}$ Ans

5) $f(x) = \frac{x^2-2x+4}{x^2+2x+4}$

$y = \frac{x^2-2x+4}{x^2+2x+4}$

$yx^2+2yx+4y = x^2-2x+4$

$(y-1)x^2+(2y+2)x+(4y-4)=0$

$\therefore (2y+2)^2 - 4(y-1)(4y-4) \geq 0$

$4y^2+8y+4-16y^2+32y-16 \geq 0$

$-3y^2+10y-3 \geq 0$

$3y^2-10y+3 \leq 0$

$(3y-1)(y-3) \leq 0$

$\therefore y \in \left[\frac{1}{3}, 3\right]$ Range Ans

6) $f(x) = \frac{x^2-6x+1}{x^2+6x+1}$

$yx^2+6xy+y = x^2-6x+1$

$x^2(y-1)+6x(y+1)+(y-1)=0$

$\therefore (6y+6)^2 - 4(y-1)^2 \geq 0$

$36y^2+72y+36-4y^2+8y-4 \geq 0$

$2y^2+5y+2 \geq 0$

$(y+2)(2y+1) \geq 0$

$y \in (-\infty, -2] \cup \left[-\frac{1}{2}, \infty\right)$ Ans

7) $f(x) = \frac{x^2-5x+4}{x^2+2x-3}$

$= \frac{(x-4)\cancel{(x-1)}}{(x+3)\cancel{(x-1)}}$, $x \neq 1$

$f(x) = \frac{(x-4)}{(x+3)}$

$\therefore$ Range : $R - \left\{-\frac{3}{4}, 1\right\}$ Ans

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(Solution to KTK)

QUESTION

(KTK 1)

If range of function f(x) whose domain is set of all real numbers is [–2, 4], then range of function $g(x) = \frac{1}{2}f(2x + 1)$ is equal to :

(A) [–2, 4]

(B) [–1, 2]

(C) [–3, 9]

(D) [–2, 2]

Ans. B

KTK-1

If range of f^n $f(x)$ whose domain is set of all real nos is $[-2,4]$, then range of f^n $g(x) = \frac{1}{2} f(2x+1)$ is equal to :-

$D_f \longrightarrow R$, $R_f \longrightarrow [-2,4]$

$$g(x) = \frac{1}{2} f \underbrace{(2x+1)}_{[-2,4]}$$

$\hookrightarrow R$

Parwez
Bihar

$$y = \frac{1}{2}[-2,4]$$

$$y = [-1,2]$$

QUESTION

(KTK 2)

Let two functions $f(x)$ and $g(x)$ are defined on $R \to R$ such that

$$f(x) = \begin{cases} x^2, & x \in \text{irrational} \\ 2 - x^2, & x \in \text{rational} \end{cases} \text{ and } g(x) = \begin{cases} 2 - x^2, & x \in \text{irrational} \\ x^2, & x \in \text{rational} \end{cases}.$$

Then the function $f + g : R \to R$ is

(A) injective as well as surjective.

(B) injective but not surjective.

(C) surjective but not injective.

(D) neither surjective nor injective.

Ans. D

KTK-2

Let two fns $f(x)$ & $g(x)$ are defined on $R \to R$

s.t. $f(x) = \begin{cases} x^2, & x \in \text{irrational} \\ 2-x^2, & x \in \text{rational} \end{cases}$ & $g(x) = \begin{cases} 2-x^2, & x \in \text{Irrational} \\ x^2, & x \in \text{rational} \end{cases}$

Then the fn $f+g : R \to R$ is-

$$(f+g)x = \begin{cases} 2, & x \in \text{Irr.} \\ 2, & x \in \text{Rational} \end{cases}$$

$x \in R$, $y = 2$

↳ into, many-one

$x \in IR$, $y = 2$

(D) neither surjective nor injective

QUESTION

(KTK 3)

Let $f(x)$ be a real valued function defined on $f : R \to R$ such that $f(x) = [x]^2 + [x+1] - 3$, where $[x]$ = the greatest integer $\leq x$. Then

(A) $f(x)$ is a many-one and into function

(B) $f(x) = 0$ for infinite number of values of x

(C) $f(x) = 0$ for only two real values

(D) none of these

Ans. A, B

(3) Let $f(x)$ be a real valued function defined on $f: \mathbb{R} \to \mathbb{R}$ such that $f(x) = [x]^2 + [x+1] - 3$ where $[x]$ denotes greatest integer $\le x$. Then

(A) $f(x)$ is a many one & into f^n

(B) $f(x) = 0$ for infinite no. of points

(C) $f(x) = 0$ for only two real values

(D) none of these.

KTK 3

$$f(x) = [x]^2 + [x] - 2$$

clearly, $f\left(\frac{3}{2}\right) = f\left(\frac{4}{3}\right) = 0 \Rightarrow$ many one f^n.

clearly. $f(x) \in \mathbb{Z} \Rightarrow$ range $\ne$ codomain $\Rightarrow$ into f^n.

Jaskaran Singh
From Samba J&K

Now, $f(x) = 0 \Rightarrow [x]^2 + [x] - 2$

$\Rightarrow ([x]-1)([x]+2) = 0$

$\Rightarrow [x] = 1, -2$

$\Rightarrow x \in [1,2) \cup [-2,-1)$

$\hookrightarrow$ infinite sol^{n}s.

KTK-3

Date: ______ Page: ______

Given $f(x) = [x]^2 + [x+1] - 3 = [x]^2 + [x] - 2$

$f(x) = [x]^2 + [x] - 2 \quad : \mathbb{R} \to \mathbb{R}$

$f(1.2) = 0$
$f(1.4) = 0$ $\Bigg\rangle$ Hence $f(x)$ is many one function Ans

$f(x) = \underbrace{[x]^2}_{I} + \underbrace{[x]}_{I} \underbrace{-2}_{I} \rightarrow$ Hence this gives only Integral Solutions.

Therefore non Integral numbers are in codomain but not in Range
So it is not onto function. Ans

every number between $[1,2)$ satisfies $f(x)$
Hence $f(x) = 0$ has infinite solutions Ans

Akash gupta
Uttar Pradesh

KTK-3

Let $f(x)$ be a real valued fn defined on $f: R \to R$ such that $f(x) = [x]^2 + [x+1] - 3$, where $[x]$ = the greatest integer $\leq x$, Then

a) $f(x)$ is a many-one & into fn

b) $f(x) = 0$ for infinite no of values of x

c) $f(x) = 0$ for only two real values

D) none of these

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$$f(x) = [x]^2 + [x] + 1 - 3 = 0$$
$$= [x]^2 + [x] - 2 = 0$$
$$= [x]^2 + 2[x] - [x] - 2 = 0$$
$$= [x]([x]+2) - 1([x]+2) = 0$$
$$= ([x]+2)([x]-1) = 0$$

$[x] = -2 \qquad [x] = 1$

$x \in [-2, -1) \cdot x \in [1, 2)$

$\therefore$ A & B are correct.

QUESTION

(KTK 4)

Let $f(x) = \sqrt{\frac{1}{x^2 + 2\sqrt{c}x + 1}}$. If domain of $f(x)$ is $(-\infty, \infty)$, then the number of integers in the range of 'c' is

(A) 3

(B) 2

(C) 1

(D) 0

Ans. C

KTK-4

$$f(x) = \sqrt{\frac{1}{x^2+2\sqrt{c}\,x+1}} \qquad \text{Domain} \to R$$

$f(x)$ to be defined: $\quad x^2+2\sqrt{c}\,x+1 > 0 \to D < 0$

$$4c-4<0$$

$$(c-1)<0$$

$$c \in (-\infty, 1)$$

also $\quad c \geq 0$

Hence the only integral value c can have is zero

only one solution.

QUESTION

(KTK 5)

Classify the following functions as injective, surjective, both or none.

(a) $f : R \to R$, be a function defined by $f(x) = \frac{x^2+4x+30}{x^2-8x+18}$.

(b) $f : R \to R$, be a function defined by $f(x) = x^3 - 6x^2 + 11x - 6$

(c) $f : R \to R$, be a function defined by $f(x) = (x^2 + x + 5)(x^2 + x - 3)$

(d) $f : R \to \{x \in R : -1 < x < 1\}$, be a function defined by $f(x) = \frac{x}{1+|x|}$

(e) $f : [-1, 3] \to [-37, 27]$, be a function defined by $f(x) = 2x^3 - 6x^2 - 18x + 17$

Ans. (a) neither surjective nor injective;
(b) surjective but not injective;
(c) neither injective nor surjective;
(d) injective and surjective;
(e) injective and surjective

KTK 8

(a) $f: R \to R$ $\quad f(x) = \dfrac{x^2+4x+80}{x^2-8x+18}$

Numerator: $D<0, a>0$ always +ve.

Denominator: $D<0, a>0 \to$ always +ve

$\Downarrow$

$\therefore$ $f(x)$ is always +ve.

Range $\neq$ codomain.

$\Downarrow$

Into

$$\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \frac{1+\frac{4}{x}+\frac{80}{x^2}}{1-\frac{8}{x}+\frac{18}{x^2}} = 1.$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{1+\frac{4}{x}+\frac{80}{x^2}}{1-\frac{8}{x}+\frac{18}{x^2}} = 1.$$

$\therefore$ graph must have bent

$\Downarrow$

many one

Ans many on & into.

(b) $f: R \to R$ $\quad f(x) = x^3 - 6x^2 + 11x - 6.$

odd degree polynomial.

Range $= R$ (codomain)

$\Downarrow$

Onto

$\dfrac{dy}{dx} = f'(x) = 3x^2 - 12x + 11$ — many be Inc & Dec.

$D>0$.

$\Downarrow$

becomes zero @ $x = \dfrac{12 \pm \sqrt{12}}{6}$.

Not always +ve or -ve.

$\therefore$ many one

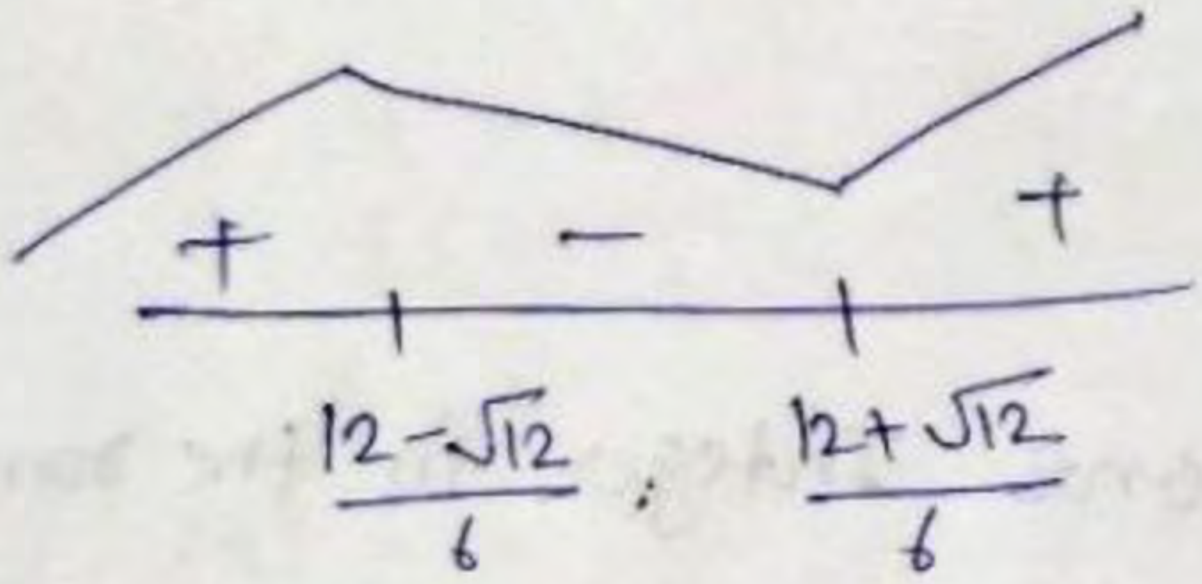
Ans many-one & Onto

$f(x) = x^3 - 6x^2 + 11x - 6$

It is polynomial fn so range $\in R \to$ Surjective

$f'(x) = 3x^2 - 12x + 11$

$$x = \frac{12 \pm \sqrt{144-132}}{6} \Rightarrow \frac{12+\sqrt{12}}{6}, \frac{12-\sqrt{12}}{6}$$



$\to$ Not Injective

(c) $f : R \to R$ be a funⁿ defined by $f(x) = \dfrac{(x^2+x+5)}{(x^2+x-3)}$

Solⁿ :- $f(x) = (x^2+x+5)(x^2+x-3)$

$\underbrace{\begin{matrix} D<0 \\ a>0 \end{matrix}}_{\text{Always +ve}}$ $\qquad$ $\underbrace{\begin{matrix} D>0 \\ a>0 \end{matrix}}_{\text{No real roots}}$

$\therefore$ Range of $f(x) \neq R$ $\quad \therefore$ Not Surjective.

$f(0) = (5)(-3) = -15$

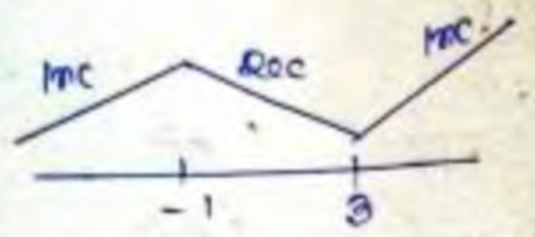
$f(-1) = (+1-1+5)(+1-1-3) = 5 \times (-3) = -15$ $\quad$ Many-one

$\therefore$ Neither Surjective nor Injective.

(e) $f : [-1, 3] \to [-37, 27]$.

$$f(x) = 2x^3 - 6x^2 - 18x + 17$$

$$f'(x) = 6x^2 - 12x - 18$$
$$= 6(x^2 - 2x - 3)$$
$$= 6(x-3)(x+1).$$



maxima at -1 and minima at 3.

Dec. in $[-1, 3]$.

$\Downarrow$

One-one

values at optimum points.

$f(-1) = 27 \qquad f(3) = 54 - 54 - 54 + 17$
$\qquad\qquad\qquad\quad = -37.$

$\therefore$ Range: $[-37, 27]$ = codomain

$\Downarrow$

Onto

Ans One-one & onto.

QUESTION

(KTK 6)

If domain of $y = f(x)$ is $[-3, 2]$, then domain of $f(|[x]|)$ is equal to
[Note: [k] denotes greatest integer function less than or equal to k]

(A) $[-3, 2]$

(B) $[-2, 3)$

(C) $[-3, 3]$

(D) $[-2, 3]$

Ans. B

KTK : 6

If domain of $y = f(x)$ is $[-3, 2]$ then domain of $f(|[x]|)$ is equal to.

Sol^n

$D_{f(x)} = [-3, 2]$

$x \in [-3, 2]$

$[x]$

$f(x) \to [-3, 2]$

$y = f(|[x]|)$

$-3 \le |[x]| \le 2$

$|x| \ge 0$

$0 \le |[x]| \le 2$

$-2 \le [x] \le 2$

$x \in [-2, 3)$

$[x] \ge -2$

$x \ge -2$

$x \in [-2, \infty)$

&

$[x] \le 2$

$x < 3$

$x \in (-\infty, 3)$

Ritesh Singh , From Devbhoomi Uttarakhand

KTK-6

If domain of $y = f(x)$ is $[-3, 2]$, then domain of $f(|[x]|)$ is equal to

Domain of $y = f(x)$ is $[-3, 2]$

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$f(|[x]|)$

$|[x]| \in [-3, 2]$

$\boxed{|x| \geq 0}$

$|[x]| \in [0, 2]$

$[x] = -2, -1, 0, 1, 2$

$x \in [-2, 3)$ Ans

QUESTION

(KTK 7)

Find domain of $f(x)=\sqrt{\log_{1/3}\left(\log_4\left([x]^2-5\right)\right)}$ (Where $[\cdot]$ denotes G.I.F.)

Ans. $[-3,-2)\cup[3,4)$

KTK-7

Find domain of $f(x) = \sqrt{\log_{4/3}(\log_4([x]^2 - 5))}$.

$\log_{4/3}(\log_4([x]^2-5)) \geq 0$; $\log_4([x]^2-5) > 0$

$\log_4([x]^2-5) \leq 1$ $\qquad$ $[x]^2 - 5 > 1$

$[x]^2 - 5 \leq 4$ $\qquad$ $[x]^2 - 6 > 0$

$[x]^2 - 9 \leq 0$ $\qquad$ $[x] \in (-\infty, -\sqrt{6}) \cup (\sqrt{6}, \infty)$ — (ii)

$([x]-3)([x]+3) \leq 0$

$[x] \in [-3, 3]$ — (i)

& $[x]^2 - 5 > 0$

$[x]^2 > 5$

$[x] \in (-\infty, -\sqrt{5}) \cup (\sqrt{5}; \infty)$ — (iii)

$\therefore$ (i) $\cap$ (ii) $\cap$ (iii)

$[x] \in [-3, -\sqrt{6}) \cup (\sqrt{6}, 3]$

$x \in [-3, -2) \cup [3, 4)$ Ans

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QUESTION

(KTK 8)

Let $f(x) = \sqrt{\log_2\left(\frac{10x-4}{4-x^2}\right) - 1}$. Then sum of all integers in domain of $f(x)$ is

(A) -15

(B) -16

(C) -17

(D) -18

Ans. C

(8) Let $f(x) = \sqrt{\log_2\left(\frac{10x-4}{4-x^2}\right)-1}$ then sum of all integral values of x in domain is ___ .

$\log_2\left(\frac{10x-4}{4-x^2}\right)-1 \geq 0$ & $\frac{10x-4}{4-x^2} > 0$

no need

$\log_2 \frac{10x-4}{4-x^2} \geq 1$

$\frac{10x-4}{4-x^2} \geq 2$

$\Rightarrow \frac{5x-2}{4-x^2} \geq 1$

$\Rightarrow \frac{5x-2}{4-x^2} - 1 \geq 0$

$\Rightarrow \frac{5x-2-4+x^2}{4-x^2} \geq 0$

$\Rightarrow \frac{x^2+5x-6}{x^2-4} \leq 0$

$\Rightarrow \frac{(x+6)(x-1)}{(x-2)(x+2)} \leq 0$

$+ \quad -6 \quad - \quad -2 \quad + \quad 1 \quad - \quad 2 \quad +$

$\Rightarrow x \in [-6,-2) \cup [1,2)$

integers $= -6, -5, -4, -3, 1$

sum $= -17$

KTK 8

Jaskaran Singh
From Samba J&K

QUESTION

(KTK 9)

The domain of the function, $f(x) = \frac{\sqrt{\sin x}}{\sqrt{(x-2)(8-x)}}$ is

A $[0, \pi] \cup [2\pi, 8)$

B $(2, \pi] \cup [2\pi, 8)$

C $(2, 8)$

D $(0, 8)$

Ans. B

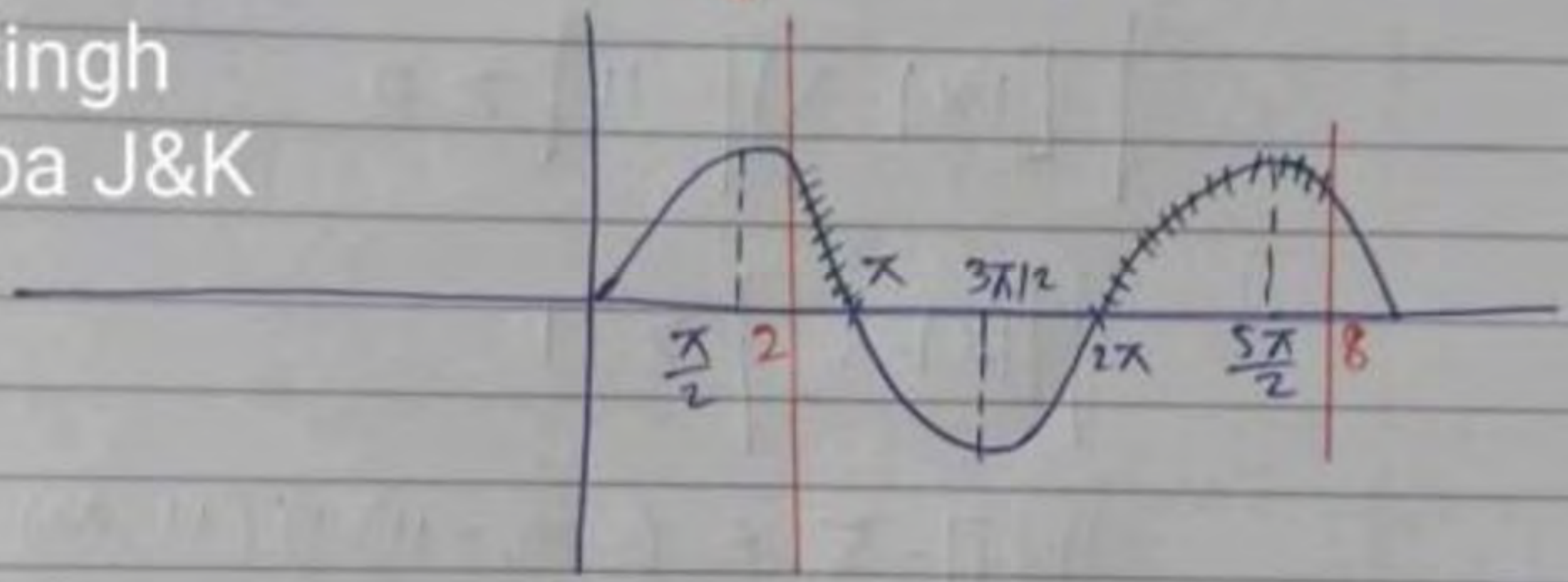
(9) The domain of f^n $f(x) = \dfrac{\sqrt{\sin x}}{\sqrt{(x-2)(8-x)}}$ is ____.

$(x-2)(x-8) < 0$ & $\sin x \geq 0$

$x \in (2,8)$

KTK 9

Jaskaran Singh
From Samba J&K



so $x \in (2,\pi] \cup [2\pi, 8)$

QUESTION

(KTK 10)

The domain of the function $f(x)=\sqrt{10-\sqrt{x^4-21x^2}}$ is

(A) $[5,\infty)$

(B) $[-\sqrt{21},\sqrt{21}]$

(C) $[-\sqrt{5},\sqrt{21}]\cup[\sqrt{21},\sqrt{5}]\cup\{0\}$

(D) $(-\infty,-5)$

KTK-10

The domain of the fn $f(x) = \sqrt{10 - \sqrt{x^4 - 21x^2}}$ is

$10 - \sqrt{x^4 - 21x^2} \geq 0$ & $x^4 - 21x^2 \geq 0$

$x^2(x^2 - 21) \geq 0$

$\sqrt{x^4 - 21x^2} \leq 10$

$x^2 - 21 \geq 0$, x can be '0'

$x^4 - 21x^2 \leq 100$

$x \in (-\infty, -\sqrt{21}] \cup [\sqrt{21}, \infty] \cup \{0\}$

$(x^2 - 25)\underbrace{(x^2 + 4)}_{\text{always +ve}} \leq 0$

$(x - 5)(x + 5) \leq 0$

$x \in [-5, 5]$

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$x \in [-5, -\sqrt{21}] \cup [\sqrt{21}, 5] \cup \{0\}$ Ans

KTK-10,

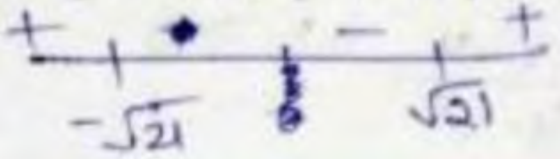
Sol^n,

$$10-\sqrt{x^4-21x^2}\geq 0 \quad \& \quad x^4-21x^2\geq 0$$

$$-\sqrt{x^4-21x^2}\geq -10 \quad \& \quad x^2(x^2-21)\geq 0$$

$$\sqrt{x^4-21x^2}\leq 10 \quad \& \quad x^2(x-\sqrt{21})(x+\sqrt{21})\geq 0$$

$$x^4-21x^2-100\leq 0 \qquad (x-\sqrt{21})(x+\sqrt{21})\geq 0,\ x=0\ ✓$$



Let $x^2 = t$,

$$t^2-21t-100\leq 0$$

$$t^2-25t+4t-100\leq 0$$

$$(t+4)(t-25)\leq 0$$

$$t\in[-4,25]$$

$$x^2\in[-4,25]$$

$$x^2\in[0,25]$$

$$\boxed{x\in[-5,5]}$$

$$\boxed{x\in(-\infty,-\sqrt{21}]\cup[\sqrt{21},\infty)}\cup\{0\}$$

Aryan tomar
Ktk 10.

$$x\in[-5,-\sqrt{21}]\cup[\sqrt{21},\sqrt{5}]\cup\{0\}$$

QUESTION

(KTK 11)

Find the domain $f(x) = \frac{1}{\sqrt{|[|x|-5]|-11}}$ where [.] denotes greatest integer function.

KTK-11

Find the domain $f(x)=\dfrac{1}{\sqrt{|[|x|-5]|-11}}$ where $[\cdot]$ denotes greatest Integer fn.

$$|[|x|-5]|-11>0$$

$$|[|x|-5]|>11$$

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* $[|x|-5]>11$

$|x|-5\geq 12$

$|x|\geq 17$

* $[|x|-5]<-11$

$|x|<-6$ (which is impossible since $|x|\geq 0$)

$\therefore\ x\in(-\infty,-17]\cup[17,\infty)$ Ans

Q) KTK-11

Find the domain $f(x) = \frac{1}{\sqrt{|[|x|-5]|-11}}$ where

[.] denotes greatest integer function

Soln) $f(x) = \frac{1}{\sqrt{|[|x|-5]|-11}}$

Suraj Pandey

$|[|x|-5]|-11 > 0$

$|[|x|-5]| > 11$

$[|x|-5] > 11$, $[|x|-5] < -11$

$|x|-5 \geq 12$, $|x|-5 \leq -10$

$|x| \geq 17$, $|x| \leq -5$

$\Downarrow$ $\Downarrow$

$x \geq 17$, $x \leq -17$ Not possible $\times$

$\Downarrow$

$\boxed{x \in (-\infty, -17] \cup [17, \infty)}$

(Solution to RPP)

QUESTION

Let $\sum_{n=1}^{\infty}\left(\frac{n}{n^4+4}\right)=\frac{p}{q}$,

where p & q are coprime natural numbers then $|2p-q|$ is equal to

(A) 3

(B) 2

(C) 8

(D) 9

Ans. B

RPP-1

Let $\sum_{n=1}^{\infty}\left(\frac{n}{n^4+4}\right)=\frac{p}{q}$ where p & q are coprime natural no's then $|2p-q|$ is equal to

$$T_n=\sum_{n=1}^{\infty}\frac{n}{n^4+4n^2+4-4n^2}=\sum_{n=1}^{\infty}\frac{n}{(n^2+2)^2-(2n)^2}$$

$$=\sum_{n=1}^{\infty}\frac{n}{(n^2+2+2n)(n^2+2-2n)}=\sum_{n=1}^{\infty}\frac{(n^2+2+2n)-(n^2+2-2n)}{4(n^2+2n+2)(n^2+2-2n)}$$

$$T_n=\sum_{n=1}^{\infty}\frac{1}{4}\left[\frac{1}{n^2-2n+2}-\frac{1}{n^2+2n+2}\right]$$

$$T_1=\frac{1}{4}\left[1-\cancel{\frac{1}{5}}\right]$$

$$T_2=\frac{1}{4}\left[\frac{1}{2}-\cancel{\frac{1}{10}}\right]$$

$$T_3=\frac{1}{4}\left[\cancel{\frac{1}{5}}-\cancel{\frac{1}{17}}\right]$$

$$T_4=\frac{1}{4}\left[\cancel{\frac{1}{10}}-\cancel{\frac{1}{26}}\right]$$

$$T_5=\frac{1}{4}\left[\cancel{\frac{1}{17}}-\cancel{\frac{1}{37}}\right]$$

$$\vdots$$

$$T_\infty=\frac{1}{4}\left[\cancel{\frac{1}{\infty}}-\cancel{\frac{1}{\infty}}\right]$$

$$\therefore S_\infty=\frac{1}{4}\left[1+\frac{1}{2}\right]$$

$$S_\infty=\frac{1}{4}\times\frac{3}{2}=\frac{3}{8}$$

$$\therefore \frac{p}{q}=\frac{3}{8}$$

$p=3$ & $q=8$

$$\therefore |2p-q|=|6-8|=|-2|=2 \text{ Ans}$$

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QUESTION

For the series,

$$S = 1 + \frac{1}{(1+3)}(1+2)^2 + \frac{1}{(1+3+5)}(1+2+3)^2 + \frac{1}{(1+3+5+7)}(1+2+3+4)^2 + \cdots$$

(A) 7^{th} term is 16

(B) 7^{th} term is 18

(C) Sum of first 10 terms is $\frac{405}{4}$

(D) Sum of first 10 terms is $\frac{505}{4}$

Ans. A, D

RPP-2

For the series

$$S = 1 + \frac{1}{(1+3)}(1+2)^2 + \frac{1}{(1+3+5)}(1+2+3)^2 + \frac{1}{(1+3+5+7)}(1+2+3+4)^2 + \dots$$

$$T_n = \frac{1}{(1+3+5+7 \dots\dots 2n-1)}(1+2+3+\dots+n)^2$$

$$T_n = \frac{\left(\frac{n(n+1)}{2}\right)^2}{n^2} = \frac{(n+1)^2}{4}$$

$$\therefore T_7 = \frac{8^2}{4} = \frac{64}{4} = 16 \text{ Ans}$$

$$S_{10} = \sum_{n=1}^{10}\left(\frac{n^2+1+2n}{4}\right) = \frac{1}{4}\left(\frac{10\times11\times21}{6}+10+10\times11\right)$$

$$= \frac{10}{4}\left(\frac{77}{2}+11\right)$$

$$= \frac{10}{4}\times\frac{99}{2} = \frac{505}{4} \text{ Ans}$$

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RPP-02

Ans: $T_7 = \frac{1}{(1+3+5+7+9+11+13)}(1+2+3+4+5+6+7)^2$

$$T_7 = \frac{1}{7^2} \times \left(\frac{7(8)}{2}\right)^2 = \frac{64}{4} = \boxed{16} \text{ Ans}$$

(a) ✓

NOW,

$$T_r = \frac{\left(\frac{r(r+1)}{2}\right)^2}{r^2} = \frac{(r+1)^2}{4}$$

$$\sum_{r=1}^{10} T_r = \sum_{r=1}^{10} \frac{(r+1)^2}{4} \Rightarrow \sum_{r=1}^{10} \frac{r^2+2r+1}{4}$$

$$\text{So, } \left[\frac{\cancel{10}^{5} \times 11 \times \cancel{21}^{7}}{\cancel{6}} + 2 \times \frac{10 \times 11}{\cancel{2}} + 1 \times 10\right] \times \frac{1}{4}$$

$$S = \frac{385 + 110 + 10}{4} = \boxed{\frac{505}{4}} \rightarrow (d) ✓$$

QUESTION

Let $a_n, n \geq 1$, be an arithmetic progression with first term 2 and common difference 4 , Let M_n be the average of the first n terms. Then the sum $\sum_{n=1}^{10} M_n$ is

(A) 110

(B) 335

(C) 770

(D) 1100

Ans. A

RPP-3

Let $a_n, n \geq 1$, be an arithmetic progression with first term 2 & common difference 4, let M_n be the average of the first n terms. $\sum_{n=1}^{10} M_n$ is

$$a_n = 2 + (n-1).4 = 4n - 2$$

$$M_n = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n}$$

$$= \frac{2n^2}{n} = 2n$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$
$$= \frac{n}{2}(2 + (4n - 2))$$
$$= \frac{n}{2} \times 4n = 2n^2$$

$$\therefore \sum_{n=1}^{10} M_n = \sum_{n=1}^{10} 2n$$
$$= 2(1 + 2 + 3 + \dots + 10)$$
$$= 2 \times \frac{10 \times 11}{2} = 110 \quad \text{Ans}$$

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RPP-3

Ans: $a_n = 2, 6, 10, 14 \ldots\ldots n.$

$$S_n = \frac{n}{2}\left[4 + (n-1)\times 4\right]$$

$$S_n = 2n\left[\cancel{1} + n \cancel{-1}\right] = \boxed{2n^2}$$

So, A·M of n terms $= \frac{2n^{\cancel{2}}}{\cancel{n}} = \boxed{2n}$

Now, $\sum_{n=1}^{10} M_n \Rightarrow \sum_{n=1}^{10} 2n = \cancel{2}\times\frac{10\times 11}{\cancel{2}}$

$= \boxed{110}$ Ans


THANK
YOU